केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

शिक्षा एवं प्रशिक्षण आंचलिक संस्थान , मैसूर

ZONAL INSTITUTE OF EDUCATION AND TRAINING, MYSORE

अध्ययन सामग्री/**STUDY MATERIAL**

सत्र/ **SESSION: 2022-23**

कक्षा/**CLASS-** बारहवीं/**TWELVE(XII)**

गणित/**MATHEMATICS**

विषय कोड/**Subject Code - 041**

तैयारकर्ता/Prepared by: डि श्रीनिवासुलु , प्रशिक्षण सहयोगी

D SREENIVASULU

TRAINING ASSOCIATE(MATHEMATICS)

शिक्षा एवं प्रशिक्षण आंचलिक संस्थान , मैसूर

ज्योति नगर मुख्य मार्ग , सिद्धार्थ नगर पी ओ, **मैसूर** - 570011

ZONAL INSTITUTE OF EDUCATION AND TRAINING, MYSORE

JYOTHI NAGAR MAIN ROAD, SIDDHARTH NAGAR PO, MYSORE-570011

website:zietmysore.kvs.gov.in

email: kvszietmysore@gmail.com, Tlephone number 0821-2470345

INDEX

CURRICULUM: CLASS – XII SUBJECT: MATHEMATICS (Code No. 041)

Session – 2022-23

One Paper **Max Marks: 80**

UNIT NO	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	Total	240	80
	Internal Assessment		20

Unit-I: Relations and Functions

Relations and Functions 15 Periods

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

Inverse Trigonometric Functions 15 Periods

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

Matrices 25 Periods

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. On- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Determinants **25 Periods**

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

Continuity and Differentiability **20 Periods**

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, *like* $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

Applications of Derivatives **10 Periods**

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real- life situations).

Integrals **20 Periods**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{1}{x^2 \pm a^2} dx, \int \frac{1}{a^2 - x^2} dx, \int \frac{1}{\sqrt{x^2 \pm a^2}} dx, \int \frac{1}{\sqrt{a^2 - x^2}} dx, \int \sqrt{x^2 \pm a^2}\, dx,$$

$$\int \sqrt{a^2 - x^2}\, dx, \int \frac{1}{ax^2+bx+c} dx, \int \frac{1}{\sqrt{ax^2+bx+c}} dx, \int \sqrt{ax^2+bx+c}\, dx,$$

$$\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

Applications of the Integrals 15 Periods

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

Differential Equations 15 Periods

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + P(x)y = Q(x) \quad \text{and} \quad \frac{dx}{dy} + P(y)x = Q(y)$$

Unit-IV: Vectors and Three-Dimensional Geometry

Vectors 15 Periods

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

Three - dimensional Geometry 15 Periods

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

Linear Programming 20 Periods

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

Probability 30 Periods

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

MATHEMATICS (Code No. - 041)

QUESTION PAPER DESIGN CLASS - XII(2022-23)

Time: 3 hours **Max. Marks: 80**

s.no	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

No chapter wise weightage. Care to be taken to cover all the chapters

Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.

<u>Choice(s):</u>

There will be no overall choice in the question paper. However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT 20 MARKS
Periodic Tests (Best 2 out of 3 tests conducted) 10 Marks
Mathematics Activities 10 Marks

Note: For activities NCERT Lab Manual may be referred.

Conduct of Periodic Tests:

Periodic Test is a Pen and Paper assessment which is to be conducted by the respective subject teacher. The format of periodic test must have questions items with a balance mix, such as, very short answer (VSA), short answer (SA) and long answer (LA) to effectively assess the knowledge, understanding, application, skills, analysis, evaluation and synthesis. Depending on the nature of subject, the subject teacher will have the liberty of incorporating any other types of questions too. The modalities of the PT are as follows:

Mode: The periodic test is to be taken in the form of pen-paper test.

Schedule: In the entire Academic Year, three Periodic Tests in each subject may be conducted as follows:

Test	Pre Mid-term (PT-I)	Mid-Term (PT-II)	Post Mid-Term (PT-III)
Tentative Month	July-August	November	December-January

This is only a suggestive schedule and schools may conduct periodic tests as per their convenience. The winter bound schools would develop their own schedule with similar time gaps between two consecutive tests.

Average of Marks: Once schools complete the conduct of all the three periodic tests, they will convert the weightage of each of the three tests into ten marks each for identifying best two tests. The best two will be taken into consideration and the average of the two shall be taken as the final marks for PT.

The school will ensure simple documentation to keep a record of performance as suggested in detail circular no.Acad-05/2017.

Sharing of Feedback/Performance: The students' achievement in each test must be shared with the students and their parents to give them an overview of the level of learning that has taken place during different periods. Feedback will help parents formulate interventions (conducive ambience, support materials, motivation and morale-boosting) to further enhance learning. A teacher, while sharing the feedback with student or parent, should be empathetic, non- judgmental and motivating. It is recommended that the teacher share best examples/performances of IA with the class to motivate all learners.

Assessment of Activity Work:

Throughout the year any 10 activities shall be performed by the student from the activities given in the NCERT Laboratory Manual for the respective class (XI or XII) which is available on the link: http://www.ncert.nic.in/exemplar/labmanuals.htmla record of the same may be kept by the student. An year end test on the activity may be conducted

The weightage are as under:

- The activities performed by the student throughout the year and record keeping : 5 marks
- Assessment of the activity performed during the year end test : 3 marks
- Viva-voce : 2 marks

Prescribed Books:

1) Mathematics Textbook for Class XI, NCERT Publications

2) Mathematics Part I - Textbook for Class XII, NCERT Publication

3) Mathematics Part II - Textbook for Class XII, NCERT Publication

4) Mathematics Exemplar Problem for Class XI, Published by NCERT

5) Mathematics Exemplar Problem for Class XII, Published by NCERT

6) Mathematics Lab Manual class XI, published by NCERT

7) Mathematics Lab Manual class XII, published by NCERT

CHAPTER: RELATIONS AND FUNCTIONS

FORMULAE AND DEFINITIONS

- **TYPES OF RELATIONS**:
  - **EMPTY RELATION**: A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \emptyset \subset A \times A$.
  - **UNIVERSAL RELATION**: A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.
  - **TRIVIAL RELATIONS**: Both the empty relation and the universal relation are sometimes called trivial relations.
  - A relation R in a set A is called
    - a) **Reflexive**, if $(x, x) \in R$ $for\ every\ x \in A$
    - b) **Symmetric**, $if\ (x, y) \in R\ implies\ that\ (y, x) \in R\ for\ all\ x, y\ \in A$
    - c) **Transitive**, $if\ (x, y) \in R\ and\ (y, z) \in R implies\ that\ (x, z) \in R for\ all\ x, y, z\ \in A$
  - **EQUIVALENCE RELATION**: A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.
- **EQUIVALENCE CLASS**: Let R be an equivalence relation on a non-empty set A and $a \in A$. Then the set of all those elements of A which are related to a, is called the equivalence class determined by a and is denoted by $[a]$.i.e $[a] = \{x \in A : (x, a) \in R\}$
- **TYPES OF FUNCTIONS**:
  - **ONE-ONE (INJECTIVE) FUNCTION**: A function $f : X \to Y$ is defined to be one-one (or injective), if the images of distinct elements of X under f are distinct, i.e., for every $x_1, x_2 \in X, f(x_1) = f(x_2)\ implies\ x_1 = x_2$. Otherwise, f is called many-one.
  - **ONTO (SURJECTIVE) FUNCTION**: A function $f : X \to Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under f. i.e., for every $y \in Y$, there exists an element $x\ in\ X$ such that $f(x) = y$.

    NOTE: **f : X → Y is onto if and only if Range of f = Codomain**.
  - **BIJECTIVE FUNCTION**: A function $f : X \to Y$ is said to be bijective, if f is both one-one and onto.

MULTIPLE CHOICE QUESTIONS

1) Let set $X = \{1, 2, 3\}$ and a relation R is defined in X as $R = \{(1,3), (2,2), (3,2)\}$, then minimum order pairs which should be added in relation R to make it reflexive and symmetric are
   (a) $\{(1,1), (2,3), (1,2)\}$ (b) $\{(3,3), (3,1), (1,2)\}$
   (c) $\{(1,1), (3,3), (3,1), (2,3)\}$ (d) $\{(1,1), (3,3), (3,1), (1,2)\}$
2) The relation "less than" in the set of natural numbers is
   (a) Only symmetric (b) Only transitive
   (c) Only reflexive (d) Equivalence relation
3) If $R = \{(x, y): x, y \in Z,\ x^2 + y^2 \le 4\}$ is a relation in set Z, then domain of R is
   (a) $\{0, 1, 2\}$ (b) $\{-2, -1, 0, 1, 2\}$
   (c) $\{0, -1, -2\}$ (d) $\{-1, 0, 1\}$

4) Let $X = \{x^2 : x \in N\}$ and the function $f: N \to N$ is defined by $f(x) = x^2, x \in N$. Then the function is

(a) injective only (b) not bijective

(c) surjective only (d) bijective

5) A function $f: N \to N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$. Then the function is

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

6) Let R be the relation in the set {1,2,3,4} given by R={(1,2), (2,2), (1,1),(4,4), (1,3), (3,3),(3,2)}. Choose the correct answer.

a) R is reflexive and symmetric but not transitive.
b) R is reflexive and transitive but not symmetric.
c) R is symmetric and transitive but not reflexive.
d) R is an equivalence relation.

7) Let R be the set N given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer.

a) $(2,4) \in R$
b) $(3,8) \in R$
c) $(6,8) \in R$
d) $(8,7) \in R$

8) Let $f: R \to R$be defined as $f(x) = 3x$. Choose the correct answer.

a) f is one-one onto
b) f is many-one onto
c) f is one-one but not onto
d) f is neither one-one nor onto.

ASSERTION AND REASONING QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1) ASSERTION: A relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$ is reflexive defined on the set $\{1,2,3\}$

REASON: A relation R in a set A is said to be reflexive if $(a, a) \in R$, for every $a \in A$.

Ans: (a)

2) ASSERTION: A relation $R = \{(L1, L2): L1 \text{ } is \text{ } perpendicular \text{ } to \text{ } L2\}$ is an equivalence relation defined on the set L of all lines in a plane

REASON: A relation R in a set A is said to be an *equivalence relation* if R is reflexive, symmetric and transitive.

Ans: (d)

3) ASSERTION:A relation R in the set **R** defined as $R = \{(a, b): a \leq b)\}$ is reflexive and transitive but not symmetric.

REASON: A relation R in a set A is said to be reflexive if $(a, a) \in R$, for every $a \in A$.

Ans: (b)

4) ASSERTION: A relation R in the set {1,2,3} given by $R = \{(1,2),(2,1)\}$ is symmetric and transitive but not reflexive.
REASON: A relation R in the set {1,2,3} given by $R = \{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive.

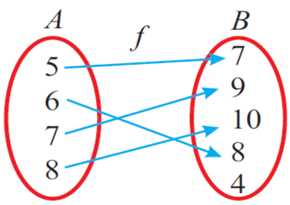
Ans: (d)

5) ASSERTION: A relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b): |a-b| \text{ is even}\}$ is an equivalence relation.
REASON: No element of {1,3,5} is related to any element of {2,4}
Ans: (b)

6) ASSERTION: The Modulus function $f: R \to R$, given by $f(x) = |x|$ is not onto
REASON: A function $f: X \to Y$ is onto if and only if Range of $f = Y$
Ans: (a)

7) ASSERTION: The Modulus function $f: R \to R$, given by $f(x) = [x]$ is one-one
REASON: A function $f: X \to Y$ is said to be one-one if for every $x, y \in X, f(x) = f(y)$
Ans: (d)

8) ASSERTION: A mapping shown in the arrow diagram, the function $f: A \to B$, is injective



REASON: A function $f: A \to B$ is said to be onto if every element of B has a pre-image in A
Ans: (b)

9) ASSERTION: A function is said to be bijective if it is one-one and onto
REASON: The signum function $f: R \to R$, given by

$$f(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ -1, if\ x < 0 \end{cases}$$ is bijective.

Ans: (c)

10) ASSERTION: Let A and B be two sets. A function $f: A \times B \to B \times A$ such that $F(a,b) = (b,a)$ is one one.
REASON: A function $f: X \to Y$ is said to be one-one if for every $x, y \in X, f(x) = f(y)$
Ans:(a)

SHORT ANSWER TYPE QUESTIONS

1) Let $A = \{x \in Z: 0 \le x \le 12\}$ and given by $R = \{(a,b): a, b \in A, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related 1.
2) Let $A = \{x \in Z: 0 \le x \le 12\}$ and given by $R = \{(a,b): a, b \in A, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Write the equivalence class[2].
3) Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. whether f is one-one or not
4) State whether the function $f: N \to N$ given by $f(x) = 7x$ is injective, surjective or both.
5) Show that the function $f: \boldsymbol{R} \to \boldsymbol{R}$ defined by $f(x) = 3 - 4x$ is one-one and onto.

6) Show that the function $f: \boldsymbol{R} \rightarrow \boldsymbol{R}$ be defined as $f(x) = x^4$ is neither one-one nor onto.

7) Let the function $f: \boldsymbol{R} \rightarrow \boldsymbol{R}$ defined by $f(x) = [x]$. Check whether f is one-one and onto.

8) Let the function $f: \boldsymbol{N} \rightarrow \boldsymbol{N}$ defined by $f(x) = |x|$. Check whether f is one-one and onto.

9) Show that the function $f: \boldsymbol{R} \rightarrow \boldsymbol{R}$ defined by $f(x) = 1 + x^2$ is neither one-one nor onto.

10) Determine whether the relation R in the set N of natural numbers defined as $R = \{(x, y): y = x + 5 \ and \ x < 4\}$ is reflexive. Justify your answer.

11) If $n(A) = 4$, then write the number of one-one functions from $A \ to \ A$

12) Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \ is \ perpendicular \ to \ L_2\}$. Whether R is transitive or not

13) State the reason for the relation R in the set $\{1,2,3\}$ given by $\{(1,2), (2,1)\}$ not to be transitive

LONG ANSWER TYPE QUESTIONS

1) Show that the relation R on the set Z of all integers, given by $R = \{(a, b): 2 \ divides (a - b)\}$ is an equivalence relation

2) Let Z be the set of all integers and R be relation on Z defined as $R = \{(a, b): a, b \in Z \ and \ (a - b)$is divisible by 5$\}$. Prove that R is an equivalence relation.

3) Show that the relation S in the set $A = \{x \in Z: 0 \leq x \leq 12\}$ given by $S = \{(a, b): a, b \in Z, |a - b| \ is \ divisible \ by \ 3\}$is an equivalence relation.

4) Prove that the relation R in the set A ={1, 2, 3, 4, 5} given by R = {(a, b):$|a - b|$is even}, is an equivalence relation

5) Let T be the set of all triangles in a plane with R as a relation in T given by R = $\{(T_1, T_2): T_1 \cong T_2\}$. Show that R is an equivalence relation.

6) Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b): b = a + 1\}$ is reflexive, symmetric or transitive.

7) Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b): a \leq b\}$, is reflexive, and transitive but not symmetric

8) Show that the relation R in the set **R** of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

9) Check whether the relation R in **R** defined as $R = \{(a, b): a \leq b^3\}$ is reflexive, symmetric or transitive. (Hint: R is neither reflexive, nor symmetric, nor transitive.)

10) Prove that the function $f: N \rightarrow N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

11) Show that the function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall \ x \in R$ is neither one-one nor onto.

12) Let A = R – {2} and B = R – {1}. If $f: A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto.

13) Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $(x) = \frac{4x}{3x+4}$. Show that f is one-one function. Also check whether f is an onto function or not.

14) Consider $f: \mathrm{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathrm{R} - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective

15) Show that $f: N \rightarrow N$, given by $f(x) = \begin{cases} x+1, if\ x\ is\ odd \\ x-1\ if\ x\ is\ even \end{cases}$ $\quad is\ both\ one - one\ and\ onto.$

16) Let f: **N** → **N** be defined by $f(n) = \begin{cases} \frac{n+1}{2}\ , if\ n\ is\ odd \\ \frac{n}{2}\ , if\ n\ \ is\ even \end{cases}$

State whether the function f is bijective. Justify your an hat f is bijective.

17) Show that the relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b): |a-b|\ is\ divisible\ by\ 2\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other, but no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

18) Determine whether the relation R defined on the set R of all real numbers as $R = \{(a,b): a,b \in R\ and\ a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive.

19) Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a,b)\ R\ (c,d)\ if\ ad(b + c) = bc(a + d)$. Show that R is an equivalence relation

20) Show that the relation R defined by $(a,b)R(c,d) \Leftrightarrow a + d = b + c$ on the $A \times A$, where $A = \{1,2,3, \ldots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3,4)]; a,b,c,d \in A$.

CHAPTER: INVERSE TRIGONOMETRIC FUNCTION

FORMULAE AND DEFINITIONS

PRINCIPAL VALUE BRANCHES:

FUNCTION	DOMAIN	RANGE (Principal Value Branch)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\mathrm{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$(0, \pi)$

- $\sin(\sin^{-1} x) = x\ ,\ \ x \in [-1, 1]$
- $\sin^{-1}(\sin x) = x\ ,\ \ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\sin^{-1}\frac{1}{x} = \ \text{cosoc}^{-1} x\ ,\ \ x \geq 1\ or\ x \leq -1$
- $\cos^{-1}\frac{1}{x} = \ \sec^{-1} x\ ,\ \ x \geq 1\ or\ x \leq -1$
- $\tan^{-1}\frac{1}{x} = \ \cot^{-1} x\ ,\ \ x > 0$

ASSERTION AND REASONING QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1) ASSERTION:The Principal Value of $\cot^{-1}\left(-\frac{1}{3}\right) = \frac{2\pi}{3}$

   REASON: The range of Principal value Branch of $\cot^{-1}\ is\ (0, \pi)$

   Ans: (a)
2) ASSERTION:If $\cos^{-1} x = y, then\ (0, \pi)$

   REASON: If $\sin^{-1} x = y, then -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

   Ans: (d)
3) ASSERTION:$\tan^{-1}(\tan x) = x, if\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

   REASON:$\tan(\tan^{-1} x) = x,\ \ if\ x \in R$

   Ans: (b)
4) ASSERTION: $\sin^{-1} x = \frac{1}{sinx}$

   REASON: $(sinx)^{-1} = \frac{1}{sinx}$

   Ans: (d)
5) ASSERTION: $\sin^{-1}\left(sin\frac{3\pi}{5}\right) = \frac{2\pi}{5}$

   REASON: $\sin^{-1}(sin\theta) = \theta, where\ \theta\epsilon\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

   Ans: (a)

MULTIPLE CHOICE QUESTIONS

1) If $sin^{-1}x = y$, then

   a) $0 \leq y \leq \pi$
   b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
   c) $0 < y < \pi$
   d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

2) The principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{12}$

(b) π

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

3) $tan^{-1}\sqrt{3} - sec^{-1}(-2)$ is equal to

a) π

b) $-\frac{\pi}{3}$

c) $\frac{\pi}{3}$

d) $\frac{2\pi}{3}$

4) The principal value of $\tan^{-1}\left(tan\frac{9\pi}{8}\right)$ is

(a) $\frac{\pi}{8}$

(b) $\frac{9\pi}{8}$

(c) $-\frac{\pi}{8}$

(d) $-\frac{3\pi}{8}$

5) $cos^{-1}\left(cos\frac{7\pi}{6}\right)$ is equal to

a) $\frac{7\pi}{6}$

b) $\frac{5\pi}{6}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{6}$

6) $\sin\left(\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) 1

7) $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$is equal to

a) π

b) $-\frac{\pi}{2}$

c) 0

d) $2\sqrt{3}$

8) $\sin(tan^{-1}x), |x| < 1$is equal to

a) $\frac{x}{\sqrt{1-x^2}}$

b) $\frac{1}{\sqrt{1-x^2}}$

c) $\frac{1}{\sqrt{1+x^2}}$

d) $\frac{x}{\sqrt{1+x^2}}$

VERY SHORT ANSWER QUESTIONS

1) Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ (Ans: $-\frac{\pi}{6}$)
2) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ (Ans:$\frac{2\pi}{3}$)
3) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ (Ans:$\frac{2\pi}{3}$)
4) Find the principal value of $\tan^{-1}(-1)$ (Ans; $-\frac{\pi}{4}$)
5) Find the principal value of $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ (Ans: $\frac{5\pi}{6}$)

SHORT ANSWER QUESTIONS

1) Find the value of $\cos^{-1}\left(cos\frac{2\pi}{3}\right)+\sin^{-1}\left(sin\frac{2\pi}{3}\right)$ (Ans: π)
2) Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ (Ans: $\frac{3\pi}{4}$)
3) Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ (Ans : $-\frac{\pi}{3}$)
4) Find the value of) $\cos^{-1}\left(cos\frac{13\pi}{6}\right)$ (Ans: $\frac{\pi}{6}$)
5) Find the value of $\tan^{-1}\left(tan\frac{7\pi}{6}\right)$ (Ans: $\frac{\pi}{6}$)
6) Find the value of $tan^{-1}(\sqrt{3})-cot^{-1}(-\sqrt{3})$ $\left(Ans:\ -\frac{\pi}{2}\right)$
7) Find the value of $2\cos^{-1}\frac{1}{2}+3\sin^{-1}\frac{1}{2}$ (Ans : $\frac{7\pi}{6}$)

8) Write the value of $tan^{-1}\left[2sin\left(2cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ $\left(Ans:\frac{\pi}{3}\right)$
9) Write the value of $tan^{-1}\left[2cos\left(2sin^{-1}\frac{1}{2}\right)\right]$ $\left(Ans:\ \frac{\pi}{4}\right)$

CHAPTER:MATRICES

FORMULAE AND DEFINITIONS

➢ **ORDER OF A MATRIX** : A general matrix of order $m \times n$ can be written as

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{pmatrix}$$

$$= [a_{ij}]_{m\times n}, where\ i = 1,2, \dots m\ and\ j = 1,2, \dots n$$

$$Number\ of\ rows = m\ and\ \ Number\ of\ columns = n$$

➢ **TYPES OF MATRICES:**

- **COLUMN MATRIX**: A matrix is said to be a column matrix if it has only one column.

  Examples: $A = \begin{bmatrix} 2 \\ -9 \end{bmatrix}$ $order\ of\ matrix\ A\ is\ 2 \times 1$

  $and\ B = \begin{bmatrix} -\sqrt{5} \\ 0 \\ -12 \end{bmatrix}$ $order\ of\ matrix\ B\ is\ 3 \times 1$

- **ROW MATRIX:** A matrix is said to be a row matrix if it has only one row

  Examples: $A = [14 \quad 26]$ $order\ of\ matrix\ A\ is\ 1 \times 2$

  $\boldsymbol{B} = [0 \quad \sqrt{7} \quad 12]$ $order\ of\ matrix\ B\ is\ 1 \times 3$

- **SQUARE MATRIX:** A matrix in which the number of rows is equal to the number of columns, is said to be a square matrix.

  Examples: $A = \begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}$ $order\ of\ matrix\ A\ is\ 2 \times 2$

  $X = \begin{bmatrix} 5 & 0 & -8 \\ \frac{1}{2} & \sqrt{2} & 14 \\ 7 & -8 & 4 \end{bmatrix}$ order of matrix X is 3×3

- **DIAGONAL MATRIX:** A square matrix A is said to be a diagonal matrix if all its non-diagonal elements are zero

  Example: $A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & 9 \end{bmatrix}$

- **SCALAR MATRIX** : A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal

Example : $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

- **IDENTITY MATRIX:** A square matrix in which all the elements in the diagonal are all equal 1 and rest are all zero is called an identity matrix.

  Example : $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, generally it is denoted by I.

- **ZERO MATRIX:** A matrix is said to be zero matrix or null matrix if all its elements are zero.

  Examples : $[0]$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are all Zero matrices, generally denoted by $\boldsymbol{O}$.

➢ **EQUALITY OF MATRICES**: Two matrices $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are said to be equal if (i) they are of the same order (ii) each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ $for\ all\ i\ and\ j$.

Example: Let $A = \begin{bmatrix} 2 & 1 \\ 8 & 6 \\ 4 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 8 & 6 \\ 4 & -5 \end{bmatrix}$, we say that $A = B$

➢ **OPERATION OF MATRICES:**

- **ADDITION OF MATRICES:** Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two matrices of the same order. Then $A + B$ is defined to be the matrix of order of $m \times n$ obtained by adding corresponding elements of A and B

  i.e $A + B = [a_{ij} + b_{ij}]_{m\times n}$

- **DIFFERENCE OF MATRICES**: Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two matrices of the same order. Then $A - B$ is defined to be the matrix of order of $m \times n$ obtained by subtracting corresponding elements of A and B

  i.e $A - B = [a_{ij} - b_{ij}]_{m\times n}$

- **MULTIPLICATION OF MATRICES:** The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$. Then the product of the matrices A and B is the matrix C of order m × p

Example: Let $\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{5} \\ \mathbf{1} & \mathbf{6} & \mathbf{8} \end{bmatrix}$ $\boldsymbol{and}\ \boldsymbol{B} = \begin{bmatrix} \mathbf{4} & \mathbf{3} \\ \mathbf{6} & \mathbf{9} \\ \mathbf{5} & \mathbf{8} \end{bmatrix}$

$$\boldsymbol{AB} = \begin{bmatrix} \mathbf{2\times4+3\times6+5\times5} & \mathbf{2\times3+3\times9+5\times8} \\ \mathbf{1\times4+6\times6+8\times5} & \mathbf{1\times3+6\times9+8\times8} \end{bmatrix}$$

$$= \begin{bmatrix} 8+18+25 & 6+27+40 \\ 4+36+40 & 3+54+64 \end{bmatrix} = \begin{bmatrix} 51 & 73 \\ 80 & 121 \end{bmatrix}$$

- **MULTIPLICATION OF A MATRIX BY A SCALAR**: Let $A = [a_{ij}]_{m\times n}$ and k is a scalar, then $kA = k[a_{ij}]_{m\times n} = [k.a_{ij}]_{m\times n}$

  Example: $A = \begin{bmatrix} 2 & 4 & -5 \\ y & z & x \end{bmatrix} \Rightarrow 3A = \begin{bmatrix} 3(2) & 3(4) & 3(-5) \\ 3y & 3z & 3x \end{bmatrix} = \begin{bmatrix} 6 & 12 & -15 \\ 3y & 3z & 3x \end{bmatrix}$

- **TRANSPOSE OF A MATRIX**: If $A = [a_{ij}]_{m\times n}$ be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or A^{T} .

  If $A = [a_{ij}]_{m\times n}$, then $A' = [a_{ji}]_{n\times m}$

  Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 7 & 9 \\ 5 & 1 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 7 & 1 \\ 3 & 9 & 0 \end{bmatrix}$

- **SYMMETRIC MATRIX**: A square matrix If $A = [a_{ij}]$ is said to be symmetric if $A' = A$, that is, $[a_{ij}]$ =$[a_{ji}]$ for all possible values of i and j

  Example: $A = \begin{bmatrix} 2 & 5 & 12 \\ 5 & 7 & 3 \\ 12 & 3 & 6 \end{bmatrix}$, $clearly\ A' = A.$

- **SKEW-SYMMETRIC MATRIX**: A square matrix $A = [a_{ij}]$ is said to be skew symmetric matrix if $A' = -A$, that is $a_{ij} = -a_{ji}$ for all possible values of i and j and $a_{ii} = 0$ for all i.(all the diagonal elements are zero).

- **Example:** $A = \begin{bmatrix} 0 & 5 & -12 \\ -5 & 0 & -3 \\ 12 & 3 & 0 \end{bmatrix}$, $clearly\ A' = -A$

- **PROPERTIES OF MATRICES**:
  - $A + B = B + A$
  - $A - B \neq B - A$
  - $AB \neq BA$
  - $(AB)C = A(BC)$
  - $(A')' = A$
  - $AI = IA = A$
  - $AB = BA = I, then\ A^{-1} = B\ and\ B^{-1} = A$
  - $AB = 0 \Rightarrow$ *it is not necessary that one the matrix is zero.*
  - $A(B + C) = AB + AC$
  - Every square matrix can possible to express as the sum of symmetric and skew-symmetric matrices.

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where $(A + A')$ is symmetric matrix and $(A - A')$ is skew-symmetric matrices.

ASSERTION AND REASONING QUESTIONS

1) ASSERTION: The number of elements in a matrix of order 3×2 *and* 2×3 are equal
REASON: Two matrices are same if their order is either 3×2 *or* 2×3
Ans: (c)

2) ASSERTION: $A = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$ is a scalar matrix
REASON: A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a scalar matrix if
$a_{ij} = 0, when\ i \neq j\ and\ a_{ij} = k\ \ when\ i = j, for\ some\ constant\ k$
Ans: (a)

3) ASSERTION: If $\begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 5 & 8 \end{bmatrix}$, then
$x = 4, y = 2, z = 0\ or\ x = 2, y = 4, z = 0$
REASON: Two matrices $A = [a_{ij}]_{m \times n}\ and\ B = [b_{ij}]_{m \times n}$ are said to be equal if $a_{ij} = b_{ij}$ for all $i\ and\ j$
Ans: (a)

4) ASSERTION: $A = \begin{bmatrix} -7 & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ is a diagonal matrix
REASON: A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a scalar matrix if
$a_{ij} = 0, when\ i \neq j\ and\ a_{ij} = k\ \ when\ i = j, for\ some\ constant\ k$
Ans: (b)

MULTIPLE CHOICE QUESTIONS

1) Which of the given values of $x\ and\ y$ make the following pair of matrices equal
$\begin{bmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{bmatrix}, \begin{bmatrix} 0 & y - 2 \\ 8 & 4 \end{bmatrix}$
   a) $x = \frac{-1}{3}, y = 7$
   b) Not possible to find
   c) $y = 7, x = \frac{-2}{3}$
   d) $x = \frac{-1}{3}, y = \frac{-2}{3}$

2) If $\begin{bmatrix} 3c + 6 & a - d \\ a + d & 2 - 3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is
   a) 4
   b) 16
   c) -4
   d) -16

3) The number of all possible matrices of order 3x3 with each entry 0 or 1 is:
   a) 27
   b) 18
   c) 81
   d) 512

4) For two matrice $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ $and\ Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ the $P - Q$ is

a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

5) The restriction on $n, k\ and\ p$ so that PY + WY will be defined are:

a) $k = 3, p = n$

b) k is arbitrary, $p = 2$

c) p is arbitrary, $k = 3$

d) $k = 2, p = 3$

6) If $n = p$, then the order of the matrix 7X – 5Z is:

a) $p \times 2$

b) $2 \times n$

c) $n \times 3$

d) $p \times n$

7) For a matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $X^2 - X\ is$

a) $2I$

b) $3I$

c) I

d) $5I$

8) If A, B are symmetric matrices of same order, then AB – BA is a

a) Skew symmetric matrix

b) Symmetric matrix

c) Zero matrix

d) Identity matrix

9) If $A = \begin{bmatrix} cos\ \alpha & -sin\ \alpha \\ sin\ \alpha & cos\ \alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is

a) $\frac{\pi}{6}$

b) $\frac{\pi}{3}$

c) π

d) $\frac{3\pi}{2}$

10) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

a) $1 + \alpha^2 + \beta\gamma = 0$

b) $1 - \alpha^2 + \beta\gamma = 0$

c) $1 - \alpha^2 - \beta\gamma = 0$

d) $1 + \alpha^2 - \beta\gamma = 0$

11) If the matrix A is both symmetric and skew symmetric, then

a) A is a diagonal matrix

b) A is a zero matrix

c) A is a square matrix

d) None of these

12) If A is square matrix such that $A^2 = A, then\ (I + A)^3 - 7I$ is equal to

a) A

b) $I - A$

c) I

d) $3A$

SHORT ANSWER TYPE QUESTIONS

1) How many matrices of order 3 x 3 are possible with each entry as 0 or 1?

Ans: $2^9 = 512$

2) Write a square matrix of order 2, which is both symmetric and skew-symmetric.

Ans: Zero matrix

3) If a matrix has 5 elements, write all possible orders it can have. Ans: $1 \times 5\ and 5 \times 1$

4) Find the value of $x + y$ from the equation:

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}.$$

Ans: $2x + 3 = 7 \Rightarrow x = 2,\ 2y - 6 + 2 = 14 \Rightarrow y = 9,\ x + y = 11$

5) Write the value of $x - y + z$ from the following equation: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Ans: $x = 2, y = 4, z = 3\ ,\ x - y + z = 1$

6) If $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}\ and\ kA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$ find the values of $k\ and\ a$.

Ans: $-8k = 2 \Rightarrow k = -\frac{1}{4}\ and\ 4ka = 3 \Rightarrow a = -3$

7) If $A^T = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix}\ and\ B = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$, then find $A^T - B^T$.

8) If $A = \begin{bmatrix} cosx & -sinx \\ sinx & cosx \end{bmatrix}$, then for what value of x is A is an identity matrix. Ans: $x = 0$

9) If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}^T, find\ x.$ Ans : $3y = 6 \Rightarrow y = 2,\ 2x + y = 6 \Rightarrow x = 2$

10) If $A = [2 \quad -3 \quad -1]\ and\ B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, find\ AB. Ans: AB\ = [2\ -\ 6\ -\ 3] = [-7]$

11) Construct a 3 x 2 matrix, whose element $a_{ij}\ is\ given\ by\ a_{ij} = \frac{(i-2j)^2}{2}$

12) Construct a 3 x 2 matrix, whose element $a_{ij}\ is\ given\ by\ a_{ij} = \frac{|-i+3j|}{2}$

13) Find a matrix X such that 2A + 3X = 5B , where $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}\ and$

$B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ Ans: $X = \begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & -\frac{7}{3} \end{bmatrix}$

14) If A is a square matrix such that $A^2 = A$, then find $(I + A)^3 - 7A$.

15) If $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (b_{ij})$, find $b_{21} + b_{32}$.

16) If $A = \begin{pmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{pmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when

$A + A^T = \sqrt{2} I_2$; where A^T is transpose of A.

17) Show that the matrix A= $\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ is Skew-symmetrix.

Ans: $A^T = -A, \therefore a\ is\ skew\ symmetric\ matrix$

18) If $\begin{bmatrix} 0 & x+2 & 2-x \\ 1-2x & 0 & 2x-1 \\ 3x-8 & x-8 & 0 \end{bmatrix}$ is a skew symmetric, find value of x.

Ans: $x + 2 = -(1 - 2x) \Rightarrow x = 3$

19) For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

Ans: $x = 2$

20) Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation: $\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

21) Find the value of x, if $[1 \quad x \quad 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$

22) Let $\begin{bmatrix} cosx & sinx \\ -sinx & cos \end{bmatrix}$, show that $A^2 = \begin{bmatrix} cos2x & sin2x \\ -sin2x & cos2x \end{bmatrix}$

23) If $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$, $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ Find X and Y.

Ans: $X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$, $Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

24) If A is 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined, find order of B.

25) If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

26) Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

Ans: $2b = 3 \Rightarrow b = \frac{3}{2}$ and $3a = -2 \Rightarrow a = -\frac{2}{3}$

27) If $f(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find $f(x).f(-x)$.

28) If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, show that $A^2 = B^2$

29) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, f(x) = x^2 - 2x - 3\,, show\ that\ f(A) = 0$

30) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}, find\ 3A^2 - 2B + I$

31) If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB – BA is a skew symmetric matrix.

32) For any matrix A with real entries, $A + A'$ is symmetric matrix and $A - A'$ is skew symmetric matrix

33) If A and B are invertible matrices of same order, then $(AB)^{-1} = B^{-1}A^{-1}$

34) If A and B are both symmetric matrices of same order, then show that AB is symmetric if A and B are commute, that is AB = BA

35) Find a matrix A such that $2A - 3B + 5C = 0$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C= $\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

36) If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}, show\ that\ (A - 2I)(A - 3I) = 0.$

37) If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}\ and\ kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the value of $k, a\ and\ b$.

38) Find the value of $(x - y)$ from the matrix equation

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}.$$

LONG ANSWER TYPE QUESTIONS

1) For the matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find a and b such that $A^2 + aI = bA$, where I is 2 x 2 identity matrix.

2) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}

3) For the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 4I = O$. Hence,find A^{-1}.

4) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\ and\ A^3 - 6A^2 + 7A + kI_3 = O\ find\ k.$

5) If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and $(A + B)C$. Also verify that $(A + B)\,C = AC + BC$.

6) If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}, find\ A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

7) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\ and\ B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\ and\ (A + B)^2 = A^2 + B^2$,then find the values of a and b.

8) $If\ X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$, then find the matrix X.

9) Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}\ and\ C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find the matrix D such that $CD - AB = 0$

Ans: $D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

10) Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ Verify that $(AB)^T = B^T A.^T$

11) If $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then by induction show that $(aI + bA)^n = a^n I + n.a^{n-1}.bA$, if a & b are constants.

12) Express the following matrices as sum of a symmetric and skew symmetric matrices.

(a) A = $\begin{pmatrix} 2 & -1 & 2 \\ 3 & 4 & 0 \\ 0 & -3 & -2 \end{pmatrix}$ (b) $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

CHAPTER: DETERMINANTS

FORMULAE AND DEFINITIONS

- **DETERMINANT:**
  - $Let\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, then\ det(A) = |A| = ad - bc$
  - $Let\ A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}, then\ |A| = a\begin{vmatrix} e & f \\ h & k \end{vmatrix} - b\begin{vmatrix} d & f \\ g & k \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
- **MINORS**: Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.
- **CO-FACTORS**: Cofactor of an element a_{ij} , denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j}.M_{ij}$, where M_{ij} is minor of a_{ij}
- **ADJOINT OF A MATRIX**: The adjoint of a square matrix $A = [a_{ij}]$ is defined as the transpose of the matrix $[A_{ij}]$, where A_{ij} is the cofactor of the element a_{ij} . Adjoint of the matrix A is denoted by adj A.
- **INVERSE OF A MATRIX**: Let A be a square matrix.

  $$A^{-1} = \frac{1}{|A|} adjA$$
- **SOLUTION OF SYSTEM OF LINEAR EQUATIONS BY USING MATRIX METHOD**:

  Let the system of linear equations be

  $a_1x + b_1y + c_1z = d_1$

  $a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

These equations can be written as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

- $A^{-1} exists, if\ |A| \neq 0\ \ i.e\ the\ solution\ existsand\ it\ is\ unique.$
- $The\ system\ of\ equations\ is\ said\ to\ be\ consistent\ if\ the\ solution\ exists.$
- $if\ |A| = 0\ , then\ we\ calculate\ (adjA)B.$
- $If\ |A| = 0\ and\ \ (adjA)B \neq O$, (O being zero matrix), then solution does not exist and the system of equations is called inconsistent.
- $If\ |A| = 0\ and\ \ (adjA)B = O$, then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

➢ **IMPORTANT NOTES**:

- The matrix A is singular if $|A| = 0$
- $|\lambda A| = \lambda^n |A|, where\ n = order\ of\ matrix\ A$
- $A(adjA) = (adjA)A = |A|I$
- $|adjA| = |A|^{n-1}, where\ n = order\ of\ matrix\ A$
- $|A(adjA)| = |A|^n, where\ n = order\ of\ matrix\ A$
- $|AB| = |A||B|$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $|A^{-1}| = |A|^{-1}$
- $|A^T| = |A|$

MULTIPLE CHOICE QUESTIONS

1) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to
   a) 6
   b) ± 6
   c) -6
   d) 0
2) If $C_{ij}\ denotes\ the\ factor\ of\ elements\ p_{ij}$ of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$

   Then the value of $C_{31}C_{23}$ is
   a) 5
   b) 24
   c) -24
   d) -5

3) Let A be a square matrix of order 3 x 3, then $|kA|$ is equal to

   a) $k|A|$
   b) $k^2|A|$
   c) $k^3|A|$
   d) $3k|A|$

4) Which of the following is correct

   a) Determinant is a square matrix.
   b) Determinant is a number associated to a matrix.
   c) Determinant is a number associated to a square matrix.
   d) None of these

   5)If for a matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is

   a) ± 3
   b) -3
   c) ± 1
   d) 1

5) If area of triangle is 35sq units with vertices (2,-6),(5,4)and (k,4). Then k is

   a) 12
   b) -2
   c) -12,-2
   d) 12,-2

6) If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ $and\ A_{ij}$ is Cofactors of a_{ij}, then value of Δ is given by

   a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
   b) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
   c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
   d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

7) Let $X = [x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$. Then the matrix $Y = [m_{ij}]$, where $m_{ij} = Minor\ of\ x_{ij}$, is

   a) $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$
   b) $\begin{bmatrix} 7 & -19 & -11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$
   c) $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -5 & -1 & -1 \end{bmatrix}$
   d) $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

8) Let A be a nonsingular square matrix of order 3 x 3. Then $|adj\ A|$ is equal to

   a) $|A|$
   b) $|A|^2$
   c) $|A|^3$
   d) $3|A|$

9) If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

a) Det (A)

b) $\frac{1}{\det(A)}$

c) 1

d) 0

10) If x, y, z are nonzero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

a) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

b) $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

c) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

d) $\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11) If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is

a) 4

b) – 3

c) 2

d) 5

12) The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

a) $24 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

b) $\frac{1}{24} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\frac{1}{24} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

d) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

13) Let $A = \begin{bmatrix} 1 & sin\theta & 1 \\ -sin\theta & 1 & sin\theta \\ -1 & -sin\theta & 1 \end{bmatrix}, where\ 0 \le \theta \le 2\pi$.then

a) $Det\ (A) = 0$

b) $Det\ (A) \in (2, \infty)$

c) $Det(A) \in (2,4)$

d) $Det\ (A) \in [2,4]$

SHORT ANSWER TYPE QUESTIONS

1) If $A = \begin{bmatrix} -3 & -2 \\ 1 & -4 \end{bmatrix}$, $then\ find\ A(adjA)$

2) Write the adjoint of $A = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$

3) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, $then\ write\ A^{-1}$.

4) A is a square matrix of order 3 and $|A| = 7$. Write the value of $|adj\ A|$.

5) If A is a square matrix of order 3 such that $|adj\ \mathrm{A}|$ = 25, find $|A|$

6) A matrix A of order $3\ x\ 3$ has determinant 7, what is the value of $|3A|$?

7) For any 2 x 2 matrix A, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find |A|

8) If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$,then for any natural number n, find the value of Det (A^n).

9) If A is a skew symmetric matrix of order 3, then show that $det(A) = 0$

10) For what value(s) of k, the matrix $\begin{bmatrix} 3 & k \\ -1 & 2 \end{bmatrix}$ has no inverse.

11) For what value of x if $\begin{vmatrix} -2 & x \\ 3 & -3 \end{vmatrix} = \begin{vmatrix} -x & -1 \\ -6 & -3 \end{vmatrix}$

12) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, $write\ A^{-1}$ in terms of A.

13) If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then write the value of $a_{32}.A_{32}$.

14) If $x \in N\ and\ \begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

15) If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.

16) If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}\ and\ B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$, $find\ |AB|$.

17) If$|A| = 3\ and\ A^{-1} = \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$, then write adjA.

18) If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then find the value of $|2AB|$

19) For what value of x, the matrix $A = \begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$ is a singular?

20) Evaluate:$\begin{vmatrix} sin30^0 & cos30^0 \\ -sin60^0 & cos60^0 \end{vmatrix}$

21) If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$,then write the cofactor of the element a_{21} of its 2nd row.

22) If A is a 3 x 3 matrix and$|3A| = k|A|$, then write the value of k.

23) For what values of k, the system of linear equations

$x + y + z = 2$, $2x + y - z = 3$,$3x + 2y + kx = 4$ has a unique solution?

24) Find the value of x: $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

25) What positive value of x makes the following pairs of determinants equal? $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

26) If A and B are square matrices of the same order 3, such that $|A| = 2 \ and \ AB = 2I$, write the value of $|B|$

27) If A is a square matrix satisfying $A'A = I$, write the value of $|A|$

28) If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

29) If A is a square matrix of order 3, with $|A| = 9$, then write the value of $|2. adj\ A|$.

30) If A is a square matrix of order 2 and $|A| = 4$, then find the value of $|2. AA'|$, where A' is the transpose of matrix A.

LONG ANSWER TYPE QUESTIONS

1) Find the inverse of matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ and hence show that $A^{-1}. A = I$.

2) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}, find\ (A')^{-1}$.

3) If $A = \begin{pmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ $and\ B^{-1} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ $compute\ (AB)^{-1}$

4) Show that A = $\begin{pmatrix} -8 & 5 \\ 2 & 4 \end{pmatrix}$, satisfies the equation $x^2 + 4x - 42 = 0$. Hence find A^{-1}.

Solving the system of equations by using matrix method

1) Using matrices, solve the following system of linear equations:

$2x + 3y + 10z = 4, 4x - 6y + 5z = 1 \ and \ 6x + 9y - 20z = 2$

2) If A = $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1} and hence solve the equations:

$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3.$

3) If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$,Find BA and hence solve equations

$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$

4) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \ and \ B = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, find AB and hence solve the system of equations

$7x - 3y - 3z = -9, -x + y = 1, -x + z = 2$

5) If A = $\begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 1 \\ 1 & -2 & 3 \end{bmatrix}$ find A^{-1}. Hencesolve the equations:

$x - y + z = 4, \quad x - 2y - 2z = 9, \; 2x + y + 3z = 1$

6) If $A = \begin{bmatrix} 2 & 6 & 4 \\ 3 & 9 & -5 \\ 10 & -20 & 5 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \qquad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2, \qquad \frac{4}{x} - \frac{5}{y} + \frac{5}{z} = 1$

CHAPTER:CONTINUITY AND DIFFERENTIABLITY

FORMULAE AND DEFINITIONS

CONTINUITY: Suppose f is a real function on a subset of the real numbers and let a be a point in the domain of f. Then f is continuous at a $\lim_{x\to a} f(x) = f(a)$

i.e $LHL = RHL = f(a)$

$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

DIFFERENTIATION:

FIRST PRINCIPLE:

Let $y = f(x), then\ \frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

- $y = constant \Rightarrow \frac{dy}{dx} = 0$
- $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$
- $y = sinx \Rightarrow \frac{dy}{dx} = cosx$
- $y = cosx \Rightarrow \frac{dy}{dx} = -sinx$
- $y = tanx \Rightarrow \frac{dy}{dx} = sec^2x$
- $y = cosecx \Rightarrow \frac{dy}{dx} = -cosecx.cotx$
- $y = secx \Rightarrow \frac{dy}{dx} = secx.tanx$
- $y = cotx \Rightarrow \frac{dy}{dx} = -cosec^2x$
- $y = \sin^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$
- $y = \cos^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$
- $y = \tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$
- $y = \text{cosec}^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$

- $y = \sec^{-1} x \;\Rightarrow \frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$
- $y = \cot^{-1} x \;\Rightarrow \frac{dy}{dx} = -\frac{1}{1+x^2}$
- $y = e^x \;\Rightarrow \frac{dy}{dx} = e^x$
- $y = a^x \;\Rightarrow \frac{dy}{dx} = a^x.loga$
- $y = logx \;\Rightarrow \frac{dy}{dx} = \frac{1}{x}$
- Product Rule: $y = u.v \Rightarrow \frac{dy}{dx} = u.v' + v.u',\;\; where\; u' = \frac{du}{dx}\;, v' = \frac{dv}{dx}$
- Quotient Rule: $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v.u'-u.v\prime}{v^2},\;\; where\; u' = \frac{du}{dx}\;, v' = \frac{dv}{dx}$
- Chain Rule: Let $y = f(t) and\; x = g(t)\; then\; \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$
- $y = f(ax+b) \Longrightarrow \frac{dy}{dx} = a.f'(ax+b),$

  $Ex: y = \sin(4x+9) \Longrightarrow \frac{dy}{dx} = 4.\cos(4x+9)$
- $y = [f(x)]^n \;\Rightarrow \frac{dy}{dx} = n.[f(x)]^{n-1}.f'(x)$
- Logarithmic Differentiation: Let $y = [u(x)]^{v(x)}$

  $logy = v.logu \Longrightarrow \frac{1}{y}.\frac{dy}{dx} = \frac{v}{u}.u' + v'.logu$

  $\frac{dy}{dx} = y\left[\frac{v}{u}..u' + v'.logu\right]$

  Note: In the above all formulae $logx = log_e x$

MULTIPLE CHOICE QUESTIONS

1) For what value of 'k' is the function $f(x) = \begin{cases} \frac{sin5x}{3x} + cosx, & if\; x \neq 0 \\ k \quad\quad , & if\; x = 0 \end{cases}$ continuous at x=0

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{8}{3}$ (d) $\frac{3}{8}$

(Ans: $k = \frac{8}{3}$)

2) For what value of k is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} 2x+1; & x < 2 \\ k; & x = 2 \\ 3x-1; & x > 2 \end{cases}$$

(a) $k = 2$ (b) $k = 5$ (c) $k = 1$ (d) $k = 3$

(Ans :$k = 5$)

3) The value of $\lim_{x \to 5^-} \frac{[x]}{x}$ is

(a) 1 (b) $\frac{4}{5}$ (c) $\frac{6}{5}$ (d) not defined

(Ans: $\frac{4}{5}$)

4) If $(x^2 + y^2)^2 = xy$, then $\frac{dy}{dx}$ is

(a) $\frac{y+4x(x^2+y^2)}{4y(x^2+y^2)-x}$ (b) $\frac{y-4x(x^2+y^2)}{x+4(x^2+y^2)}$

(c) $\frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$ (d) $\frac{4y(x^2+y^2)-x}{y-4x(x^2+y^2)}$

5) The function $f(x) = \begin{cases} \frac{e^{3x}-e^{-5x}}{x}, & if\ x \neq 0 \\ k\ , & if\ x = 0 \end{cases}$

Is continuous at x=0 for the value of k, as

(a) 3 (b) 5 (c) 2 (d) 8

6) The value of $\lim_{x \to o} \frac{\sqrt{(4+x)}-2}{x}$ is

(a) ∞ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 0

(Ans: $\frac{1}{4}$)

7) If $x = 2\cos\theta - \cos 2\theta\ and\ y = 2sin\theta - sin2\theta$, then $\frac{dy}{dx}$ is

(a) $\frac{\cos\theta+\cos 2\theta}{sin\theta-sin2\theta}$ (b) $\frac{\cos\theta-\cos 2\theta}{sin2\theta-sin\theta}$

(c) $\frac{\cos\theta-\cos 2\theta}{sin\theta-sin2\theta}$ (d) $\frac{\cos 2\theta-\cos\theta}{sin2\theta+sin\theta}$

8) Let $f(x) = \begin{cases} 1, & if\ x\ is\ rational \\ 0, & if\ x\ irrational \end{cases}$, then $\lim_{x \to o} f(x)$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) None of the above

(Ans: 1)

9) Differentiate of $log[log(logx^5)]$ w.r.t x is

(a) $\frac{5}{xlog(x^5)log(logx^5)}$ (b) $\frac{5}{xlog(logx^5)}$

(c) $\frac{5x^4}{log(x^5)log(logx^5)}$ (d) $\frac{5x^4}{logx^5log(logx^5)}$

10) If $siny = x\cos(a+y)\ , then$ $\frac{dx}{dy}$ is

(a) $\frac{\cos a}{cos^2(a+y)}$ (b) $\frac{-\cos a}{cos^2(a+y)}$

(c) $\frac{\cos a}{sin^2y}$ (d) $\frac{-\cos a}{sin^2y}$

11) If a function f is defined by

$$f(x) = \begin{cases} \frac{kcosx}{\pi - 2x}, & if\ x \neq \frac{\pi}{2} \\ 3\ , & if\ x = \frac{\pi}{2} \end{cases}$$

is continous at $x = \frac{\pi}{2}$, then the value of k is

(a) 2 (b) 3 (c)6 (d) -6

CONTINUITY

1) Find the value of a and b, if f(x) = $\begin{cases} 5, & if\ x \le 2 \\ ax+b, & if\ 2 < x < 10 \\ 21, & if\ x \ge 10 \end{cases}$ is a continuous function at $x = 2\ and\ x = 10$

2) Find the relationship between 'a' and 'b' so that the function 'f' defined by is
$f(x) = \begin{cases} ax+1 & if \quad x \le 3 \\ bx+3 & if \quad x > 3 \end{cases}$ $continuous\ at\ x = 3.$

3) Find the value of a and b, if f(x) = $\begin{cases} 3ax+b, & if\ x > 1 \\ 11 & , if\ x = 1 \\ 5ax-2b, & if\ x < 1 \end{cases}$ is a continuous function at $x = 1$ (Ans: $a = 3, b = 2$)

4) Find $a, b, c\ if\ f(x)$ is continuous at $x = 0$, $f(x) = \begin{cases} \frac{\sin(a+1)x+sinx}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$

(Ans: $a = -\frac{3}{2}$, $c = \frac{1}{2}$ $and\ b\ is\ any\ arbitrary\ value$)

5) For what value of k , the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} \frac{1-cos4x}{8x^2}, & x \ne 0 \\ k, & x = 0 \end{cases}$$

6) Discuss the continuity of the function $f(x) = \begin{cases} \frac{Sinx}{x} & if \quad x < 0 \\ x+1 & if \quad x \ge 0 \end{cases}$

7) Find the value of k so that the function $f(x) = \begin{cases} \frac{1-Coskx}{xSinx} & if, x \ne 0 \\ \frac{1}{2} & if, x = 0 \end{cases}$ iscontinuous at$x = 0$.

8) Find the value of k, for which $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & if\ -1 \le x < 0 \\ \frac{2x+1}{x-1} & if\ 0 \le x < 1 \end{cases}$ is continuous at $x = 0$.

(Ans : $k = -1$)

9) Show that the function f given by: $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & if x \ne 0 \\ -1, & if x = 0 \end{cases}$ is discontinuous at $x = 0$.

10) Examine the continuity of the function $f(x) = \begin{cases} \frac{|x|}{2x} & for\ x \ne 0 \\ \frac{1}{2} & for\ x = 0 \end{cases}$ $at\ x = 0$

11) Show that the function $f(x) = 2x - |x|\ is\ continuous\ at\ x = 0$

12) Given $f(x) = \begin{cases} \frac{1-cos4x}{x^2}, & if\ x < 0 \\ a, & if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16-\sqrt{x}-4}}, & if\ x > 0 \end{cases}$ is continuous at x = 0, find the value of a.

13) Let $f(x) = \begin{cases} \frac{1-sin^3x}{3cos^2x}, & if\ x < \frac{\pi}{2} \\ a, & if\ x = \frac{\pi}{2} \\ \frac{b(1-sinx)}{(\pi-2x)^2}, & if\ x > \frac{\pi}{2} \end{cases}$ is continuous function at $x = \frac{\pi}{2}$, find a and b.

(Ans: $a = \frac{1}{2}$, $b = 4$)

14) For what value of 'a' for which the function $f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1), & if\ x \le 0 \\ \frac{tanx - sinx}{x^3}, & if\ x > 0 \end{cases}$ is continuous at x = 0. (Ans: $a = \frac{1}{2}$)

DIFFERENTIATION

Differentiation by using Inverse TrigonometricFunctions

1) If $y = tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$, show that $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$
2) Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$with respect to $cos^{-1}x^2$.
3) Find $\frac{dy}{dx}$ if $y = tan^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right)$
4) Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w. r.t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.
5) If $y = sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right], find\ \frac{dy}{dx}$.
6) If $y = cos^{-1}\left[\frac{3x+4\sqrt{1-x^2}}{5}\right], find\,\frac{dy}{dx}$.
7) Differentiate $sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$ with respect to x.
8) Find the derivative of $sec^{-1}\left(\frac{1}{2x^2-1}\right) w.r.t\ \sqrt{1-x^2} at\ x = \frac{1}{2}$.
9) Find: $\frac{d}{dx}cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$

Differentiation by using Logarithms

1) If $x^y = e^{x-y}, show\ that\ \frac{dy}{dx} = \frac{logx}{\{log(xe)\}^2}$
2) If $xy = e^{(x-y)}$,then show that$\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.
3) Find $\frac{dy}{dx}$, if $y = x^{\cos x} + 2^x$
4) If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$
5) Find $\frac{dy}{dx}$ when $y = x^{\log x} + (\log x)^x$
6) If $y = x^x + (sin\,x)^x$ $find\ \frac{dy}{dx}$.
7) If $(\cos x)^y = (\sin y)^x, find\,\frac{dy}{dx}$.
8) Find $\frac{dy}{dx}$,when $y = x^{\cot x} + (cosx)^{\sin x}$

9) Find $\frac{dy}{dx}$ if y = $sinx^{tanx} + cosx^{secx}$

10) Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

11) Differentiate $(sin2x)^x + sin^{-1}\sqrt{3x}$ with respect to x.

12) If $y = x^{e^{-x^2}}, find \frac{dy}{dx}$.

13) If $x^x + x^y + y^x = a^b, then\ find\ \frac{dy}{dx}$.

14) If $x^p y^q = (x+y)^{p+q}$, then find $\frac{dy}{dx}$

15) Find the derivative of the following functions $f(x)\ w.r.t. x, at\ x = 1$

$$f(x) = cos^{-1}\left[sin\sqrt{\frac{1+x}{2}}\right] + x^x$$

16) Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x

Parametric forms

1) Find $\frac{dy}{dx}\ at\ t = \frac{2\pi}{3}\ when\ x = 10(t - sint) and\ y = 12(1 - cost)$.

2) Find $\frac{dy}{dx}$, if $y = e^{\theta}\left(\theta - \frac{1}{\theta}\right)\ and\ y = e^{-\theta}\left(\theta + \frac{1}{\theta}\right)$

3) If $x = a\left(cos\theta + \log tan\frac{\theta}{2}\right)\ and\ y = a\ sin\theta$, find the value of $\frac{dy}{dx} at\ \theta = \frac{\pi}{4}$.

4) If x = a (θ + sin θ) , y = a (1 − cos θ) , find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

5) If $x = a\ cos^3\theta\ and\ y = asin^3\theta$, then find the value of $\frac{d^2y}{dx^2} at\ \theta = \frac{\pi}{6}$.

6) Find the value of $\frac{dy}{dx}\ at\ \theta = \frac{\pi}{4}\ if\ x = ae^{\theta}(sin\theta - cos\theta)\ and$
$y = ae^{\theta}(sin\theta + cos\theta)$.

7) If $x = a(\cos 2t + 2t\sin 2t)\ and\ y = a(\sin 2t - 2t\cos 2t), then\ find\ \frac{d^2y}{dx^2}$.

8) If $x = \alpha\sin 2t(1 + \cos 2t)\ and\ y = \beta\cos 2t(1 - \cos 2t),\ show\ that\ \frac{dy}{dx} = \frac{\beta}{\alpha}\tan t$.

9) If $x = e^{\cos 2t}\ and\ y = e^{sin2t}, prove\ that\ \frac{dy}{dx} = -\frac{y\log x}{x\log y}$

Implicit Functions

1) Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$

2) Find $\frac{dy}{dx}$, if $xy + y^2 = tan\,x + y$.

3) If $\sin y = x\sin(a + y)$, prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$

4) *If* $x\sqrt{1+y} + y\sqrt{1+x} = 0$ *Prove that* $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

5) If $\cos y = x\cos(a+y), with \cos a \neq \pm 1,$ prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

6) If $log\sqrt{x^2 + y^2} = tan^{-1}\left(\frac{x}{y}\right), then\ show\ that\ \frac{dy}{dx} = \frac{y-x}{y+x}$.

7) If $\frac{x}{x-y} = log\frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

Second order Derivatives

1) If y = $3\,e^{2x} + 2\,e^{3x}$, prove that $\frac{d^2\,y}{dx^2} - 5\frac{dy}{dx}$ + 6y = 0

2) If $y = 2\cos(\log x) + 3\sin(\log x), prove\ that\ x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

3) If $y = \frac{\log x}{x}$, find $\frac{d^2y}{dx^2}$

4) If $y = e^{asin^{-1}x}, -1 \le x \le 1$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

5) If $x = tan\left(\frac{1}{a}\log y\right), show\ that\ (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0.$

6) If $(ax+b)e^{y/x} = x$, then show that:$x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2$

7) If $\log y = tan^{-1}x$, then show that$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0.$

8) If $y = log\left(\frac{x}{a+bx}\right)^x, prove\ that\ x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2.$

9) If $y = \sqrt{x+1} - \sqrt{x-1}, prove\ that\ (x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{1}{4}y = 0.$

10) If $x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta, show\ that\ y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$

11) If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y.$

12) If $y = x^3log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$

13) If $y = x^x, prove\ that\ \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

14) If $x\cos(a+y) = \cos y\ then\ prove\ that\ \frac{dy}{dx} = \frac{cos^2(a+y)}{\sin a}$ Hence show that $sin\ a\frac{d^2y}{dx^2} + sin\,2(a+y)\frac{dy}{dx} = 0.$

Other Problems

1) If $y = x|x|,\ find\frac{dy}{dx}\ \ for\ \ x < 0.$

2) If $y = sin^{-1}x + cos^{-1}x,\ find\frac{dy}{dx}.$

3) If $y = log(\cos e^X)$, then find $\frac{dy}{dx}.$

4) If $y = 5e^{7x} + 6e^{-7x}, show\ that\ \frac{d^2y}{dx^2} = 49y.$

5) If $f(x) = x+1, find\ \frac{d}{dx}(fof)(x)$

6) If $log(x^2+y^2) = 2\,tan^{-1}\left(\frac{y}{x}\right), show\ that\ \frac{dy}{dx} = \frac{x+y}{x-y}.$

7) If $x^y - y^x = a^b, find\frac{dy}{dx}.$

8) If $y = (sin^{-1}x)^2, prove\ that\ (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 - 0.$

9) If $x\sqrt{1+y} + y\sqrt{1+x} = 0\ and\ x \ne y, prove\ that\ \frac{dy}{dx} = -\frac{1}{(x+1)^2}.$

10) If $(\cos x)^y = (\sin y)^x, find\ \frac{dy}{dx}.$

11) If $(x-a)^2+(y-b)^2=c^2, for\ some\ c>0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

12) If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{sin^2(a+y)}{\sin a}$.

13) If $(\sin x)^y = x+y, find\ \frac{dy}{dx}$.

14) If $y=(sec^{-1}x)^2, x>0, show\ that\ x^2(x^2-1)\frac{d^2y}{dx^2}+(2x^3-x)\frac{dy}{dx}-2=0$

15) If $x=\sin t, y=\sin pt$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$.

16) Differentiate $tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to$cos^{-1}x^2$.

17) If $y=(x)^{\cos x}+(\cos x)^{\sin x}, find\ \frac{dy}{dx}$.

18) If $x^py^q=(x+y)^{p+q}, prove\ that\ \frac{dy}{dx}=\frac{y}{x}\ and\ \frac{d^2y}{dx^2}=0$.

19) Differentiate $tan^{-1}\frac{3x-x^3}{1-3x^2}, |x|<\frac{1}{\sqrt{3}} w.r.t. tan^{-1}\frac{x}{\sqrt{1-x^2}}$.

20) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y), |x|<1, |y|<1, show\ that\ \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

21) If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx}+y=0$

22) If $y=x\sqrt{1+x^2}+\log\left[x+\sqrt{x^2+1}\right]$, then show that $\frac{dy}{dx}=2\sqrt{x^2+1}$

23) If $f(x)=\sqrt{x^2+1}; g(x)=\frac{x+1}{x^2+1} and\ h(x)=2x-3, then\ find\ f'[h'\{g'(x)\}]$.

24) Let $f(x)=x-|x-x^2|, x\in[-1,1]$.find the point of discontinuity, (if any), of this function on [-1,1].

25) If function $f(x)=|x-3|+|x-4|$,then show that $f(x)$ is not differentiable at$x=3\ and\ x=4$.

26) Show that the function$f(x)=|x-1|+|x+1|$, for all $x\in R$, is not differentiable at the points $x=-1\ and\ x=1$.

27) Find whether the following function is differentiable at $x=1\ and\ x=2$ or not:

$$f(x)=\begin{cases} x, & x<1 \\ 2-x, & 1\le x\le 2 \\ -2+3x-x^2 & x>2 \end{cases}$$

28) Examine the following function $f(x)$ for continuity at $x=1$ and differentiability at $x=2$.

$$f(x)=\begin{cases} 5x-4, & 0<x<1 \\ 4x^2-3x, & 1\le x<2 \\ 3x+4, & x\ge 2 \end{cases}$$

APPLICATION OF DERIVATIVES

FORMULAE AND DEFINATIONS

RATE OF CHANGE OF QUANTITIES

1) Area of circle (A) = πr^2

   Rate of change of area = $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

2) Circumference of circle (C) = $2\pi r$

   Rate of change of Circumference = $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

3) Perimeter of a rectangle (P) = $2(x + y)$, where x = length , y = width

   Rate of change of Perimeter = $\frac{dP}{dt} = 2(\frac{dx}{xt} + \frac{dy}{dt})$

4) Area of rectangle (A) = $x.y$, where x = length , y = width

   Rate of change of area = $\frac{dA}{dt} = x.\frac{dy}{dt} + y.\frac{dx}{dt}$

5) Volume of cube (V) =x^3, where x = edge of cube

   Rate of change of Volume = $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

6) Surface area of cube (S) = $6x^2$

   Rate of change of Surface area = $\frac{dS}{dt} = 6x \frac{dx}{dt}$

7) Volume of sphere (V) = $\frac{4}{3}\pi r^3$

   Rate of change of Volume = $\frac{dV}{dt} = \frac{4}{3}(3\pi r^2)\frac{dr}{dt}$

8) Surface area of Sphere (S) = $4\pi r^2$

   Rate of change of Surface area = $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

9) Total cost = C(x), where C(x) isi Rupees of the production of x units

   Marginal cost = $\frac{dC}{dx}$

10) Total Revenue = R(x)

   Marginal Revenue = $\frac{dR}{dx}$

INCREASING AND DECREASING FUNCTION

- Let I be an interval contained in the domain of a real valued function f. Then f is said to be

(i) increasing on I $if\ x < y\ in\ I\ then\ f(x) \le f(y), for\ all\ x, y \in\ I.$

(ii) strictly increasing on I $if\ x < y\ in\ I\ then\ f(x) < f(y), for\ all\ x, y \in\ I$

(iii) decreasing on I $if\ x < y\ in\ I\ then\ f(x) \ge f(y), for\ all\ x, y \in\ I.$

(iv) strictly decreasing on I $if\ x < y\ in\ I\ then\ f(x) > f(y), for\ all\ x, y \in\ I$

- (a) f is strictly increasing in $(a, b)\ if\ f'(x)\ > 0\ for\ each\ x \in (a, b)$

  (b) f is strictly decreasing in $(a, b)\ if\ f'(x)\ < 0\ for\ each\ x \in (a, b)$
- A function will be increasing or decreasing in R if it is so in every interval of R
- f is a constant function in $[a, b]\ if\ f'(x)\ =\ 0\ for\ each\ x \in (a, b)$

TANGENTS AND NORMALS

Let given curve be y=f(x)

- Slope of tangent to the curve at $(x_1\ , y_1)\ is\ m = \left[\frac{dy}{dx}\right]_{x=x_1}$
- Slope of normal to the curve at $(x_1\ , y_1) = -\frac{1}{m}$
- Equation of tangent at $(\boldsymbol{x_1}\ , \boldsymbol{y_1})\ \boldsymbol{is\ y - y_1 = m(x - x_1)}$
- Equation of normal at $(\boldsymbol{x_1}\ , \boldsymbol{y_1})\ \boldsymbol{is\ y - y_1 = -\frac{1}{m}(x - x_1)}$

MAXIMA AND MINIMA

First Derivative Test:

Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

(i) If f '(x) changes sign from positive to negative as x increases through c, then c is a point of local maxima and maximum value of $f(x)\ =\ f(\ c)$.

(ii) If f '(x) changes sign from negative to positive as x increases through c, then c is a point of local minima and minimum value of $f(x)\ =\ f(\ c)$.

(iii) If f '(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called point of inflexion.

Second Derivative Test

Let f be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then

(i) $x\ =\ c$ is a point of local maxima if $f\ '(c)\ =\ 0$ and $f\ ''(c)\ <\ 0$ The values f (c) is local maximum value of f .

(ii) (ii) x = c is a point of local minima if $f\ '(c)\ =\ 0$ and $f\ ''(c)\ >\ 0$ In this case, f (c) is local minimum value of f .

(iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$. In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.

Absolute maxima and absolute minima (maxima and minima in a closed interval)

- Given $f(x)$ and interval $[a, b]$
- Find $f'(x)$
- Let $f'(x) = 0$
- Find critical values. (i.e find the values of x if $f'(x) = 0$). say

  $x = x_1$ and $x = x_2$
- Calculate $f(a), f(x_1), f(x_2)$ and $f(b)$.
- Identify maxima and minima values of f(x).

MULTIPLE CHOICE QUESTIONS

1) The rate of change of the area of a circle with respect to its radius r at $r = 6\ cm$ is
   a) 10π
   b) 12π
   c) 8π
   d) 11π
2) The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is
   a) 116
   b) 96
   c) 90
   d) 126
3) Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?
   a) $\cos x$
   b) $\cos 2x$
   c) $\cos 3x$
   d) $\tan x$
4) The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing on
   (a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$
   (c) $(-\infty, 2] \cup [3, \infty)$ (d) $[3, \infty)$
5) On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing?
   a) $(0,1)$
   b) $\left(\frac{\pi}{2}, \pi\right)$
   c) $\left(0, \frac{\pi}{2}\right)$
   d) None of these
6) The function $y = x^2 e^{-x}$ is decreasing in the interval
   (a) (0, 2) (b) $(2, \infty)$
   (c) $(-\infty, 0)$ (d) $(-\infty, 0) \cup (2, \infty)$

7) The interval in which $y = x^2e^{-x}$ is increasing is
   a) $(-\infty, \infty)$
   b) $(-2,0)$
   c) $(2, \infty)$
   d) $(0,2)$
8) The slope of the normal to the curve $y = 2x^2 + 3\sin x \; at \; x = 0$ is
   a) 3
   b) $\frac{1}{3}$
   c) -3
   d) $-\frac{1}{3}$
9) The equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) is
   (a) $2y - 3mx + am^3 = 0$
   (b) $2x + 3my - 3am^4 - am^2 = 0$
   (c) $2x + 3my - 3am^4 - 2am^2 = 0$
   (d) $2x + 3my + 3am^4 - 2am^2 = 0$
10) The equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses the x-axis is
   (a) $x - 5y = 2$
   (b) $5x - y = 2$
   (c) $x + 5y = 2$
   (d) $5x + y = 2$
11) The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point
   a) (1,2)
   b) (2,1)
   c) (1,-2)
   d) (-1,2)
12) For all real values of x, the maximum value of $\frac{1-x+x^2}{1+x+x^2}$ is
   a) 0
   b) 1
   c) 3
   d) $\frac{1}{3}$
13) The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \le x \le 1$ is
   a) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$
   b) $\frac{1}{2}$
   c) 1
   d) 0
14) A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of
   a) $1m^3/h$
   b) $0.1m^3/h$
   c) $1.1m^3/h$
   d) $0.5m^3/h$
15) The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2,-1) is
   a) $\frac{22}{7}$
   b) $\frac{6}{7}$
   c) $\frac{7}{6}$
   d) $\frac{-6}{7}$
16) The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$ if the value of m is

a) 1
b) 2
c) 3
d) $\frac{1}{2}$

17) The normal at the point (1,1) on the curve $2y + x^2 = 3$ is
a) $x + y = 0$
b) $x - y = 0$
c) $x + y + 1 = 0$
d) $x - y = 0$

18) The points on the curve $\frac{x^2}{9} + \frac{y^2}{25} = 1$, where tangent is parallel to $x - axis$ are
(a) $(\pm 5, 0)$ (b) $(0, \pm 5)$ (c) $(0, \pm 3)$ (d) $(\pm 3, 0)$

19) The maximum value of $\left(\frac{1}{x}\right)^x$ is
(a) $e^{1/e}$ (b) e (c) $\left(\frac{1}{e}\right)^{1/e}$ (d) e^e

20) The normal to the curve $x^2 = 4y$ passing (1,2) is
a) $x + y = 3$
b) $x - y = 3$
c) $x + y = 1$
d) $x - y = 1$

21) The absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $\left[-2, \frac{9}{2}\right]$ is
(a) 8 (b) 9 (c) 6 (d) 10

22) The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are
a) $\left(4, \pm\frac{8}{3}\right)$ b) $\left(4, \frac{-8}{3}\right)$ c) $\left(4, \pm\frac{3}{8}\right)$ d) $\left(\pm 4, \frac{3}{8}\right)$

SHORT/LONG ANSWER TYPE QUESTIONS

Rate of change of quantities

1) The side of an equilateral triangle is increasing at the rate of 2 cm/s. at what rate is its area increasing when the side of the triangle is 20 cm?
2) The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8\ cm\ and\ y = 6\ cm$, find the rate of change of a) the perimeter, b) the rate of the rectangle.
3) A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. how fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?
4) A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing? $(Ans: 80\pi cm^2/s)$
5) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?$\left(Ans: \frac{8}{3} cm/s\right)$

6) The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres ? $(Ans: 3.6\ cm^3/s)$

7) Sand is pouring from a pipe at the rate of 12 cm3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm? $\left(Ans: \frac{1}{48\pi} cm/sec\right)$

8) The total cost C(x) in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02\,x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output. $(Ans: Rs. 30.02)$

Increasing and Decreasing Functions

1) Find the intervals in which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ are (i) increasing (ii) decreasing.

2) Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ are (i) increasing (ii) decreasing.($Ans: Strictly\ increasing\ in\ (-2,-1)$ and $Strictly\ decreasing\ in\ (-\infty,-2) \cup (-1,\infty)$)

3) Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is strictly increasing or strictly decreasing.

4) Find the intervals in which $f(x) = \sin 3x - \cos 3x, 0 < x < \pi$, is strictly increasing or strictly decreasing

5) Find whether the function $f(x) = cos\left(2x + \frac{\pi}{4}\right)$; is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

6) Show that $y = \log(1 + x) - \frac{2x}{2+x}, x > -1$, is increasing of xthroughout its domain.

7) Find the intervals on which the function $f(x) = \sin x + \cos x,\ 0 \leq x \leq 2\pi$ is strictly increasing or decreasing..($Ans: Strictly\ increasing\ in\ [0,\frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$ and $Strictly\ decreasing\ in\ \left(\frac{\pi}{4},\frac{5\pi}{4}\right)$)

8) Find the values of x for which $f(x) = x^2(x-2)^2$ is an increasing function.

9) Find the intervals in which the function $f(x) = \frac{4\ sinx}{2+\cos x} - x; 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

Tangents and Normals

1) Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(-1,4)$.

2) Find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

3) Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

4) At what points will the tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to x-axis? Also, find the equations of tangents to the curve at those points.

5) The equation of tangent at (2,3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

6) Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 5$, which is parallel to the line $4x - 2y + 5 = 0$.

7) Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$.

8) Find the value of p for which the curves $x^2 = 9p(9-y)$ and $x^2 = p(y+1)$ cut each other at right angles.

9) Find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis $(Ans: 20y - x + 7 = 0)$

10) Find the equations of the tangents to the curve $3x^2 - y^2 = 8$, which pass through the point $\left(\frac{4}{3}, 0\right)$

11) Find the equations of the tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $\left(\sqrt{2}a, b\right)$.

12) Find the equation of tangent to thecurve given by $x = a\sin^3 t$ and $y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$ $(Ans: y = 0)$

Maxima and Minima

1) A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs.70 per square meter for the base and Rs. 45 per square meter for the sides, what is the cost of least expensive tank?

2) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

3) Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

4) An isosceles triangle of vertical angle 2 θ is inscribed in a circle of radius a. show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

5) Find the minimum value of $(ax + by)$, where $xy = c^2$.

6) Show that the volume of the greatest cylinder that can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

7) Show that the semi-vertical angle of the right circular cone of given slant height and maximum volume is $\tan^{-1}\sqrt{2}$.

8) Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

9) A window has the shape of a rectangle surmounted by a equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

10) Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

11) An open box with a square base is to be made out of a given quantity of sheet of area c^2. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.

12) Show that the altitude of a right circular cone of maximum volume which can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

13) Find the coordinates of a point of the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

14) Find the point on the curve $y = \frac{x}{1+x^2}$, where the tangent to the curve has the greatest slope.

15) Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with its vertex at one end of the major axis.

16) Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

17) Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3r}$.

18) Tangent to the circle $x^2 + y^2 = 4$ at any point on it in the first quadrant makes intercepts OA and OB on x *and* y axes respectively, O being the Centre of the circle. Find the minimum value of (OA + OB).

19) If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

20) Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r *is* $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

21) A wire of length 34 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a rectangle whose length is twice its breadth. What should be the lengths of the two pieces, so that the combined area of the square and the rectangle is minimum?

22) Show that semi-vertical angle of a cone maximum volume and given slant height is$cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

23) Find the maximum and minimum values of $f(x) = \sec x + \log cos^2 x, 0 < x < 2\pi$

24) Find the local maxima and minima, of the function $f(x) = \sin x - \cos x, 0 < x < 2\pi$. Also find the local maximum and local minimum values.

25) Find the absolute maximum and absolute maximum values of the function f given by $f(x) = cos^2\, x + \sin x, x \in [0, \pi]$.

26) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = sin^2 x - \cos x, x \in [0, \pi]$.

27) If the function $f(x) = 2x^3 - 9mx^2 + 12m^2 x + 1, where\ m > 0$ attains its maximum and minimum at p and q respectively such that$p^2 = q$, then find the value of m.

28) If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show the area of the triangle is maximum when the angle between them is $\frac{\pi}{3.}$.

29) A manufacturer can sell x items at a price of $Rs.\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is $Rs.\left(\frac{x}{5} + 500\right)$ Find the number of items he should sell to earn maximum profit.

30) If the length of three sides of a trapezium other than the base is 10 cm each, find the area of the trapezium, when it is maximum.

CHAPTER: INTEGRALS

FORMULAE AND DEFINITIONS

INDEFINITE INTEGRALS

1) $\int 1\, dx = x + c$

2) $\int x\, dx = \frac{x^2}{2} + c$

3) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$

4) $\int sinx\, dx = -cosx + c$

5) $\int cosx\, dx = sinx + c$

6) $\int tanx\, dx = log|secx| + c$

7) $\int cosecx\, dx = log|cosecx - cotx| + c$

8) $\int secx\, dx = log|secx + tanx| + c$

9) $\int cotx\, dx = log|sinx| + c$

10) $\int sec^2x\, dx = tanx + c$

11) $\int cosec^2x\, dx = -cotx + c$

12) $\int secx.tanx\, dx = secx + c$

13) $\int cosecx.cotx\, dx = -cosecx + c$

14) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c \quad or \quad -\cos^{-1} x + c$

15) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \ \ or \ \ -\cot^{-1} x + c$

16) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c \quad or \quad -\mathrm{cosec}^{-1} x + c$

17) $\int e^x\, dx = e^x + c$

18) $\int \frac{1}{x} dx = log|x| + c$

19) $\int a^x\, dx = a^x loga + c$

20) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} log\left|\frac{x-a}{x+a}\right| + c$

21) $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} log\left|\frac{a+x}{a-x}\right| + c$

22) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

23) $\int \frac{1}{\sqrt{x^2-a^2}} dx = log\left|x + \sqrt{x^2 - a^2}\right| + c$

24) $\int \frac{1}{\sqrt{x^2+a^2}} dx = log\left|x + \sqrt{x^2 + a^2}\right| dx = +c$

25) $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + c$

26) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + c$

27) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + c$

28) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

29) $\int e^x[f(x) + f'(x)]\, dx = e^x f(x) + c$

30) $\int u.v\, dx = u\int vdx - \int[u'\int vdx]\, dx + c, where\ u = u(x)\ and\ v = v(x)$

31) $\int[f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

32) $\int k.f(x)dx = k\int f(x)dx, where\ k\ is\ constant.$

Note: Let antiderivative of $f(x) = F(x)$

i.e. $\int f(x)dx = F(x) + c$, then

$\int f(ax + b)dx = \frac{1}{a}F(ax + b) + c$

Partial fractions

- The rational function $\frac{P(x)}{Q(x)}$ is said to be proper if the degree of $Q(x)$ is less than the degree of $P(x)$
- Partial fractions can be used only if the integrand is proper rational function

S.No	Form of rational function	Form of Partial fraction
1	$\frac{1}{(x-a)(x-b)}$	$\frac{A}{x-a} + \frac{b}{x-b}$
2	$\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{x-a} + \frac{b}{x-b}$
3	$\frac{1}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{b}{x-b} + \frac{1}{x-c}$
4	$\frac{px+q}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{b}{x-b} + \frac{1}{x-c}$
5	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{b}{x-b} + \frac{1}{x-c}$
6	$\frac{1}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
7	$\frac{px+q}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

8	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
9	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ where x^2+bx+ccannot be factorized further	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

Integral of the type

$\frac{Linear}{Quadratic}$, $\frac{Linear}{\sqrt{Quadratic}}$,$Linear\sqrt{Quadratic}$

$Linear = A\frac{d}{dx}(Quadratic) + B$

DEFINITE INTEGRALS

Properties Of Definite Integrals

1) $\int_a^a f(x)dx = 0$
2) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$
3) $\int_a^b f(x)\,dx = -\int_b^a f(x)dx$
4) $\int_a^b f(x)\,dx = \int_a^c f(x)dx + \int_c^b f(x)dx, where\ a < c < b$
5) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
6) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
7) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
8) $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx\ , if\ f(2a-x) = f(x) \\ 0\ , \quad if\ \ f(2a-x) = -f(x) \end{cases}$
9) $\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx\,,\ \ if\ f(x) is\ even. i.e\ f(-x) = f(x) \\ 0\ , \quad if\ f(x) is\ odd. i.e\ f(-x) = -f(x) \end{cases}$

MULTIPLE CHOICE QUESTIONS

1) The anti-derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$equals

(a) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (b) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(c) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (d) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

2) If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$is

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

3) $\int \frac{10x^9 + 10^x log_e{}^{10} dx}{x^{10} + 10^x}$ equals

(a) $10^x - x^{10} + C$ (b) $10^x + x^{10} + C$

(c) $(10^x - x^{10})^{-1} + C$ (d) $log(10^x + x^{10}) + C$

4) $\int \frac{dx}{sin^2 x cos^2 x}$ equals

(a) $\tan x + \cot x + C$ (b) $\tan x - \cot x + C$

(c) $\tan x \cot x + C$ (d) $\tan x - \cot 2x + C$

5) $\int \frac{sin^2 x - cos^2 x}{sin^2 x cos^2 x} dx$ is equal to

(a) $\tan x + \cot x + C$ (b) $\tan x + cosec\, x + C$

(c) $-\tan x + \cot x + C$ (d) $\tan x + \sec x + C$

6) $\int \frac{e^x(1+x)}{cos^2(e^x x)} dx$ equals

(a) $-cot(ex^x) + C$ (b) $tan(xe^x) + C$

(c) $tan(e^x) + C$ (d) $cot(e^x) + C$

7) $\int \frac{dx}{x^2 + 2x + 2}$ equals

(a) $xtan^{-1}(x+1) + C$ (b) $tan^{-1}(x+1) + C$

(c) $(x+1)tan^{-1}x + C$ (d) $tan^{-1}x + C$

8) $\int \frac{dx}{\sqrt{9x - 4x^2}}$ equals

(a) $\frac{1}{9} sin^{-1}\left(\frac{9x-8}{8}\right) + C$ (b) $\frac{1}{2} sin^{-1}\left(\frac{8x-9}{9}\right) + C$

(c) $\frac{1}{3} sin^{-1}\left(\frac{9x-8}{8}\right) + C$ (d) $\frac{1}{2} sin^{-1}\left(\frac{9x-8}{9}\right) + C$

9) $\int \frac{x\, dx}{(x-1)(x-2)}$ equals

(a) $log\left|\frac{(x-1)^2}{x-2}\right| + C$ (a) $log\left|\frac{(x-2)^2}{x-1}\right| + C$

(c) $log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$ (d) $log|(x-1)(x-2)| + C$

10) $\int \frac{dx}{x(x^2+1)}$ equals

(a) $log|x| - \frac{1}{2} log(x^2+1) + C$ (b) $log|x| + \frac{1}{2} log(x^2+1) + C$

(c) $-log|x| + \frac{1}{2} log(x^2+1) + C$ (d) $\frac{1}{2} log|x| + log(x^2+1) + C$

11) $\int x^2 e^{x^3} dx$ equals

(a) $\frac{1}{3} e^{x^3} + C$ (a) $\frac{1}{3} e^{x^2} + C$

(c) $\frac{1}{2} e^{x^3} + C$ (d) $\frac{1}{2} e^{x^2} + C$

12) $\int e^x \sec x(1 + \tan x) dx$ equals

(a) $e^x \cos x + C$ (b) $e^x \sec x + C$

(c) $e^x \sin x + C$ (d) $e^x \tan x + C$

13) $\int \sqrt{1+x^2}\, dx$ is equal to

(a) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2} log\left|\left(x + \sqrt{1+x^2}\right)\right| + C$ (b) $\frac{2}{3}(1+x^2)^{\frac{3}{2}} + C$

(c) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}} + C$ (d) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2 log\left|x + \sqrt{1+x^2}\right| + C$

14) $\int \sqrt{x^2 - 8x + 7}\, dx$ is equal to

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} + 9\, log\left|x - 4 + \sqrt{x^2-8x+7}\right| + C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7} + 9\, log\left|x + 4 + \sqrt{x^2-8x+7}\right| + C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - 3\sqrt{2}\, log\left|x - 4 + \sqrt{x^2-8x+7}\right| + C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2}\, log\left|x - 4 + \sqrt{x^2-8x+7}\right| + C$

15) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

16) $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

17) The value of the integral $\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$ is

(a) 6 (b) 0 (c) 3 (d) 4

18) If $f(x) = \int_0^x t\sin t\, dt,\ then\ f'(x)$is

(a) $cosx + x\sin x$ (b) $x\ sinx$ (c) $x\ cosx$ (d) $sinx + x\ cosx$

19) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^3 + x\ cosx + tan^5x + 1)dx$ is

(a) 0 (b) 2 (c) π (d) 1

20) The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) -2

21) $\int \frac{dx}{e^x+e^{-x}}$ is equal to

(a) $tan^{-1}(e^x) + C$ (b) $tan^{-1}(e^{-x}) + C$

(c) $\log(e^x - e^{-x}) + C$ (d) $log(e^x + e^{-x}) + C$

22) $\int \frac{cos2x}{(\sin x+\cos x)^2} dx$ is equal to

(a) $\frac{-1}{\sin x+\cos x} + C$ (b) $log|\sin x + \cos x| + C$

(c) $log|\sin x - \cos x| + C$ (d) $\frac{1}{(\sin x+\cos x)^2}$

23) If $f(a + b - x) = f(x), then\ \int_a^b x\, f(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b+x)dx$

(c) $\frac{b-a}{2}\int_a^b f(x)dx$ (d) $\frac{a+b}{2}\int_a^b f(x)dx$

24) The value of $\int_0^1 tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is

(a) 1 (b) 0 (c) -1 (d) $\frac{\pi}{4}$

SHORT/LONG ANSWER TYPE QUESTIONS

Using Partial Fractions

Evaluate the following integrals:

(1) $\int \frac{2cosx}{(1-sinx)(1+sin^2x)} dx$

Ans: $log\frac{\sqrt{1+sin^2x}}{1-\sin x} + \tan^{-1}(\sin x) + c$

$Hint: Let\ \sin x = t \Longrightarrow \cos x\ dx = dt$

$$\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}\ ,A = 1, B = 1, C = 1$$

(2) $\int \frac{4}{(x-2)(x^2+4)} dx$

(3) $\int \frac{cos\theta}{(4+sin^2\theta)(5-4cos^2\theta)} d\theta$

(4) $Ans: -\frac{1}{30}\tan^{-1}\left(\frac{sin\theta}{2}\right) + \frac{2}{15}\tan^{-1}(2sin\theta) + C$

(5) $\int \frac{2x}{(x^2+1)(x^2+2)^2}\ dx$

Ans: $log|x^2+1| - log|x^2+1| + \frac{1}{x^2+2} + C$

Hint: $Let\ x^2 = t,\quad 2x\,dx = dt,\quad \int \frac{dt}{(t+1).(t+1)^2} = \int \frac{dt}{t+1} - \int \frac{dt}{t+2} - \int \frac{dt}{(t+1)^2}$

(6) $\int \frac{e^x}{(2+e^x)(4+e^{2x})}\,dx$

$Ans: \frac{1}{8}log|2+e^x| - \frac{1}{16}log|4+e^{2x}| + \frac{1}{8}\tan^{-1}\left(\frac{e^x}{2}\right) + c$

(7) $\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$

Hint: $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

(8) $\int \frac{1}{(2-x)(x^2+3)}\,dx$

Hint: $\frac{1}{(2-x)(x^2+3)} = \frac{A}{2-x} + \frac{Bx+C}{x^2+3}$

(9) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$ $\left(Ans: \frac{3}{5}log|x+2| + \frac{1}{5}log|x^2+1| + \frac{1}{5}\tan^{-1}x + C\right)$

Hint: $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}\ , A = \frac{3}{5}, B = \frac{2}{5}, C = \frac{1}{5}$

(10) $\int \frac{3x-2}{(x+1)^2(x+3)}\,dx$ $\left(Ans: \frac{11}{4}log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)}\right)$

Hint: $\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}\ , A = \frac{11}{4}\ , B = \frac{-5}{2}\ , C = \frac{-11}{4}$

(11) Evaluate : $\int \frac{dx}{1+x+x^2+x^3}$

Hint : $\frac{1}{1+x+x^2+x^3} = \frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

(12) Evaluate $\int \frac{3x+1}{(x-2)^2(x+2)}\,dx.$

Hint: $\frac{3x+1}{(x-2)^2(x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$

(13) Evaluate : $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$

Hint: Let $x^2 = t, 2x\,dx = dt,\ \frac{1}{(t+1)(t+3)} = \frac{A}{t+1} + \frac{B}{t+3}$

(14) Evaluate : $\int \frac{x^2}{(1+x^3)(2+x^3)}\,dx$

Hint: Let $x^3 = t, 3x^2dx = dt, x^2dx = \frac{1}{3}dt\ , I = \frac{1}{3}\int \frac{dt}{(1+t)(2+t)}$

(15) Evaluate: $\int \frac{11x+8}{x^2(3x+8)}\,dx$

Hint: $\frac{11x+8}{x^2(3x+8)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x+8}$

(16) Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\,dx$ $\left(Ans: log\left|\frac{2-\sin x}{1-\sin x}\right| + C\right)$

Hint: Let $\sin x = t\ , \cos x\,dx = dt,\ \frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}, A = 1,\ B = -1$

(17) Evaluate $\int \frac{e^x}{(1+e^x)(2+e^x)}\,dx$

Hint: Let $e^x = t \Longrightarrow e^x dx = dt,\quad I = \int \frac{dt}{(1+t)(2+t)}$

(18) $\int \frac{(3sin\phi-2)cos\phi}{5-cos^2\phi-4\sin\phi}\,d\phi$ $\left(Ans: 3\log(2-\sin\emptyset) + \frac{4}{2-\sin\emptyset} + C\right)$

Hint: Let $x = \sin\emptyset, then\ I = \int \frac{3x-2}{(x-2)^2}\,dx\ ,\ \frac{3x-2}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}\ , A = 3,\ B = 4$

Improper Rational Functions

Evaluate the following integrals:

(1) $\int \frac{x^2+1}{(x-2)(x-3)}dx$ $(Ans: x-5log|x-2|+10log|x-3|+C)$

Hint: Divide Numerator by denominator, we get

$\frac{x^2+1}{(x-2)(x-3)} = \frac{x^2+1}{x^2-5x+6} = 1+\frac{5x-5}{(x-2)(x-3)} = 1+\left(\frac{-5}{x-2}\right)+\left(\frac{10}{x-3}\right)$

(2) $\int \frac{x^3+x^2+1}{x^2-1}dx$ $\left(Ans: \frac{x^2}{2}+x+\frac{1}{2}log|x^2-1|+log\left|\frac{x-1}{x+1}\right|+C\right)$

Hint: Divide Numerator by denominator, we get

$\frac{x^3 + x^2 + 1}{x^2-1} = \frac{(x^2-1)(x+1)+(x+2)}{x^2-1} = (x+1)+\frac{(x+2)}{x^2-1}$

$= (x+1)+\frac{x}{x^2-1}+\frac{2}{x^2-1}$

(3) Evaluate the following: $\int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Hint: $\int_1^2 \frac{5x^2}{x^2+4x+3}dx = 5\int_1^2 \frac{x^2}{x^2+4x+3}dx = 5\int_1^2 \frac{(x^2+4x+3)-(4x+3)}{x^2+4x+3}dx$

$=5\int_1^2 \frac{(x^2+4x+3)}{x^2+4x+3}dx - 5\int_1^2 \frac{4x+3}{x^2+4x+3}dx = 5\int_1^2 dx - 5\int_1^2 \frac{4x+3}{(x+1)(x+3)}dx$

$\frac{4x+3}{(x+1)(x+3)} = \frac{A}{x+1}+\frac{B}{x+3}$

(4) $\int \frac{x^4}{(x-1)(x^2+1)}dx \left(Ans: \frac{x^2}{2}+x+\frac{1}{2}log|x-1|-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+C\right)$

Hint: Divide Numerator by denominator, we get

$\frac{x^4}{(x-1)(x^2+1)} = \frac{x^4}{x^3-x^2+x-1} = (x+1)+\frac{1}{x^3-x^2+x-1}$

$\frac{1}{x^3-x^2+x-1} = \frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}, A=\frac{1}{2}, B=-\frac{1}{2}, C=-\frac{1}{2}$

Linear/Quadratic or Linear/$\sqrt{Quadratic}$ or Linear.$\sqrt{Quadratic}$

Evaluate the following integrals:

(1) $\int \frac{dx}{5-8x-x^2}$ Ans: $\frac{1}{2\sqrt{21}}log\left|\frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)}\right|+C$

(2) $\int \frac{dx}{\sqrt{3-2x-x^2}}$ Ans: $\sin^{-1}\left(\frac{x+1}{2}\right)+C$

(3) $\int \frac{dx}{3x^2+13x-10}$ $\left(Ans: \frac{1}{17}log\left|\frac{3x-2}{x+5}\right|+C,\ where\ C=\frac{1}{17}log\frac{1}{3}+C_1\right)$

Hint: $3x^2+13x-10 = 3\left[\left(x+\frac{13}{6}\right)^2-\left(\frac{17}{6}\right)^2\right]$

(4) Find: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

(5) $\int \frac{dx}{\sqrt{(x+5)(x+1)}}$

(6) $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

(7) $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx.$

(8) $\int \frac{2x+3}{\sqrt{x^2+4x+5}}dx.$

(9) $\int \frac{x+2}{\sqrt{(x-2)(x-3)}}dx$

(10) $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$

(11) $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

(12) $\int \frac{5x-2}{1+2x+3x^2}dx$

(13) $\int \sqrt{3-2x-x^2}dx$

(14) $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

Integrals using the formula $\int e^x\{f(x)+f'(x)\}dx = e^x.f(x)+c$

Evaluate the following integrals:

(1) Evaluate $\int \frac{(x-3)e^x}{(x-1)^3}dx$

(2) Evaluate:$\int \frac{(x-4)e^x}{(x-2)^3}dx$

(3) Evaluate $\int \left(\frac{1-\sin x}{1-\cos x}\right)e^x dx$

(4) Evaluate: $\int e^x\left(\frac{2+sin2x}{1+cos2x}\right)dx$

(5) Evaluate:$\int e^x\left(\frac{sin4x-4}{1-cos4x}\right)dx$

Integration by Parts

Evaluate the following integrals:

(1) $\int e^{2x}\sin(3x+1)dx$ Ans: $\frac{e^{2x}}{13}\{2\sin(3x+1)-3\cos(3x+1)\}+C$

(2) Evaluate: $\int x\, sin^{-1}x\, dx$

(3) $\int_0^1 x(tan^{-1}x)^2dx.$

(4) $\int \frac{x\,sin^{-1}x}{\sqrt{1-x^2}}dx$

(5) $\int x\, sin^{-1}x\, dx$

(6) $\int xcos^{-1}xdx$

(7) $\int (sin^{-1}x)^2dx$

(8) $\int tan^{-1}xdx$

(9) $\int \left[log(\log x)+\frac{1}{(\log x)^2}\right]dx$

(10) Evaluate :$\int_0^1 x(tan^{-1}x)^2dx.$

Properties of Integrals

Evaluate the following integrals:

(1) Evaluate$\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{sin^4x+cos^4x}dx$

(2) Evaluate : $\int_0^{\pi} \frac{xtanx}{\sec x+\tan x}dx$ Ans: $\frac{\pi(\pi-2)}{2}$

(3) Evaluate: $\int_1^4\{|x-1|+|x-2|+|x-4|\}dx$ Ans: $\frac{23}{2}$

(4) Evaluate: $\int_0^{\pi} \frac{x\,sinx}{1+cos^2x}dx$ Ans: $\frac{\pi^2}{4}$

(5) Evaluate: $\int_0^{3/2} |x \sin \pi x|\, dx$

Ans: $\int_0^1 x \sin \pi x\, dx - \int_1^{3/2} x \sin \pi x\, dx = \frac{2}{\pi} + \frac{1}{\pi^2}$

(6) Evaluate: $\int_{-2}^{1} |x^3 - x| dx$

Ans: $\int_{-2}^{-1} (x - x^3)\, dx + \int_{-1}^{0} (x^3 - x)\, dx - \int_0^1 (x^3 - x)\, dx = \frac{11}{4}$

(7) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, hence evaluate $\int_0^\pi \frac{x \sin x}{1+cos^2 x} dx$.

(8) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and hence evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$.

(9) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$.

(10) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, and hence evaluate $\int_0^1 x^2(1-x)^n\, dx$.

(11) Find: $\int_{-\frac{\pi}{4}}^{0} \frac{1+tanx}{1-tanx} dx$

(12) Evaluate $\int_0^{\frac{\pi}{4}} \log(1 + tanx)\, dx$

(13) Evaluate: $\int_0^{\frac{\pi}{2}} logsinx\, dx\, dx$.

(14) Evaluate: $\int_0^{\pi} \log(1 + cosx\,)\, dx$

(15) Evaluate: $\int_0^{\pi/2} (2\, logsin\, x - \log \sin 2x) dx$

(16) Evaluate : $\int_0^\pi \frac{x}{a^2 cos^2 x + b^2 sin^2 x} dx$ (Ans: $\frac{\pi^2}{2ab}$)

(17) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + cosx} dx$

(18) Evaluate : $\int_0^\pi \frac{x \sin x}{1+cos^2 x} dx$.

(19) Evaluate: $\int_0^\pi \frac{x}{1+sinx} dx$

(20) Evaluate: $\int_0^a sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$

(21) Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$.

(22) Evaluate: $\int_0^1 cot^{-1}(1 - x + x^2) dx$

(23) Evaluate : $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$

(24) Evaluate $\int_0^2 |x-1|\, dx$

(25) Evaluate: $\int_{-5}^{0} f(x)dx, where\ f(x) = |x| + |x+2| + |x+5|$

(26) $\int_{-1}^{\frac{3}{2}} |x\, sin(\pi\, x)| dx$ Ans: $\frac{3}{\pi} + \frac{1}{\pi^2}$

Other Integrals

Evaluate the following integrals:

(1) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{sinx + sosx}{16 + 9sin2x} dx$

(2) Show that: $\int_0^{\pi/2} \left(\sqrt{tanx} + \sqrt{cotx}\right) dx = \sqrt{2\pi}$

(3) $\int_0^{\frac{\pi}{4}} \left(\sqrt{tanx} + \sqrt{cotx}\right) dx = \frac{\pi}{\sqrt{2}}$

(4) $\int_0^{\frac{\pi}{2}} 2\sin x \cos x \, tan^{-1}(\sin x) dx$

(5) $\int \sin x . \sin 2x . sin3x dx$

(6) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}} dx$

(7) Evaluate: $\int \frac{Sin(x-a)}{Sin(x+a)} dx$

(8) Evaluate: $\int \frac{\sin x}{\sin(x+a)} dx$

(9) Evaluate $\int \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}} \, dx$

(10) Evaluate $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} \, dx$

(11) Evaluate: $\int \frac{dx}{5+4\cos x}$

(12) Evaluate: $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4} dx$

(13) Evaluate $\int \frac{1}{x^4+1} dx$

(14) Evaluate : $\int \sqrt{tanx} dx$

(15) $\int \frac{1}{1+\tan x} dx$

(16) $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha} dx$

(17) $\int \frac{1}{cos(x-a)cos(x-b)} dx$

(18) Find: $\int \frac{3-5\sin x}{cos^2 x} dx$

(19) Evaluate: $\int \frac{cos2x +2sin^2 x}{cos^2 x} \, dx$

(20) Find : $\int \frac{sin^2 x-cos^2 x}{sinxcosx} \, dx$

(21) Ans: $-log|sin2x| + c$ or $log|secx| - log|sinx| + c$

(22) Evaluate: $\int_2^3 3^x \, dx$

Ans: $\frac{18}{log3}$

(23) Evaluate $\int_0^{2\pi} cos^5 x \, dx$

Ans: $Zero$

(24) $Evaluate$: $\int_0^{\frac{\pi}{2}} (cos^2 \frac{x}{2} - sin^2 \frac{x}{2}) \, dx$

(25) Write the value of : $\int_0^1 \frac{2x}{1+x^2} dx$

(26) $\int e^x \left(tan^{-1}x + \frac{1}{1+x^2}\right) dx$

(27) Find the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ (Ans : Zero)

(28) Evaluate $\int \sqrt{x^2 - 4} \, dx$

(29) Write the value of $\int \frac{dx}{x^2+16}$.

(30) Evaluate: $\int \sqrt{4 + x^2} dx$

(31) Write the value of $\int e^{3\log x} . x^4 dx$ (Ans: $\frac{x^8}{8} + C$)

(32) Evaluate: $\int \frac{e^{4\log x} + e^{3\log x}}{e^{2\log x} + e^{\log x}} \, dx$ (Ans: $\frac{x^3}{3} + C$)

(33) Evaluate $\int e^x(\log x + \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3})dx$.

(34) Evaluate: $\int \frac{2cosx}{3sin^2x} dx$.

(35) Evaluate : $\int_0^{\frac{\pi}{4}} \sqrt{1 - sin2x} dx$

(36) Evaluate : $\int_0^1 \frac{2x}{1+x^2} dx$

(37) Evaluate : $\int \frac{x^2}{1+x^3} dx$

(38) Evaluate: $\int_0^1 \frac{dx}{1+x^2}$

(39) Evaluate: $\int \frac{x+cos6x}{3x^2+sin6x} dx$

(40) Evaluate: $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}} dx$.

(41) Evaluate: $\int \frac{cos\sqrt{x}}{\sqrt{x}}$ dx.

(42) Evaluate: $\int \frac{sin\sqrt{x}}{\sqrt{x}} dx$.

(43) Evaluate: $\int \frac{sec^2\sqrt{x}}{\sqrt{x}} dx$.

(44) Evaluate: $\int \frac{sec^2x}{3+tanx} dx$.

(45) If $\int_0^1 (3x^2 + 2x + k)dx = 0$,find the value of k

(46) Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} dx$

(47) Evaluate $\int sec^2(7 - x)dx$.

(48) Evaluate: $\int \frac{(1+\log x)^2}{x}$ dx.

(49) Evaluate: $\int \frac{\log x}{x} dx$.

(50) Evaluate: $\int \frac{1-x^2}{x(1-2x)} dx$.

(51) Evaluate : $\int \frac{dx}{\sqrt{1-x^2}}$

(52) Evaluate : $\int \frac{(\log x)^2}{x} dx$

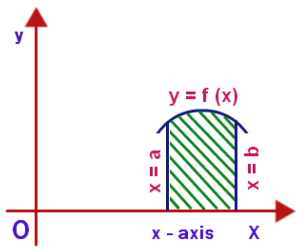
CHAPTER: APPLICATION OF INTEGRALS

- CURVE - LINE
  - CIRCLE – LINE
  - PARABOLA - LINE
  - ELLIPSE - LINE
- AREA OF TRIANGLE
  - CO-ORDINATES OF VERTICES ARE GIVEN
  - EQUATIONS OF SIDES ARE GIVEN

STEPS

- DRAW THE DIAGRAM
- MAKE A SHADED REGION
- FIND INTERSECTION POINTS
- IDENTIFY THE LIMITS
- WRITE THE INTEGRAL(S) FOR THE REGION
- EVALUATE THE INTEGRAL
- THE VALUE SHOULD BE POSITIVE

$Area\ of\ shaded\ region = \int_a^b f(x)dx$



MULTIPLE CHOICE QUESTIONS

1) Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0\ and\ x = 2$ is
   (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$
2) Area of the region bounded by the curve $y^2 = 4x, y$ −axis and the line $y = 3$ is
   (a) 2 (b) $\frac{9}{4}$ (c) $\frac{9}{3}$ (d) $\frac{9}{2}$
3) Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is
   (a) $2(\pi - 2)$ (b) $\pi - 2$ (c) $2\pi - 1$ (d) $2(\pi + 2)$
4) Area lying between the curves $y^2 = 4x\ and\ y = 2x$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

5) Area bounded by the curve $y = x^3$, the x −axis and the ordinates $x = -2$ and $x = 1$ is
(a) -9 (b) $\frac{-15}{4}$ (c) $\frac{15}{4}$ (d) $\frac{17}{4}$

6) The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by
(a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

LONG ANSWER TYPE QUESTIONS

1) Using integration, find the area of triangle ABC, whose vertices are A(2,5), B(4,7) and C(6,2).

2) Using the method of integration, find the area of the region bounded by the lines
$3x - 2y + 1 = 0, 2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

3) Using integration, find the area of triangle ABC bounded by the lines
$4x - y + 5 = 0, x + y - 5 = 0$ and $x - 4y + 5 = 0$.

4) Using method of integration, find the area of the triangle whose vertices are $(1,0), (2,2)$ and $(3,1)$.

5) Find the area of the triangle whose vertices are $(-1, 1), (0,5)$ and $(3,2)$, using integration.

6) Find the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

7) Find the area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$.

8) Find the area of the smaller region between the two curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x}{a} + \frac{y}{b} = 1$ in the first quadrant

9) Using integration, find the area of the triangular region with vertices (1, 0), (2,2) and (3, 1).

10) Using the method of integration, find the area of the region bounded by the lines
$2x + y = 4, 3x - 2y = 6$ and $x - 3y + 5 = 0$.

11) Using integration, find the area bounded by the lines $x + 2y = 2, y - x = 1$ and $2x + y = 7$

12) Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

13) Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$

14) Using the method of integration, find the area of the region bounded by the lines
$2x + y = 4,\ 3x - 2y = 6$ and $x - 3y + 5 = 0$

15) Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

16) Using integration, find the area of the triangle ABC, the coordinates of whose vertices are A (4, 1), B (6, 6) and C (8, 4).

17) Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

18) Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$.

CHAPTER: DIFFERENTIAL EQUATIONS

FORMULAE AND DEFINITIONS

Methods of solving First Order and First Degree Differential Equations

- Differential Equations with Variables seperables
- Homogeneous differential equations
- Linear differential equations.

Differential Equations with Variables separables

- Let the differential equation be $\frac{dy}{dx} = \frac{f(x)}{g(y)}$

  then $g(y)dy = f(x)dx$

  then integrate on both sides $\int g(y)dy = \int f(x)dx$
- Let the differential equation be $\frac{dy}{dx} = \frac{g(y)}{f(x)}$

  then$\frac{dy}{g(y)} = \frac{dx}{f(x)}$

  then integrate on both sides $\int \frac{dy}{g(y)} = \int \frac{dx}{f(x)}$
- Let the differential equation be $\frac{dy}{dx} = f(x).g(y)$

  then$\frac{dy}{g(y)} = f(x)dx$

  then integrate on both sides $\int \frac{dy}{g(y)} = \int f(x)dx$

Homogeneous differential equations

- A function $F(x, y)$ is said to be homogeneous function of degree n if

  $F(\lambda x, \lambda y) = \lambda^n F(x, y)$
- A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is saidto be homogeneous if F(x,y) is a homogeneous function of degree zero

  i.e. if $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$

Steps to solve the homogeneous differential equation of the type: $\frac{dy}{dx} = f(\frac{y}{x})$

- Let $y = vx$
- $\frac{dy}{dx} = v + x\frac{dv}{dx}$
- Substitute $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in $\frac{dy}{dx} = f(\frac{y}{x})$
- Then use variables and separables in terms of y and v only

Steps to solve the homogeneous differential equation of the type: $\frac{dx}{dy} = f(\frac{x}{y})$

- Let $x = vy$
- $\frac{dx}{dy} = v + y\frac{dv}{dy}$
- Substitute $x = vy$ and $\frac{dx}{dy} = v + y\frac{dv}{dy}$ in $\frac{dx}{dy} = f(\frac{x}{y})$

- Then use variables and separables in terms of x and v only

Lineardifferential equation

Steps to solve the Linear differential equation of the type: $\frac{dy}{dx} + P(x)y = Q(x)$

- $\frac{dy}{dx} + P(x)y = Q(x)$
- $Integratin\ Factor\ (IF) = e^{\int p(x)dx}$
- $Solution\ is\quad y.(IF) = \int (IF).Q(x)dx$

Steps to solve the Linear differential equation of the type: $\frac{dx}{dy} + P(y)x = Q(y)$

- $\frac{dx}{dy} + P(y)x = Q(y)$
- $Integratin\ Factor\ (IF) = e^{\int p(y)dy}$
- $Solution\ is\quad x.(IF) = \int (IF).Q(y)dy$

MULTIPLE CHOICE QUESTIONS

1) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin\left(\frac{dy}{dx}\right) + 1 = 0$ is
   (a) 3 (b) 2 (c) 1 (d) Not defined
2) The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$is
   (a) 2 (b) 1 (c) 0 (d) Not defined
3) The number of arbitrary constants in the general solution of a differential equation of fourth order are:
   (a) 0 (b) 2 (d) 3 (d) 4
4) The number of arbitrary constants in the particular solution of a differential equation of third order are:
   (a) 3 (b) 2 (c) 1 (d) 0
5) Which of the following differential equations has $y = c_1e^x + c_2e_x$ as the general solution?
   (a) $\frac{d^2y}{dx^2} + \mathrm{y} = 0$ (b) $\frac{d^2y}{dx^2} - \mathrm{y} = 0$ (c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$
6) Which of the following differential equations has $y = x$ as one of its particular solution?
   (a) $\frac{d^2y}{dx^2} - \mathrm{x}^2\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{xy} = \mathrm{x}$ (b) $\frac{d^2y}{dx^2} + \mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{xy} = x$
   (c) $\frac{d^2y}{dx^2} - \mathrm{x}^2\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{xy} = 0$ (d) $\frac{d^2y}{dx^2} + \mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{xy} = 0$
7) The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is
   (a) $e^x + e_y = \mathrm{C}$ (b) $e^x + e^y = C$ (c) $e_x + e^y = C$ (d) $e_x + e_y = C$
8) A homogeneous differential equation of the from $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution.
   (a) $y = vx$ (b) $v = yx$ (c) $x = vy$ (d) $x = v$
9) Which of the following is a homogeneous differential equation?
   (a) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$
   (b) $(xy)dx - (x^3 + y^3)dy = 0$
   (c) $(x^3 + 2y^2)dx + 2xy\ dy = 0$

(d) $y^2dx + (x^2 - xy - y^2)dy = 0$

10) The integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(a) e^x (b) e^y (c) $\frac{1}{x}$ (d) x

11) The integrating Factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay(-1 < y < 1)$is

(a) $\frac{1}{y^{2-1}}$ (b) $\frac{1}{\sqrt{y^2-1}}$ (c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{1-y^2}}$

SHORT ANSWER TYPE QUESTIONS

1) Find the order and the degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$.

2) Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2log\left(\frac{d^2y}{dx^2}\right)$$

3) Write the order and the degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$.

4) Write the order and degree of the differential equation $y = x\frac{d^2x}{dx^2} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

5) Find the order of the differential equation of the family of circles of radius 3 units.

LONG ANSWER TYPE QUESTIONS

6) Solve the following differential equation:$\frac{dy}{dx} + y = \cos x - \sin x$

7) Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

8) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

9) Solve the following differential equation:

$$(y + 3x^2)\frac{dy}{dx} = x.$$

10) Find the particular solution of the differential equation : $(1 + e^{2x})dy + (1 + y^2)e^xdx = 0$, given that $y(0) = 1$.

11) Find the particular solution of the differential equation:

$$x\frac{dy}{dx}sin\left(\frac{y}{x}\right) + x - y\ sin\left(\frac{y}{x}\right) = 0, given\ that\ y(1) = \frac{\pi}{2}$$

12) Solve the differential equation :$\frac{dy}{dx} = \frac{x+y}{x-y}$

13) Solve the differential equation : $(1 + x^2)dy + 2xy\,dx = \cot x\,dx$

14) Solve the differential equation : $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$

15) Solve the differential equation: $\frac{dy}{dx} = -\left[\frac{x+y\cos x}{1+\sin x}\right]$

16) Solve the differential equation : $x\,dy - y\,dx = \sqrt{x^2 + y^2}dx$, given that $y = 0\ when\ x = 1$.

17) Solve the differential equation: $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$, subject to the initial condition $y(0)=0$.

18) Solve the following differential equation : $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$

19) Solve the following differential equation : $x^2\frac{dy}{dx}=y^2+2xy$,given that y = 1, when x = 1.

20) Solve the following differential equation: $\frac{dy}{dx}+2y\tan x=\sin x$.

21) Solve the following differential equation: $cos^2x\frac{dy}{dx}+y=\tan x$.

22) Solve the following differential equation :
$(x^2-y^2)dx+2\,xy\,dy=0$ given that $y=1$ when $x=1$.

23) Solve the following differential equation: $\frac{dy}{dx}=\frac{x(2y-x)}{x(2y+x)}, if\ y=1$ When $x=1$

24) Solve for the following differential equation : $x\frac{dy}{dx}+y=x\log x; x\neq 0$.

25) Solve the following differential equation: $(3xy+y^2)dx+(x^2+xy)dy=0$.

26) Solve the following differential equation: $(1+x^2)\frac{dy}{dx}+y=tan^{-1}x$.

27) Solve the following differential equation: $x\log x\frac{dy}{dx}+y=2\log x$.

28) Solve the following differential equation: $\frac{dy}{dx}+2y\tan x=\sin x$.

29) Solve the following differential equation: $cos^2x\frac{dy}{dx}+y=\tan x$.

30) Solve the following differential equation: $(1+x^2)\frac{dy}{dx}+y=tan^{-1}x$.

31) Solve the following differential equation: $x\log x\frac{dy}{dx}+y=2\log x$.

32) Solve the following differential equation : $x\frac{dy}{dx}=y-x\,tan\left(\frac{y}{x}\right)$

33) Solve the following differential equation: $\frac{dy}{dx}+y=\cos x-\sin x$

34) Find the particular solution, satisfying the given condition, for the following differential equation : $\frac{dy}{dx}-\frac{y}{x}+cosec\left(\frac{y}{x}\right)=0; y=0\ when\ x=1$.

35) If $y=3e^{2x}+2e^{3x}$,prove that $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$.

36) If $y=e^x(\sin x+\cos x)$,prove that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$.

37) Solve the following differential equation: $(x^2-1)\frac{dy}{dx}+2xy=\frac{1}{x^2-1}; |x|\neq 1$

38) Solve the following differential equation: $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

39) Show that the differential equation $(x-y)\frac{dy}{dx}=x+2y$, is homogeneous. Solve it.

40) Find the general solution of the differential equation $x\log x.\frac{dy}{dx}+y=\frac{2}{x}.\log x$

41) Find the particular solution of the differential equation satisfying the given conditions:
$\frac{dy}{dx}=y\tan x$, given that $y=1$, when $x=0$.

42) Find the particular solution of the differential equation satisfying the given conditions: $x^2dy+(xy+y^2)dx=0, given\ that\ x=1, when\ y=1$.

43) Solve the following differential equation : $x\,dy-y\,dx=\sqrt{x^2+y^2}dx$

44) Solve the following differential equation : $x\,dy-(y+2x^2)\,dx=0$

45) Find the particular solution of the differential equation
$2y\,e^{x/y}\,dx+(y-2x\,e^{x/y})\,dy=0$, given that x = 0 when y = 1.

46) Solve the following differential equation : $x\frac{dy}{dx} + y - x + xy\cot x = 0, x \neq 0$

47) Solve: $x\frac{dy}{dx} - y = \sqrt{x^2 - y^2}$, $x > 0$

48) Solve the differential equation $(x + y)dy + (x - y)dx = 0$.given that $y(0) = 1$

49) Solve: $\frac{dy}{dx} = \cos(x + y)$.

50) Solve the following differential equation: $xy(x + y)dy = (x^3 + y^3)dx$

51) Verify that $y = e^x \cdot \cos x$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

52) Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$.

53) Solve the differential equation $x\frac{dy}{dx} . - y + x \, \text{Sin}(\frac{y}{x}) = 0$

54) Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$.

55) Solve the differential equation $(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$,given that when $x = 1$ then $y = 1$.

56) Solve, $y\,dx - (x + 2y^2)\,dy = 0$

57) Solve the differential equation: $x^2\frac{dy}{dx} = y^2 + 2xy$, given that y=1when x = 1.

58) Solve the differential equation: $\frac{dy}{dx} = (4x + y + 1)^2$

CHAPTER: VECTORS

MULTIPLE CHOICE QUESTIONS

1) If $\bar{a}$ and $\bar{b}$ are two collinear vectors, then which of the following are incorrect:
   (a) $\bar{b} = \lambda\bar{a}$, for some scalar λ
   (b) $\bar{a} = \pm\bar{b}$
   (c) The respective components of $\bar{a}$ and $\bar{b}$ are proportional
   (d) Both the vectors $\bar{a}$ and $\bar{b}$ have same direction, but different magnitudes.

2) If $\bar{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\bar{a}$ is unit vector if
   (a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = |\lambda|$ (d) $a = 1|\lambda|$

3) If θ is the angle between any two vectors $\bar{a}$ and $\bar{b}, then |\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$ when θ is equal to
   (a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

4) If $|\vec{a}| = 13$, $|\vec{b}| = 19$ and $|\vec{a} + \vec{b}| = 24$ find $|\vec{a} - \vec{b}|$
   (a) 31 (b) 22 (c) 37 (d) 5

SHORT/LONG ANSWER TYPE QUESTIONS

1) If the sum of two units vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

2) Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}, where$ $\vec{a} = \hat{\imath} - 7\hat{\jmath} - 7\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

3) The scalar product of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

4) Using vectors, prove that the points $(2,-1,3), (3,-5,1)$ and $(-1,11,9)$ are collinear.

5) For any two vectors $\vec{a}$ and $\vec{b}$ prove that $(\vec{a} \times \vec{b})^2 = \overrightarrow{a^2}\,\overrightarrow{b^2} - (\vec{a}.\vec{b})^2$

6) **X** and **Y** are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point **Z** which divides the line segment **XY** in the ratio 2 : 1 externally.

7) Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

8) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ $along$ $\vec{a}$ is equal to the projection of $\vec{c}$ $along$ $\vec{a}$; and $\vec{b}, \vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

9) Let $\vec{a} = 4i + 5j - k, \vec{b} = i - 4j + 5k$ and $\vec{c} = 3i + j - k$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a} = 21$.

10) Show that the points A, B, C with position vectors $2i - j + k, i - 3j - 5k$ and $3i - 4j - 4k$ respectively, are the vertices of a right-angled triangle.

11) If $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = 225$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$

12) Find unit vector in the direction of vector $\vec{a} = 2i + 3j + k$. $\left(Ans: \frac{2}{\sqrt{14}} i + \frac{3}{\sqrt{14}} j + \frac{1}{\sqrt{14}} k\right)$

13) Find λ, if $\lambda(\hat{\imath} + \hat{\jmath} + \hat{k})$ has the magnitude $(\ Ans : \lambda = \pm 1\ )$

14) Find the unit vector in the direction of the sum of the vectors

$\vec{a} = 2i + 2j - 5k$ and $\vec{b} = 2i + j + 3k$. $\left(Ans: \frac{4}{\sqrt{29}} i + \frac{3}{\sqrt{29}} j - \frac{2}{\sqrt{29}} k\right)$

15) Find a vector in the direction of the vector $5i - j + 2k$ which has magnitude 8 units.

$$\left(Ans: \frac{40}{\sqrt{30}} i - \frac{8}{\sqrt{30}} j + \frac{16}{\sqrt{30}} k\right)$$

16) If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

17) Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $i + 2j - k$ and $i + j + k$ respectively, in the ratio 2:1

(i) Internally (ii) externally (Ans: (i) $-\frac{1}{3} i + \frac{4}{3} j + \frac{1}{3} k$ (ii) $-3i + 3k$)

18) Find the position vectors of the points which divide internally and externally in the ratio 2: 3, the join of the points with position vectors $2\vec{a} - 3\vec{b}$ and $3\vec{a} - 2\vec{b}$.

19) Verify that the points A,B and C with posit $\vec{a} = 3i - 4j - 4k, \vec{b} = 2i - j + k$ and $\vec{c} = i - 3j - 5k$, respectively form the vertices of a right angled triangle.

20) Find the projection of the vector $\vec{a} = 2i + 3j + 2k$ on the vector $\vec{b} = i + 2j + k$

$\left(\text{Ans:} \frac{5}{3}\sqrt{6}\right)$

21) Find the projection of $\vec{b} + \vec{c}$ on $\vec{a}, where \vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

22) Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$.(Ans: $\sqrt{5}$)

23) If $\vec{a} = 5i - j - 3k$ and $\vec{b} = i + 3j - 5k$,then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

24) Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}, where$ $\vec{a} = i + j + k, \vec{b} = i + 2j + 3k$ $\left(\text{Ans:} \frac{-1}{\sqrt{6}}i + \frac{2}{\sqrt{6}}j - \frac{1}{\sqrt{6}}k\right)$

25) If $\vec{a} = 5\hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \lambda\hat{k}$, find λ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

26) Write the value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors.

27) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$. (Ans:$5\sqrt{2}$)

28) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a},$ if $|\vec{a}| = 1, |\vec{b}| = 4$ and $|\vec{c}| = 2$ (Ans:$\frac{-21}{2}$)

29) Express the vector $\vec{a} = 2i + j - 3k$ as sum of two vectors such that one is parallel to the vector $\vec{b} = 3i - j$ and other is perpendicular to $\vec{b}$. (Ans:$\frac{3}{2}i - \frac{1}{2}j$ and $\frac{1}{2}i + \frac{3}{2}j - 3k$.)

30) Let $\vec{a} = i + 4j + 2k, \vec{b} = 3i - 2j + 7k$ and $\vec{c} = 2i - j + 4k$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$. (Ans: $\frac{1}{3}(160i - 5j - 70k)$)

31) The scalar product of the vector $i + j + k$ with a unit vector along the sum of vectors $2i + 4j - 5k$ and $\lambda i + 2j + 3k$ is equal to one. Find the value ofλ. (Ans:$\lambda = 1$)

32) If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

33) The dot product of a vector with the vectors $i - 3k, i - 2k$ and $i + j + 4k$ are 0, 5 and 8 respectively. Find the vector. (Ans: $15i - 27j + 5k$)

34) If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

35) If $\vec{a} = 2i + 2j + 3k, \vec{b} = -i + 2j + k$ and $\vec{c} = 3i + j$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find the value of λ. (Ans: $\lambda = 8$.)

36) If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

37) If the sides BC, CA and AB represent vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively, prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

CHAPTER:THREE DIMENSIONAL GEOMETRY

FORMULAE AND DEFINATION

- Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.
  Let a line making the angles with x, y, z axis are α, β, γ repectively.
  Direction cosines are $l = cos\alpha, m = cos\beta, \; n = cos\gamma$
- If $l, m, n$are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.
- Direction ratios of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are
  $$a = x_2 - x_1, \quad b = y_2 - y_1, \quad a = y_2 - y_1$$
- If l, m, n are the direction cosines and $a, b, c$are the direction ratios of a line then
  $$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$
- Direction cosines of a line joining two points
  $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$
  where PQ= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- Direction ratios of a line are the numbers which are proportional to the direction cosines of a line.
- Skew lines are lines in space which are neither parallel nor intersecting. They lie in different planes.
- Angle between skew lines is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.
- If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then $cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$
- If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two lines and θ is the acute angle between the two lines; then $cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$
- Equation of a line through a point (x_1, y_1, z_1) and having direction cosines l, m, n is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$
- Cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.
- If θ is the acute angle between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, $then$ $cos\theta = \left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$
- If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are the equations of two lines, then the acute angle between the two lines is given by $cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$.
- Shortest distance between two skew lines is the line segment perpendicular to both the lines.
- Shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is $\left|\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$
- Shortest distance between the lines: $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is
  - $$\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

❖ Distance between parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ is $\left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}|}\right|$

MULTIPLE CHOICE QUESTIONS

1) Distance between the two planes: $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is
   (a) 2 units (b) 4 units (c) 8 units (d) $\frac{2}{\sqrt{29}}$ units

2) The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are
   (a) Perpendicular (b) Parallel (c) Intersect y-axis (d) Passes through $\left(0,0,\frac{5}{4}\right)$

SHORT ANSWER TYPE QUESTIONS

1) If a line makes angles $90^0, 135^0, 45^0$ with the x, y and z axes respectively, find its direction cosines.
2) Find the vector equation of the line which passes through the point $(3, 4, 5)$ and is parallel to the vector $2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.
3) A line passes through the point with position vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and is the direction of the vector $\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find the equation of the line in Cartesian form.
4) If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?
5) Find the Cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.
6) If the equation of a line AB is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, find the direction ratios of a line parallel to AB. Also write in the vector form.
7) Find the Cartesian and vector equation of the line which passes through the point(–2, 4, –5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.
8) Find the angle between the pair of lines: $\frac{x-5}{-3} = \frac{y+3}{-2} = \frac{z-5}{0}$ and $\frac{x}{1} = \frac{y-1}{-3} = \frac{z-5}{2}$.
9) Find the co-ordinates of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

LONG ANSWER TYPE QUESTIONS

1) Find the shortest distance between the following lines :
$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$
2) Find the shortest distance between the following two lines :
$$\vec{r} = (1+\lambda)\hat{\imath} + (2-\lambda)\hat{\jmath} + (\lambda+1)\hat{k} \text{ and } \vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k}).$$
3) Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.
4) If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.
5) Find the value of λ for which the following lines are perpendicular to each other :

$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$ hence, find whether the lines intersect or not.

6) Find the vector equation of a line passing through the point (2,3,2) and parallel to the line $\vec{r} = (-2\hat{\imath} + 3\hat{\jmath}) + \lambda(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$. Also, find the distance between these two lines.

7) Find the vector equation of the line passing through $(2,1,-1)$ and parallel to the line $\vec{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$. Also, find the distance between these two lines.

8) Find the equation of the line passing through $(2,-1,2)$ *and* $(5,3,4)$ and of the plane passing through $(2,0,3), (1,1,5)$ *and* $(3,2,4)$. Also, find their point of intersection.

9) Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

10) Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ *and* $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

11) Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P (1, 3, 3).

12) Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$; $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ are intersecting. Hence find their point of intersection.

CHAPTER: PROBABILITY

MULTIPLE CHOICE QUESTIONS

1) If $P(A) = \frac{1}{2}, P(B) = 0, then\ P((A|B))$ is
 (a) 0 (b) $\frac{1}{2}$ (c) Not defined (d) 1

2) If A and B are events such that $P(A|B) = P(B|A)$, then
 (a) $A \subset B\ but\ A \neq B$ (b) $A = B$ (c) $A \cap B = \phi$ (d) $P(A) = P(B)$

3) The probability of obtaining an even prime number on each die, when a pair of dice is rolled is
 (a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{36}$

4) Two events A and B will be independent, if
 (a) A and B are mutually exclusive
 (b) P(A'B')=[1-P(A)] [1-P(B)]
 (c) P(A)=P(B)
 (d) P(A)+P(B)=1

5) Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is
 (a) $\frac{4}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

6) If A and B are two events such that $A \subset B\ and\ P(B) \neq 0$, then which of the following is correct?
 (a) $P(A|B) = \frac{P(B)}{P(A)}$ (b) $P(A|B) < P(A)$
 (c) $P(A|B) \geq P(A)$ (d) None of these

7) If A and B are two events such that $P(A) \neq 0\ and\ P(B|A) = 1$, then
 (a) $A \subset B$ (b) $B \subset A$ (c) $B = \phi$ (d) $A = \phi$

8) If $P(A|B) > P(A)$,then which of the following is correct:
 (a) $P(B|A) < P(B)$ (b) $A(A \cap B) < P(A).P(B)$
 (c) $P(B|A) > P(B)$ (d) $P(B|A) = P(B)$

9) If A and B are any two events such that $P(A) + P(B) - P(A\ and\ B) = P(A)$, then

(a) $P(B|A) = 1$ (b) $P(A|B) = 1$ (c) $P(B|A) = 0$ (d) $P(A|B) = 0$

SHORT/LONG TYPE QUESTIONS

1) If $P(not\ A) = 0.7, P(B) = 0.7\ and\ P(B/A)0.5, then\ find\ P(A/B)$.

2) A coin is tossed 5 times. What is the probability of getting (i) 3 heads, (ii) at most 3 heads?

3) Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

4) A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event ''number is even'' and B be the event ''number is marked red''. Find whether the events A and B are independent or not.

5) The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number. Determine the value of 'k'.

$$P(X = x) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1 \\ 3k, & if\ x = 2 \end{cases} .$$
0, otherwise

6) There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin?

7) Mother, father and son line up at random for a family photo. If A and B are two events given A = Son on one end, B = Father in the middle, find P(B/A).

8) 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

9) Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

10) In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

11) Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$ find the probability distribution of X.

12) A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads, and

(ii) at most 4 heads.

13) There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.

14) An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and

0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist?

15) If $P(A) = 0.6, P(B) = 0.5$ and $P(B/A) = 0.4, find$ $P(A \cup B)$ and $P(A/B)$.

16) Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamonds.

17) In answering a question on a multiple choice questions test with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability the copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question, given that he correctly answered it.

18) The probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of the them coming to school on time.

19) A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.(Ans: 1/9)

Hint: A: sum=8, $A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$ B: red die number less than 4,

$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3), \\ (4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3), \end{Bmatrix}$

$A \cap B = \{(5,3),(6,2)\}$, $\therefore P(A/_B) = \frac{P(A \cap B)}{p(B)} = \frac{2/36}{18/36} = \frac{1}{9}$

20) A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A?

21) Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5, or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ? (Ans: 8/11)

22) Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean of X. (Ans: Mean = 4)

23) Two groups are competing for the positions of the Board of directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3if the second group wins. Find the probability that the new product introduced was by the second group.

24) From a lot of 20 bulbs which include 5 defectives, a sample of 3 bulbs is drawn at random, one by one with replacement. Find the probability distribution of the number of defective bulbs. Also, find the mean of the distribution.

25) A die whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red" . Find if A and B are independent events.

Ans: $P(A) = \frac{1}{2},\ P(B) = \frac{1}{2}\ and\ P(A \cap B) = \frac{1}{6}\ \ \therefore P(A) \neq P(B)$

$\therefore A\ and\ B\ are\ not\ independent\ event$

26) There are four cards 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and X.

Ans:

+	1	3	5	7
1	*	4	6	8
3	4	*	8	10
5	6	8	*	12
7	8	10	12	*

X	4	6	8	10	12
P(X)	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$

Mean =8

27) The random variable X can take only the values 0 , 1 , 2 , 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\sum p_i x_i^2 = 2\sum p_i x_i$, find the value of p.

Ans: Let $P(X = 2) = P(X = 3) = k,\ \sum P(X) = 1 \Rightarrow 2p - 2k = 1 \Rightarrow k = \frac{1}{2} - p$

$$Given\ \sum p_i x_i^2 = 2\sum p_i x_i \Rightarrow \frac{5}{2} - 4p = \frac{13}{2} - 12p \Longrightarrow p = \frac{3}{8}$$

28) Of the students in school, it is known that 30% have 100% attendance and 70%students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have A grade. What is the probability that the student has 100% attendance? Is regularity only in school? Justify your answer.

Ans: $P\left({}^{E_1}/_{A}\right) = \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100}+\frac{70}{100}\times\frac{10}{100}} = \frac{3}{4}$

29) Prove that if E and F are independent events, then the event E and F' are also independent.

Ans: $(E \cap F') = P(E) - P(E \cap F) = P(E) - P(E)P(F) = P(E)\{1 - P(F)\} = P(E).P(E')$

$P(E \cap F') = P(E).P(E')\quad \therefore E\ and\ F'are\ independent\ events$

30) Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is six. Find the probability that it is actually six.

Ans: E_1= 6 appear on throwing a die, E_2= 6 dos not appear on throwing a die

$$P\left({}^{E_1}/_{A}\right) = \frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}} = \frac{4}{4+5} = \frac{4}{9}$$

31) If P (A) = 0.4, P (B) = p, P (A U B) =0.6 and A and B are given to be independent events, find the value of 'p'.

Ans: $p = \frac{1}{3}$

32) There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X.

$$Ans: Mean = 5, variance = \frac{160}{3} - 5^2 = \frac{5}{3}$$

33) In a shop X, 30 tins of pure ghee and 40 tins of adulterated ghee which look alike, are kept for sale while in shop Y, similar 50 tins of pure ghee and 60 tins of adulterated ghee are there. One tin of ghee is purchased from one of the randomly selected shops and is found to be adulterated. Find the probability that it is purchased from shop Y

Ans: $E_1 = Gheepurchased\ from\ shop\ X$

$E_2 = Gheepurchased\ from\ shop\ Y$

$A = Getting\ adulterated\ ghee$

$$P\left({E_2}/{A}\right) = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}} = \frac{21}{43}$$

34) In a game, a man wins Rs. 5 for getting a number greater than 4 and loses Rs.1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

Ans: Rs.$\frac{19}{9}$

35) A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

Ans: $\frac{3}{5}$

36) Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean of the distribution.

Ans: Probability distribution is

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Mean $= \frac{4}{5}$

37) A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, then that at least one girl must be there in the committee.

38) A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

Find the value of C and also calculate mean of the distribution

39) A,B and C throw a pair of dice in that order alternately till one them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first

40) A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement)at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

41) A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first

42) Three numbers are selected at random (without replacement) from first six position integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean of the distribution

43) There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag B Ans: $\frac{3}{5}$

44) Three numbers are selected at random(without replacements) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also find the mean of the distribution.

Ans: $Mean = \frac{7}{4}$

45) A man takes a step forward with probability 0.4 and backward with probability 0.6.

Find the probability that at the end of 5 steps, he is one step away from the starting point.

46) Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of 'tails'. If she gets 3, 4, 5 or 6 she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4 5 or 6 with the die?

An urn contains 5 red and 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes, find the mean of X.