केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN

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ZONAL INSTITUTE OF EDUCATION AND TRAINING, MYSORE

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गणित/MATHEMATICS
विषय कोड/Subject Code - 041

सुश्री मीनाक्षी जैन
निदेशक, केवीएस ज़िएट मैसूरु

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D SREENIVASULU
TRAINING ASSOCIATE(MATHEMATICS)

DIRECTOR’S MESSAGE

It is with profound delight and utmost pride that I announce the publication of our study material for class XII (MATHEMATICS) for the session 2023-24. It’s my firm belief that access to quality education should know no boundaries, transcending social and economic constraints. Our collective vision is to empower all students with the tools for success and intellectual growth.

With their steadfast dedication PGT-MATHEMATICS of Bangalore, Chennai, Ernakulam & Hyderabad regions of Kendriya Vidyalaya Sangathan have invested their knowledge, expertise, and passion into meticulously crafting these study materials to complement the classroom learning experience of the students. These materials serve as invaluable aids for self-study since they are comprehensive, well-structured, and presented in a manner that is easy to comprehend.

It is with pleasure that I place on record my commendation for the commitment and dedication of the team of teachers which included the Training Associate (MATHEMATICS) from ZIET Mysore who has been the Coordinator of this assignment and all the concerned PGT- Mathematics subject experts from the four feeder regions of ZIET Mysore.

Wishing you all the very best in your academic journey!

MENAXI JAIN
DIRECTOR
ZIET MYSORE

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SYLLABUS

MATHEMATICS (XII)
(Code No. 041)
Session – 2023-24

No.	Units	Marks
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Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3×3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. **Continuity and Differentiability**

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, $like\ x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{a^2 - x^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$, $\int \sqrt{x^2 \pm a^2}\, dx$, $\int \sqrt{a^2 - x^2}\ dx$,
$\int \sqrt{ax^2 + bx + c}\ dx$ $\int \frac{px+q}{ax^2+bx+c} dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}$ *+py=q, where p and q are functions of x or constants.*

$\frac{dx}{dy}$ *+ px=q, where p and q are functions of y or constants.*

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

CHAPTER : RELATIONS AND FUNCTIONS

SYLLABUS: *Types of relations: Reflexive, Symmetric, transitive and equivalence relations.*

One to one and onto functions.

Definitions and Formulae:

- **Types of Relations:**

- **Empty Relation:** A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \emptyset \subset A \times A$.

- **Universal Relation:** A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

- **Trivial Relations:** Both the empty relation and the universal relation are sometimes called trivial relations.

- A relation R in a set A is called

  a) **Reflexive**, if $(x,x) \in R$ $for\ every\ x \in A$

  b) **Symmetric**, $if\ (x,y) \in R\ implies\ that\ (y,x) \in R\ for\ all\ x,y\ \in A$

  c) **Transitive,** $if\ (x,y) \in R\ and\ (y,z) \in R$ implies that $(x,z) \in R$ for all

  $x,y,z \in A$

- If A is a finite set with n elements,then

  i) Total number of relations in A=2^{n^2}.

  ii) Total number of reflexive relations in A = 2^{n^2-n}.

  iii) Total number of symmetric relations in A = $2^{\frac{n(n+1)}{2}}$.

- **Equivalence Relation:**A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

- **Equivalence Class**: Let R be an equivalence relation on a non-empty set A and $a \in A$. Then the set of all those elements of A which are related to a, is called the equivalence class determined by a and is denoted by $[a]$.i.e$[a] = \{x \in A : (x,a) \in R\}$

- **Types of Functions:**

- **One-One (Injective) Function**: A function $f : X \rightarrow Y$ is defined to be one-one (or injective), if the images of distinct elements of X under f are distinct, i.e., for every $x_1, x_2 \in X, f(x_1) = f(x_2)\ implies\ x_1 = x_2$. Otherwise, f is called many-one.

- **Onto (Surjective) Function**: A function $f : X \rightarrow Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under f. i.e., for every $y \in Y$, there exists an element $x \ in \ X$ such that $f(x) = y$.

NOTE: $\boldsymbol{f}: \boldsymbol{X} \rightarrow \boldsymbol{Y}$ is onto if and only if Range of $\boldsymbol{f} = \boldsymbol{Codomain}$.

- **Bijective Function:** A function $f : X \rightarrow Y$ is said to be bijective, if f is both one-one and onto.

- If A and B are two finite sets having $m \ and \ n$ elements respectively, then

i) Number of functions from A to B is n^m.

ii) Number of one-one functions from A to $B = \begin{cases} n_{p_m}, if \ m \leq n \\ 0, if \ m > n \end{cases}$

iii) If the function is bijective i. e both one-one and onto then $m = n$.

MULTIPLE CHOICE QUESTIONS

Q.No	QUESTIONS AND SOLUTIONS
1	If $A = \{5,6,7\}$ and let R $= \{(5,5), (6,6)), (7,7), (5,6), (6,5), (6,7), (7,6)\}$. Then R is a) Reflexive, symmetric but not Transitive b) Symmetric, transitive but not reflexive c) Reflexive, Transitive but not symmetric d) an equivalence relation **Answer: A** (5,6) ∈ R and (6,7) ∈ R but (5,7) does not belong to R
2	Let R be a relation defined on Z as follows: $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25. Then \ Domain \ of \ R \ is$ a) {3,4,5} b) {0,3,4,5} c) {0,±3,±4,±5} d) None of these **Answer: C** R={(0, ,±5), (,±5,0),(±3, ,±4),(,±4, ±3)} Domain of R is the set of all first elements of R.
3	The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ is a) 1 b) 2 c) 3 d)5 **Answer: D** Possible equivalence relations are R_1 = { (1,1) , (2,2) , (3,3) } R_2 = { (1,1) , (2,2) , (3,3), (1,2) ,(2,1)} R_3 = { (1,1) , (2,2) , (3,3), (1,3), (3,1)} R_4 = { (1,1) , (2,2) , (3,3), (2,3),(3,2) } R_5 = A×A
4	Consider the set $A = \{1, 2\}$. The relation on A which is symmetric but neither transitive nor reflexive is a) {(1,1) (2,2)} b) { } c) {(1,2)} d) { (1,2) (2,1) } **Answer: D**

	R is not reflexive since (1,1) and (2,2) are not there in R R is not transitive since (1,2) and (2,1) belong to R but (1,1) does not belong to R.
5	If $A = \{d, \mathrm{e}, \mathrm{f}\}$ and let R $= \{(\mathrm{d},\mathrm{d}), (\mathrm{d},\mathrm{e}), (\mathrm{e},\mathrm{d}), (e,e)\}$. Then R is a) Reflexive, symmetric but not Transitive b) Symmetric, transitive but not reflexive c) Reflexive, Transitive but not symmetric d) an equivalence relation **Answer: B** R is not reflexive because (f,f) is not present in R
6	Let R be a reflexive relation on a finite set A having n elements and let there be m,minimum number of ordered pairs in R, then a) m < n b) m > n c) m = n d)none of these **Answer: C** A relation on a set A is reflexive if every element of A is related to itself i.e. $(a,a) \in R, for\ all\ a \in R$
7	The number of elements in set A is 3.The number of possible relations that can be defined on A is a) 8 b) 4 c) 64 d) 512 **Answer: D** The number of possible relations on a set having n elements is 2^{n^2} as every relation is a subset of A× A.
8	The number of elements in Set A is 3.The number of possible reflexive relations that can be defined in A is a) 64 b) 8 c) 512 d) 4 **Answer: A** If a set has A has n elements then the number of possible reflexive relations on A is $2^{n(n-1)}$.
9	The number of elements in set P is 4.The number of possible symmetric relations that can be defined on P is a) 16 b) 32 c) 512 d) 1024 **Answer: D** If a set has A has n elements then the number of possible symmetric relations on A is $2^{\frac{n(n+1)}{2}}$
10	Let R be a relation on the set N of natural numbers defined by aRb if and only if $a\ divides\ b$.Then R is a) Reflexive, symmetric but not Transitive b) Symmetric, transitive but not reflexive c) Reflexive, Transitive but not symmetric d) an equivalence relation **Answer C** R is reflexive, since every natural number divides itself. If a divides b and b divides c then a divides c So R is transitive $a\ divides\ b$ need not imply that $b\ divides\ a$. So R is not symmetric.

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
RELATIONS AND FUNCTIONS	https://youtu.be/hbZIyGzmI0w	

EXERCISE

1.	Let $f: R \rightarrow R$ *be defined by* $f(x) = \frac{1}{x}\ \forall\, x \in R$. *Then* f *is* a) One-one b) Onto c) Bijective d) f is not defined **Answer: d**
2.	Set A has 4 elements and set B has 5 elements. Then the number of bijective mappings from A to B is a) 120 b) 20 c) 0 d) 625 **Answer: c**
3.	Set A has 3 elements and set B has 4 elements. Then the number of injective mappings from A to B is a) 144 b) 12 c) 24 d) 64 **Answer: c**
4.	The function $f: [\pi, 2\pi] \rightarrow R$ defined by $f(x) = cosx$ is a) one – one but not onto b) onto but not one – one c) many – one function d) bijective function **Answer: d**
5.	Let $f: R \rightarrow R$ *be defined by* $f(x) = x^3 + 4$,then f is a) Injective b) Surjective c) Bijective d) None of these **Answer: c**

6.	Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Based on the given information f is best defined as a) Surjective function b) Injective function c) Bijective function d) Function **Answer: b**
7.	Let $A = \{1,2,3,\ldots,n\} and\ B = \{p.q\}$.Then the number of onto functions from A to B is a) 2^n b) 2^n-2 c) 2^n-1 d) None of these **Answer: b**

ASSERTION AND REASONING QUESTIONS

	In the following question a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option: A) Both A and R are true and R is the correct explanation of A. B) Both A and R are true but R is NOT the correct explanation of A. C) A is true but R is false. D) A is false but R is true.
1.	**Assertion (A):** *If n (A) =p and n (B) = q then the number of relations from A to B is* 2^{pq}. **Reason(R):** *A relation from A to B is a subset of* $A \times B$. **Answer. A** **Solution:** Every relation from set A to set B is a subset of $A \times B$. So R is true The number of elements in $A \times B$ is $p \times q$.So number of subsets of $A \times B$.i.e no of relations from A to B is 2^{pq} . So A is true.
2.	**Assertion (A):** *If n (A) =m, then the number of reflexive relations on A is m* **Reason(R) :** *A relation R on the set A is reflexive if (a, a)* $\in R, \forall a \in A$. **Answer: D** **Solution:** A relation R is reflexive on the set A iff (a,a) ∈ R ∀ a ∈ A. So R is true. $n(A) = m$ then the number of reflexive relations on A is 2^{m^2-m}. So A is false.
3.	**Assertion (A):** Domain and Range of a relation $R = \{(x,y): x - 2y = 0\}$defined on the set A $= \{1,2,3,4\}$ are respectively $\{1,2,3,4\}$and $\{2,4,6,8\}$ **Reason(R):** Domain and Range of a relation R are respectively the sets $\{a: a \in A\ and\ (a,b) \in R.\}\ and\ \{b: b \in A\ and\ (a,b) \in R\}$ **Answer: D** **Solution:** Domain of a relation R is $\{x: x \in A\ and\ (x,y) \in R.\}$. Range of a relation R is $\{y: y \in A\ and\ (x,y) \in R.\}$. So R is true.

	$R = \{(2,1),(4,2)\}$ So A is false
4.	**Assertion (A):** A relation $R = \{(1,1),(1,2),(2,2),(2,3)(3,3)\}$ defined on the set $A = \{1,2,3\}$ is reflexive. **Reason(R):** A relation R on the set A is reflexive if $(a,a) \in R, \forall\, a \in A$ **Answer: A** **Solution:**A relation R on the set A is reflexive if $(x,x) \in R, \forall\, x \in A$ So R is true For , $\forall\, x \in A\ (x,x) \in R$ so R is reflexive and thus A is true. Therefore answer is A.
5.	**Assertion (A):** A relation $R = \{(1,1),(1,2),(2,2),(2,3)(3,3)\}$ defined on the set $A = \{1,2,3\}$ is symmetric **Reason(R):** A relation R on the set A is symmetric if $(a,b) \in R \Longrightarrow (b,a) \in R$ **Answer:D** **Solution:**A relation R on the set A is symmetric if $(x,y) \in R \Longrightarrow (y,x) \in R$ So,R is true $(1,2) \in R\ but\ (2,1)$ does not belong to R so R is not symmetric . So A is false
6.	**Assertion (A):** A relation $R = \{(1,1),(1,3),(1,5),(3,1)(3,3),(3,5)\}$ defined on the set $A = \{1,3,5\}$ is transitive. **Reason(R):** A relation R on the set A symmetric if $(\mathrm{a,b}) \in \mathrm{R}$ and $(\mathrm{a,c}) \in \mathrm{R} \Longrightarrow (\mathrm{a,c}) \in \mathrm{R}$ **Answer:C** **Solution:** A relation R on the set A transitive iff $(\mathrm{a,b}) \in \mathrm{R}$ and $(\mathrm{a,c}) \in \mathrm{R} \Longrightarrow (\mathrm{a,c}) \in \mathrm{R}$. So R is false As the condition of transitivity is satisfied, so A is true.
7.	**Assertion (A):** $A = \{1,2,3\}, B = \{4,5,6,7\}, f = \{(1,4),(2,5),(3,6)\}$ is a function from A to B.Then $f\ is\ one - one$ **Reason(R):** A function $f\ is\ one - one$ if distinct elements of A have distinct images in B. **Answer:A** **Solution:** A function f is one –one if distinct elements of A have distinct images in B. So R is true As distinct elements of Set A have distinct images in the set B. So, A is true Thus A and R are true and R is the correct explanation of A.
8.	**Assertion (A):** Consider the function $f: R \rightarrow R$ defined by $f(x) = x^3$. Then f is one-one **Reason(R):** Every polynomial function is one-one **Answer: C** **Solution:** Every polynomial function is not one-one as $f(x) = x^2$ is not one one. So R is false. A function f is one-one if distinct elements have distinct images So A is true Thus A is true but R is false.

9.	**Assertion (A):** If $X = \{0,1,2\}$ and the function $f: X \to Y$ defined by $f(x) = x^2 - 2$ is surjection then $Y = \{-2,-1,0,2\}$ **Reason(R):** If $f: X \to Y$ is surjective if for all $y \in Y$ there exists $x \in X$ such thtat $y = f(x)$ **Answer: D** **Solution:** A function is surjective or onto if $range = co - domain$,i.e $f: X \to Y$ is surjective if for all $y \in Y$ there exists $x \in X$ such thtat $y = f(x)$ So R is true. There is no $x\ in\ X$ such that $f(x) = 0$, so range of f is not equal to the codomain, i.e f is not surjective So, A is false. Thus A is false but R is true.
10.	**Assertion (A):** A, B are two sets such that $n(A) = m\ and\ n(B) = n$. The number of one-one functions from A to B is n_{p_m} ,if $n \geq m$ **Reason(R):** A function f is one –one if distinct elements of A have distinct images in B **Answer: A** **Solution:**A function f is one –one if distinct elements of A have distinct images in B. So R is true. For a function from set A to B is one-one iff $n(A) \leq n(B)$ So A is true.

EXERCISE

1	Assertion (A): A function $f: A \to B$, cannot be an onto function if $n(A) < n(B)$. Reason(R): A function f is onto if every element of co-domain has at least one pre-image in the domain **Answer:A**
2	Assertion (A): Consider the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$ .Then f is one – one Reason(R): f(4)=4/17 and f(1/4)=4/17 **Answer: D**
3	Assertion (A): $n(A) = 5, n(B) = 5$ and $f : A \to B$ is one-one then f is bijection Reason(R): If $n(A) = n(B)$ then every one-one function from A to B is onto **Answer: A**
4	Assertion (A): Set A has 4 elements and set B has 5 elements. Then the number of bijective mappings from A to B is 5^4 Reason(R): A mapping from A to B cannot be bijective if n(A) is not equal to n(B). Answer: D
5	Assertion (A):The identity relation on a set A is an equivalence relation. Reason (R): The Universal relation on a set A is an equivalence relation. **Answer: B**

2 MARKS QUESTIONS

1.	The relation R defined by $(a,b)R(c,d) \Rightarrow\ a+d=b+c$ on the A×A. Where $A=\{1,2,3,\ldots\ldots,10\}$ is an equivalence relation. Find the equivalence class of the element (3,4). **Solution:** Let (3,4) R (a,b) on $A\times A$ where $A=\{1,2,3,\ldots\ldots.10\}$ $\Rightarrow 3+b=4+a\ \rightarrow\ b-a=1$ $[(3,4)]_R=\{(1,2).(2,3),(3,4),(4,5),(6,7),(7,8),(8,9),(9,10)\}$
2.	Let $f:N\rightarrow N$ be defined by $f(n)=\begin{cases}\frac{n+1}{2}, \text{if n is odd}\\ \frac{n}{2}, \text{if n is even}\end{cases}$ for all n ∈ N. Is the function f one-one or not ? Justify your answer. **Solution:** Given function is not one-one, because 1 and 2 have the same image. $f(1)=\frac{1+1}{2}=1\,, f(2)=\frac{2}{2}=1$
3.	Consider f : $R_+\rightarrow[-9,\infty)$ given by f(x) = 5x^2 + 6x – 9. Show that f is one-one. **Solution:** One-one: Let $f(x)=f(y)\ \Rightarrow 5x^2+6x-9=5y^2+6y-9$ $\Rightarrow 5(x-y)(x+y)+6(x-y)=0$ $\Rightarrow (x-y)(5x+5y+6)=0$ $\Rightarrow x=y. Since\ 5x+5y+6\neq 0\ for\ all\ x,y\in\ R_+$ $Given\ function\ is\ one-one.$
4.	Let R be the relation defined in the set $A\ =\ \{1,2,3,4,5\}$ by $R\ =\ \{(x,y):x,y\ \epsilon\ A, x\ and\ y\ are\ either\ both\ odd\ or\ both\ even\}$. Find R in Roster form. **Solution:** $R=\{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5),(2,2),(2,4),(4,2),(4,4)\}$
5.	Check whether the following relation $R\ =\ \{(a,b): a\ \leq b\}$ defined on set of real numbers are reflexive and symmetric or not. **Solution:** for each $a\in R, a\leq a$ is true. Given relation is reflexive. (2,3)∈ $R\ but\ (3,2)\notin R$ thus,for each (a,b)∈ $R\ \ \not\Rightarrow (b,a)\in R$ Hence given relation is not symmetric.
6.	Prove that the greatest integer function $f:R\rightarrow R$, given by $f(x)=[x]$, is neither one-one nor onto. **Solution:** $f(1.1)=1, f(1.3)=1, but\ 1.1\neq 1.3, \therefore f\ is\ not\ one-one.$ Range is set of integers only whereas codomain is set of real numbers. Range ≠ codomain , ∴ f is not onto
7.	Let $\mathrm{A}=\mathrm{R}-\{1\}$. If $f:A\rightarrow A$ is a mapping defined by $\mathrm{f(x)}=\frac{\mathrm{x}-2}{\mathrm{x}-1}$, show that f is one-one. **Solution:** One-One: Let $f(x)=f(y)\Rightarrow\frac{x-2}{x-1}=\frac{y-2}{y-1}$ $(x-2)(y-1)=(y-2)(x-1)$ $\Rightarrow xy-x-2y+2=xy-2x-y+2$ $\Longrightarrow x-y=0$ $\Longrightarrow x=y$ $Given\ function\ is\ one-one.$
8.	Let $\mathrm{A}=\mathrm{R}-\{1\}$. If $f:A\rightarrow A$ is a mapping defined by $\mathrm{f(x)}=\frac{\mathrm{x}-2}{\mathrm{x}-1}$, show that f is onto. **Solution:** $Onto$: Let $\frac{x-2}{x-1}=y\ \ \Longrightarrow x-2=y(x-1)$

	$\Rightarrow x - 2 = xy - y$ $x(1 - y) = 2 - y$ $x = \frac{2-y}{1-y}$ $for\ each\ y \in A, there\ exist\ x = \dfrac{2-y}{1-y} \in A\ such\ that\ f(x) = y.$ $Given\ function\ is\ onto.$
9.	A function $f: A \to B$ defined as $f(x) = 2x$, is both one-one and onto.If $A = \{1,2,3,4\}$,then find the set B. **Solution:** $f = \{(1,2), (2,4), (3,6), (4,8)\}$. Range of $f = \{2,4,6,8\}$. As function is onto $range\ = codomain.$ So $B = \{2,4,6,8\}$.
10.	Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2){:}\ L_1$ is perpendicular to $L_2\}$. Is the relation R transitive? Justify your answer. **Solution:** R is not transitive Let $(L_1, L_2) \in R\ and\ (L_2, L_3) \in R$ $\Rightarrow L_1\ is\ perpendicular\ to\ L_2\ and\ L_2\ is\ perpendicular\ to\ L_3$ $\Rightarrow\ L_1 is\ parallel\ to\ L_3\ \ \Rightarrow L_1\ is\ not\ perpendicular\ to\ L_3$ $\Rightarrow\ (L_1, L_3) \notin\ R$ Hence, R is not transitive.

EXERCISE

1.	Is the relation R = {(a, b): a$\leq b^2$} defined on set of real numbers transitive? Justify your answer. **Answer:** No, it is not transitive (5,3) ∈ R and (3,2) ∈ R but (5,2) ∉ R.
2.	Determine whether the relation R defined on the set of all real numbers as R = {(a, b): a, b and a – b + $\sqrt{3}$ ∈ S, where S is the set of all irrational numbers}, is symmetric. **Answer:** $(4\sqrt{3}, 3\sqrt{3})$ ∈ R but $(3\sqrt{3}, 4\sqrt{3})$ ∉ R. So, R is not symmetric.
3.	Is the relation R defined on the set of all real numbers as $R\ = \{(a, b){:}\ a > b\}$, reflexive,symmetric and trasitive? Justify your answer. **Answer:** $(a, a) \notin R$ .So, R is not reflexive. Let $(a, b) \in R \Longrightarrow a > b, but\ b\ is\ not\ greater\ than\ a \Rightarrow (b, a) \notin R$ Therefore R is not symmetric. $a > b\ and\ b > c \Longrightarrow a > c$ Therefore R is transitive.
4.	Show that the relation R defined on the set A of all triangles in a plane as $R = \{(T_1, T_2){:}\ T_1\ is\ similar\ to\ T_2\}$ is an equivalence relation.
5.	Show that the relation R in the set of ℝ real numbers, defined as $R = \{(a, b){:}\ a \leq\ b^3\}$ is not transitive. **Answer:** (9,4) ∈ R and (4,2) ∈ R but (9,2) ∉ R .Hence R is not transitive

3 MARKS QUESTIONS

<table>
<tr><td>1.</td><td>Show that the relation R in the set $\{1, 2, 3\}$ given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

Solution: Let A = {1, 2, 3}
The relation R is defined on A is given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}
Now, we have to show that R is reflexive but neither symmetric nor transitive.
<u>Reflexive</u>:
Clearly, $(a, a) \in R\ for\ every\ a \in A$
Hence, R is reflexive
<u>Symmetric:</u>
Clearly, $(1, 2) \in R,\ \ but\ (2, 1) \notin R$
Thus, for every $(a, b) \in R,\ \ (b, a) \notin R$
Hence, R is not symmetric
<u>Transitive:</u>
For (1, 2) $\in R$ and (2, 3) $\in R$ $\Rightarrow$ (1, 3)$\notin R$
Thus, $(a, b) \in R\ and\ (b, c) \in R\ then\ (a, c)\ ) \notin R$
Hence, R is not transitive.
Therefore, R is reflexive but neither symmetric nor transitive.</td></tr>
<tr><td>2.</td><td>Show that the relation R in the set {1, 2, 3, 4} given by
R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)} is reflexive and transitive but not symmetric.
Solution: Let A = {1, 2, 3, 4}
The relation R is defined on A is given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}
Now, we show that the relation R is reflexive and transitive but not symmetric.
<u>Reflexive</u>:
Clearly, $(a, a) \in R\ for\ every\ a \in A$
Hence, R is reflexive.
<u>Symmetric:</u>
Clearly, $(1, 2) \in R,\ \ but\ (2, 1) \notin R$
Thus, for every $(a, b) \in R,\ \ (b, a) \notin R$
Hence, R is not symmetric,
<u>Transitive:</u>
For every $(a, b) \in R\ and\ (b, c) \in R\ \Rightarrow\ (a, c) \in R$
Hence, R is transitive.
Therefore, R is reflexive and transitive but not symmetric.</td></tr>
<tr><td>3.</td><td>Check whether the relation R defined in the set A= $\{1, 2, 3, 4, 5, 6\}$ as

$R = \{(a, b): b = a + 1, a, b \in A\}$ is reflexive, symmetric or transitive.

Solution: Let A = {1, 2, 3, 4, 5, 6}
R be the relation defined as $R = \{(a, b): b = a + 1\}$
i.e; R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}
<u>To check R is reflexive:</u>
Clearly, $(a, a) \notin R\ for\ every\ a \in A$
Hence, R is not reflexive
<u>To check R is symmetric:</u>
Clearly, $(1, 2) \in R,\ \ but\ (2, 1) \notin R$</td></tr>
</table>

	Thus, for every $(a,b) \in R,\ \ (b,a) \notin R$ Hence, R is **not** symmetric **<u>To check R is Transitive:</u>** Take $(1,2) \in R,\ and\ (2,3)\ \in R\ but\ (1,3) \notin R$ Thus, $(a,b) \in R\ and\ (b,c) \in R\ then\ (a,c)\ ) \notin R$ Hence, R is not transitive. Therefore, R is neither reflexive, nor symmetric and nor transitive.
4.	Determine whether the relation R in the set $A\ =\ \{1,2,3,\ldots,13,14\}$ defined as $R\ =\ \{(x,y): 3x - y\ =\ 0, x,y \in A\}$ is reflexive or symmetric or transitive. **Solution:** Given $A\ =\ \{1,2,3,\ldots,13,14\}$ The relation R is defined as R = {(1, 3), (2, 6), (3, 9), (4, 12)} **<u>To check R is reflexive:</u>** Clearly, $(a,a) \notin R\ for\ every\ a \in A$ Hence, R is **not** reflexive **<u>To check R is symmetric:</u>** Clearly, $(1,3) \in R,\ \ but\ (3,1) \notin R$ Thus, For every $(a,b) \in R,\ \ (b,a) \notin R$ Hence, R is **not** symmetric **<u>To check R is Transitive:</u>** Take $(1,3) \in R,\ and\ (3,9)\ \in R\ but\ (1,9) \notin R$ Thus, $(a,b) \in R\ and\ (b,c) \in R\ then\ (a,c)\ ) \notin R$ Hence, R is not transitive. Therefore, R is neither reflexive, nor symmetric and nor transitive.
5.	Determine whether the relation R in the set N of natural numbers defined as $R\ =\ \{(x,y): y\ =\ x\ +\ 5\ and\ x\ <\ 4\}$ is reflexive, symmetric or transitive. **Solution:** Given N = Set of all natural numbers. The relation R is defined on the set N as $R\ =\ \{(x,y): y\ =\ x\ +\ 5\ and\ x\ <\ 4\}$ i.e; R = {(1, 6), (2, 7), (3, 8)} **<u>To check R is reflexive:</u>** Clearly, $(a,a) \notin R\ for\ every\ a \in N$ Hence, R is **not** reflexive **<u>To check R is symmetric:</u>** Clearly, $(1,6) \in R,\ \ but\ (6,1) \notin R$ Thus, For every $(a,b) \in R,\ \ (b,a) \notin R$ Hence, R is **not** symmetric **<u>To check R is Transitive:</u>** For transitive, $we\ have\ to\ show\ for\ \ (a,b) \in R\ and\ (b,c) \in R\ then\ (a,c) \in\ R$ But in this case, in the relation R, for every order pair (a, b), there exists no order pair as (b, c). In such a case, R is also transitive. Therefore, the relation R is neither reflexive nor symmetric but transitive.
6.	Prove that the Greatest Integer Function $f\ :\ R\ \rightarrow\ R$, given by $f(x) =\ [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x. **Solution:** f : R → R, given by f(x) = [x]

	Now, we prove that f is neither one-one nor onto **<u>To Prove f is not one-one:</u>** $Clearly, we\ have\ that\ f(1.1) = [1.1] = 1$ $f(1.2) = [1.2] = 1, \ldots \ldots \ldots \ldots \ldots, f(1.9) = [1.9] = 1$ From this, we conclude that different elements in the domain of f have same images in the co-domain of f. Hence, f is not one-one function. **<u>To Prove f is not onto:</u>** $we\ know\ that\ codomain\ of\ f = R\ \ (set\ of\ all\ real\ numbers)$ $Range\ of\ f = Z\ \ (set\ of\ all\ integers)$ $Clearly, codomain\ of\ f\ \neq Range\ of\ f$ Hence, f is not onto. Therefore, f is neither one-one nor onto.
7.	Show that the Modulus Function f : R → R, given by $f(x) = \mid x \mid$, is neither one-one nor onto, where $\mid x \mid is\ x$, if x is positive or 0 and $\mid x \mid is -\ x, if\ x$ is negative. **Solution:** $f: R \longrightarrow R\ is\ given\ by f(x) = \|x\| = \begin{cases} x, if\ x \geq 0 \\ -x, if\ x < 0 \end{cases}$ Now, we prove that f is neither one-one nor onto **<u>To Prove f is not one-one:</u>** $Clearly, we\ have\ that\ f(1) = 1 = f(-1)$ $f(2) = 2 = f(-2)\ and\ so\ on.$ From this, we conclude that different elements in the domain of f have same images in the co-domain of f. Hence, f is not one-one function. **<u>To Prove f is not onto:</u>** $we\ know\ that\ codomain\ of\ f = R\ \ (set\ of\ all\ real\ numbers)$ $Range\ of\ f = R^+\ \ (set\ of\ all\ non-negative\ real\ numbers)$ $Clearly, codomain\ of\ f\ \neq Range\ of\ f$ Hence, f is not onto.
8.	Show that the function $f: R \longrightarrow R\ is\ given\ by f(x) = \begin{cases} 1, if\ x > 0 \\ 0,\ if\ x = 0 \\ -1, if\ x < 0 \end{cases}$ (this function is called signum function) is neither one-one nor onto. **Solution:** The function $f: R \longrightarrow R\ is\ given\ by f(x) = \begin{cases} 1, if\ x > 0 \\ 0,\ if\ x = 0 \\ -1, if\ x < 0 \end{cases}$ Now, we prove that f is neither one-one nor onto.

	To Prove f is not one-one: $Clearly, we\ have\ that\ f(1) = 1, \quad f(2) = 1, f(3) = 1, and\ so\ on.$ $f(-1) = -1,\ f(-2) = -2,\ and\ so\ on.$ From this, we conclude that different elements in the domain of f have same images in the co-domain of f. Hence, f is not one-one function. **To Prove f is not onto:** $we\ know\ that\ codomain\ of\ f = R\ \ (set\ of\ all\ real\ numbers)$ $Range\ of\ f = \{-1, 0, 1\}$ $Clearly, codomain\ of\ f\ \neq Range\ of\ f$ Hence, f is not onto. Therefore, f is neither one-one nor onto.
9.	$Prove\ that\ the\ function\ f: N \longrightarrow N\ defined\ by\ f(x) = x^2 + x + 1$ is one-one but not onto. **Solution:** $The\ function\ \boldsymbol{f}: \boldsymbol{N} \longrightarrow \boldsymbol{N}\ \ given\ by\ \ f(x) = x^2 + x + 1$ Now we prove that f is one-one but not onto. **To prove f is one-one:** Let $x_1, x_2 \in N\ such\ that\ \ f(x_1) = f(x_2)$ $\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$ $\Rightarrow x_1^2 - x_2^2 + x_1 - x_2 = 0$ $\Rightarrow (x_1^2 - x_2^2) + (x_1 - x_2) = 0$ $\Rightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)[(x_1 + x_2) + 1] = 0$ $\Rightarrow (x_1 - x_2) = 0 \quad (\because [(x_1 + x_2) + 1] > 0\ as x_1, x_2\ in\ domain\ \boldsymbol{N})$ $\Rightarrow x_1 = x_2$ Hence, f is one-one **To prove f is onto:** we have $f(1) = 3, f(2) = 7$ and so on. Thus, $f(x) = x^2 + x + 1\ \geq 3\ for\ every\ x \in N\ (domain)$ Clearly, $f(x) not\ taking\ values\ 1\ and\ 2$ Thus, the every element of co-domain in N has no pre-image in the domain N Hence, f is not onto.
10.	$Let\ f: W \to W be\ defined\ as\ f(x) = x - 1, if\ x\ is\ odd\ and$ $f(x) = x + 1, if\ x\ is\ even.$ Show that f is both one-one and onto**.** **Solution:** $f: W \to W\ be\ defined\ as\ f(x) = x - 1, if\ x\ is\ odd$ and

	$f(x) = x + 1, if\ x\ is\ even.$ **<u>To Prove f is one-one:</u>** **<u>Case I:</u>** when $x_1\ and\ \ x_2$ are even number Now, consider $f(x_1\ ) = f(x_2\ )$ $\Rightarrow x_1\ + 1 = x_2 + 1$ $\Rightarrow x_1\ = x_2$ Hence, f is one-one **<u>Case II:</u>** when $x_1\ and\ \ x_2$ are odd number Now, consider $f(x_1\ ) = f(x_2\ )$ $\Rightarrow x_1\ - 1 = x_2 - 1$ $\Rightarrow x_1\ = x_2$ Hence, f is one-one. **<u>Case III:</u>** when $x_1\ is\ odd\ and\ \ x_2$ is even number Here, $x_1\ \neq x_2$ Also, in this case $f(x_1\ )\ is\ even\ and\ f(x_2) is\ odd.$ Hence, $f(x_1\ ) \neq\ f(x_2)$ Therefore, f is one-one. **<u>To Prove f is onto:</u>** For every even number 'y' in co-domain there exists odd number $y + 1$ in domain and for every odd number 'y' in co-domain there exists even number $y - 1$ in domain such that $f(x) =\ y.$ Hence, f is onto Therefore, f is both one-one and onto.

EXERCISE

1.	Consider $f: R^+\ \longrightarrow [4, \infty) given\ by\ f(x) =\ x^2 + 4$. Show that f is both one-one and onto.
2.	Let $\mathrm{A} = \mathrm{R} - \left\{\frac{2}{3}\right\}$ and $\mathrm{B} = \mathrm{R} - \left\{\frac{2}{3}\right\}$. If f: A→ B and $\mathrm{f(x)} = \frac{2\mathrm{x}-1}{3\mathrm{x}-2}$, then prove that the function f is one-one and onto.
3.	Show that the function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{x^2+1}$ $, \forall\ x\ \in R$ is neither one-one nor onto.
4.	Let R be the relation on set of all integer Z defined by $R\ = \{(a, b): \|a - b\| \leq 3\}$. Check whether R is an equivalence relation. Answer: R is reflexive, symmetic but not transitive

5 MARKS QUESTIONS

1.	Let R be the relation in the set Z of integers given by $R = \{(a, b): 2\ divides\ a - b\}$. Show that the relation R equivalence? Write the equivalence class [0]. **Solution :** Given R = {(a, b) : 2 divides a – b} For equivance relation we have to check : **(i) Reflexive:** If $(a - b)$ is divisible by 2 then, $\Rightarrow (a - a) = 0$ is also divisible by 2 $\Rightarrow (a, a) \in R$ Hence R is Reflexive ∀ a,b ∈ Z (ii) **Symmetric**: If $(a - b)$ is divisible by 2 then, $\Rightarrow (b - a) = -(a - b)$ is also divisible by 2 $\Rightarrow (a, b) \in R\ and\ (b, a) \in R$ Hence R is Symmetric ∀ a,b ∈ Z (iii) **Transitive**: If $(a - b) and\ (b - c)$ are divisible by 2 then, $\Rightarrow a - c = (a - b) + (b - c)$ is also divisible by 2 ⇒ (a,b) ∈ R, (b,c) ∈ R and (a,c) ∈ R. Hence R is Transitive ∀ a,b,c ∈ Z ⇒ As Relation R is satisfying**Reflexive , Symmetric** and **Transitive.** Hence R is an equivalence relation. Now equivalence class [0] $R = \{(a, b): 2\ divides\ (a - b)\} \Rightarrow (\boldsymbol{a} - \boldsymbol{b})\ is\ a\ multiple\ of\ 2.$ To find equivalence class 0**,** put b=0 So, **(**a–0) is a multiple of 2 **⇒** a is a multiple of 2 So, In the set Z of integers, all the multiple of 2 will come in equivalence class [0] Hence, equivalence class $[0] = \{2x : x \in Z\}$
2.	Show that the relation R defined by $(a, b)R\ (c, d) \Leftrightarrow a + d = b + c$ on the set × N is an equivalence relation. Also, find the equivalence classes [(2,3)] and 1,3)]. **Solution:** Given that, R be the relation in N × N defined by (a, b) R (c, d) if $a + d = b + c$ for (a, b), (c, d) in N × N. **Reflexive:** Let (a, b) R (a, b) ⇒ a + b = b + a which is true since addition is commutative on N. ⇒ R is reflexive. **Symmetric**: Let (a, b) R (c, d) ⇒ a + d = b + c ⇒ b + c = a + d ⇒ c + b = d + a ⇒ (c, d) R (a, b) ⇒ R is symmetric. **Transitive** ⇒ (a, b) R (e, f) for (a, b), (c, d),(e, f) in N × N. Let (a, b) R (c, d) and (c, d) R (e, f) ⇒ a + d = b + c and c + f = d + e ⇒ (a + d) – (d + e) = (b + c) – (c + f) ⇒ a - e = b - f ⇒ a + f = b + e ⇒ (a, b) R (e, f) ⇒ R is transitive.

	Hence, R is an equivalence relation. **Equivalence Classes:** The equivalence class of (a, b) is the set of all pairs (c, d) such that a + d = b + c. $\Rightarrow a - b = c - d$ The equivalence classes of [(2,3)] is Put $a = 2\ and\ b = 3$ c – d = 2 – 3 d – c = 1 [(2,3)] = {(c,d) : d – c = 1 ∀ c,d ∈ Z } The equivalence classes of [(1,3)] is Put a=1and b=3 c – d = 1 – 3 ⇒ d – c = 2 [(1,3)] = {(c,d) : d – c = 2 ∀c,d ∈ Z }
3.	Show that the relation R in the set of ℝ real numbers, defined as $R = \{(a, b): a \leq b^3\}$ is neither reflexive nor symmetric nor transitive. **Solution:** Given $\mathbb{R} = set\ of\ all\ real\ numbers$ The relation R in the set ℝ defined as $R = \{(a, b): a \leq b^3\}$ Now we prove that R is neither reflexive nor symmetric nor transitive. **<u>To show R is not reflexive:</u>** We know that $a \leq a^3$ is not true for any positive real number less than 1. For example, $for\ a = \frac{1}{2},\ \frac{1}{2} \nleq \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ Thus, clearly, $(a, a) \notin R\ for\ every\ a \in \mathbb{R}$ Hence, R is **not** reflexive **<u>To show R is not symmetric:</u>** Take a = 1 and b = 2 Now, $a = 1 \leq 2^3 = b^3 \Rightarrow a \leq b^3 \Rightarrow (a, b) \in R$ But, $b = 2 \nleq (1)^3 = 1 = a \Rightarrow b \nleq a^2 \Rightarrow (b, a) \notin R$ Thus, $(a, b) \in R \Rightarrow (b, a) \notin R$ Hence, R is not symmetric. **<u>To show R is not transitive:</u>** Let us take a = 10, b = 4, c = 2 $(a, b) \in R = (10, 4) \in R\ as\ 10 \leq 4^3 = 64$ $(b, c) \in R = (4, 2) \in R\ as\ 4 \leq 2^3 = 8$ But, $(a, c) \notin R\ as\ 10 \nleq 2^3 = 8$ Thus, $(a, b) \in R\ and\ (b, c) \in R \Rightarrow (a, c) \notin R$ Hence, R is not tranitive. Therefore, the relation R is neither reflexive, nor symmetric nor transitive.
4.	Let A = {1, 2, 3, ……., 9} and $(a, b)R\ (c, d)if$ $ad = bc\ for\ (a, b), (c, d)\ in$ A×A. Prove that R is an equivalence relation**.** **Solution:** Given A = {1, 2, 3, ……., 9} The relation R is defined as $(a, b)R\ (c, d) if\ ad = bc\ for\ (a, b), (c, d) in$ A × A Now, we prove that R is an equivalence relation. **<u>To show R is reflexive:</u>** Clearly, $ab = ba$ for every a, b ∈ A $\Rightarrow (a, b)R(a, b)\ for\ every\ (a, b) \in$ A × A Hence, R is reflexive. **<u>To show R is symmetric:</u>** Let (a, b) R (c, d)

	$\Rightarrow ad = bc$ $\Rightarrow da = cb$ $\Rightarrow cb = da$ $\Rightarrow$ (c, d)R (a, b) Hence, R is symmetric **<u>To show R is transitive:</u>** Let (a, b) R (c, d) $\Rightarrow ad = bc$ …….. (1) And (c, d) R (e, f) $\Rightarrow cf = de$ ………. (2) Multiply (1) and (2), we get $(ad)\,(cf) = (bc)(de)$ $\Rightarrow af = be$ $\Rightarrow (a, b)R(e, f)$ Hence, R is transitive. Therefore, R is reflexive, symmetric and transitive. Hence, R is an equivalence relation.
5.	Consider a function $f: \left[0, \frac{\pi}{2}\right] \longrightarrow R \ given\ by\ f(x) = sinx\ and\ g: \left[0, \frac{\pi}{2}\right] \longrightarrow R$ given by $g(x) = cosx$. Show that $f\ and\ g$ are one-one but $f + g$ is not one-one. **Solution:** <u>To prove f is one-one:</u> $The\ function\ f: \left[0, \frac{\pi}{2}\right] \longrightarrow R\ given\ by\ f(x) = sinx$ Clearly, different elements in the domain $\left[0, \frac{\pi}{2}\right]$ of ‘f’ have distinct images in the co-domain of ‘f’ Hence, f is one-one. <u>To prove g is one-one:</u> $The\ function\ g: \left[0, \frac{\pi}{2}\right] \longrightarrow R\ given\ by\ g(x) = cosx$ Clearly, different elements in the domain $\left[0, \frac{\pi}{2}\right]$ of ‘g’ have distinct images in the co-domain of ‘g’ Hence, ‘g’ is one-one. <u>To prove $f + g$ is one-one:</u> $(f + g)(x) = f(x) + g(x) = sinx + cosx$ $(f + g)(0) = sin0 + cos0 = 0 + 1 = 1$ $(f + g)\left(\frac{\pi}{2}\right) = sin\frac{\pi}{2} + cos\frac{\pi}{2} = 1 + 0 = 1$ From this, we conclude that different elements in the domain of $f + g$ have same images in the co-domain of $f + g$. Hence, $f + g$ is not one-one.

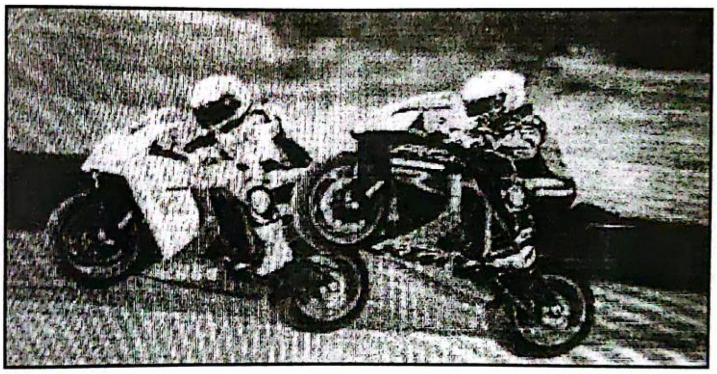
EXERCISE

1	Let f : R → R be a function defined as $f(x) = 4x + 3$, then show that f is one-one and onto.
2	A function $f: [-4,4] \to [0,4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.
3	A relation R is defined on a set of real numbers $\mathbb{R}$ as $$R = \{(x, y):\ xy\ \ is\ an\ irrational\ number\}$$ Check whether R is reflexive, symmetric and transitive or not.
4	Show that the relation R in the set A = $\{x \in Z: 0 \le x \le 12\}$, given by R = {(a, b): $\lvert a - b \rvert$ is divisible by 3} is an equivalence relation.

CASE STUDY QUESTIONS

I

Read the passage given below and answer the questions that follow:
An organisation conducted a bike race under two different categories boys and girls .Among all the participants finally three from category 1(boys) and two from category 2(girls) were selected for the final race. Let $B=\{b_1,b_2,b_3\}$,$G=\{g_1,g_2\}$, where B represents the set of boys selected and G the set of girls who were selected for the final race.



1. How many relations are possible from B to G?(1mark)
2. How many functions can be formed from B to G? (1mark)
3. How many one-one functions be formed from B to G? justify your answer. (2marks)

Solution:

(1) n(B)= 3, n(G)=2

Number of relations from B to G is 2^6 ,because every relation is a subset of B×G and there are 3×2=6 elements in B×G.

(2) Number of functions= 2^3=8.

(3) Number of one-one functions = 0 as n(B) > n(G).

II

Priya and Surya are playing monopoly in their house during COVID. While rolling the dice their mother Chandrika noted the possible outcomes of the throw every time belongs to the set $\{1, 2, 3, 4, 5, 6\}$. Let A denote the set of players and B be the set of all possible outcomes. Then $A = \{P, S\}$ $\quad B = \{1, 2, 3, 4, 5, 6\}$.Then answer the below questions based on the given information.(each question carries one mark)

(a) Let $R: B \rightarrow B$ be defined by $R = \{(a, b)\ both\ a\ and\ b\ are\ either\ odd\ or\ even\}$, then is R an Equivalence relation?

(b) Chandrika wants to know the number of functions from A to B. How many numbers of functions are possible?

(c) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then is R relexive, symmetric and transitive?

(d) Let $R: B \rightarrow B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$ then is R symmetric? Justify

Solution:

(a) Yes, it is equivalence.
The relation is reflexive,symmetric and transitive hence it is Equivalence.

(b) If $n(A) = m, n(B) = n$, then the number of functions from A to B is n^m
Ans: 6^2

(c) As $(1,1) \notin R$ It is not reflexive
As $(1,2) \in R$ but $(2,1) \notin R$ it is not symmetric
As $(1,3) \in R$ and $(3, 4) \in R$ but $(1,4) \notin R$,it is not transitive

Hence none.

(d) No, $(1,2) \in R$ but $(2,1) \notin R$ it is not symmetric

III

In two different societies, there are some school going students – including girls as well as boys. Satish forms two sets with these students, as his college project.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where $a_i's, b_i's$are the school going students of first and second society respectively.

Using the information given above, answer the following question

(a) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible? (1mark)

(b) Let $R: A \to A, R = \{(x, y): x \text{ and } y \text{ are students of same sex}\}$. Then relation is R an equivalence relation. (1 mark)

(c) Let $R: A \to B, R = \{(a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$, then is R One-one and onto ? Justify (2marks)

Solution:

(a) 2^{20}

If $n(A) = n$ then number of reflexive relations that can be defined on A is 2^{n^2-n}.here $n(A) = 5$.

(b) Yes the relation is equivalence.

As the relation is reflexive,symmetric and transitive.

(c) No R is not one-one and onto

Distinct elements of A have same image a_1 and a_2 are having same

Image b_1 ,so R is not one-one.

b_4 is not having pre-image ,so R is not onto.

EXERCISE

I

Vani and Mani are playing Ludo at home while it was raining outside.While rolling the dice Vani’s brother Varun observed and noted the possible outcomes of the throw every time belongs to the set {1,2,3,4,5,6}.Let A be the set of players while B be the set of all possible outcomes.

A= {Vani, Mani}, B= {1, 2, 3, 4, 5, 6}.

Answer the following questions:

a) Let $R: B \to B$,be defined by $R = \{(x, y): y \text{ is divisible by } x\}$.Verify that whether R is reflexive, symmetric and transitive.(2marks)

b) Is it possible to define an onto function from A to B? Justify.(1mark)

c) Which kind of relation is R defined on B given by

R={(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)}?(1mark)

Or

Find the number of possible relations from A to B.

Answer:a) R is reflexive and transitive but not symmetric.

b) No. Because n(B) is greater than n(A)

c) R is neither reflexive nor symmetric nor transitive

or

no. of relations= 2^{12}

II	 Students of class X planned to plant saplings along straight lines,parallel to each other to one side of the playground ensuring that thy had enough play area.Let us assume that they planted one of the rows of the saplings along the line $y = x + 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L. i) R = $\{(l_1, l_2): l_1 \perp l_2, where\ l_1, l_2 \in L\}$. What is the type of relation R? (1mark) ii) R = $\{(l_1, l_2): l_1 \| l_2,\ where\ \ l_1, l_2 \in L\}$. What is the type of relation R? (1mark) iii) Check whether the function $f: R \to R\ defined\ by\ f(x) = x + 4$ is one-one and onto. (2marks) **Answer:** i) R is symmetric but neither reflexive nor transitive. ii) R is an equivalence relation. iii)R is both one-one and onto

CHAPTER: INVERSE TRIGONOMETRIC FUNCTIONS

SYLLABUS: *Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.*

Definitions and Formulae:

Principal Value Branches:

FUNCTION	DOMAIN	Range (Principal Value Branch)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\mathrm{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$(0, \pi)$

$\sin^{-1}(-x) = -\sin^{-1}(x)$	$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$
$\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}(x)$	$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$
$\tan^{-1}(-x) = -\tan^{-1}(x)$	$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

$sin^{-1}(sin\, x) = x, \quad x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$cos^{-1}(cos\, x) = x, \quad x \in [0, \pi]$
$sin(sin^{-1} x) = x, \quad x \in [-1,,1]$ …

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND SOLUTIONS
1	The principal value of $cos^{-1}\left(\frac{-1}{2}\right)$`    (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (iii) $\frac{-\pi}{3}$ (d) $\frac{-\pi}{6}$    **Solution:** We have $cos^{-1}(-x) = \pi - cos^{-1}(x)$    $cos^{-1}\left(\frac{-1}{2}\right)$`$= \pi - cos^{-1}\left(\frac{1}{2}\right)$. $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ Ans: (a)
2	The principal value of $sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]$` (a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$ (iii) $\frac{-2\pi}{5}$ (d) $\frac{\pi}{5}$ **Solution:** We have $sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right] = sin^{-1}\left[sin\left(\pi - \frac{3\pi}{5}\right)\right]$ $= sin^{-1}\left[sin\left(\frac{2\pi}{5}\right)\right]$ $= \frac{2\pi}{5}$ Ans. (b)
3	The value of: $tan^{-1}\sqrt{3} - sec^{-1}(-2)$ is (a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$ (c) $\frac{-\pi}{3}$ (d) 0 **Solution:** We have $sec^{-1}(-x) = \pi - sec^{-1}(x)$. $\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - (\pi - \frac{\pi}{3}) = \frac{-\pi}{3}$ **Ans: (c)**
4	The value of $sin\left[\frac{\pi}{3} - sin^{-1}\left(\frac{-1}{2}\right)\right]$ is (a) 0 (b) 1 (c) -1 (d) 2 **Solution:** We have $sin^{-1}(-x) = -sin^{-1}(x)$

	$\therefore \sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \left(\frac{-\pi}{6}\right)\right] = \sin\left(\frac{\pi}{2}\right) = 1$ Ans: (b)
5	The principal value of $cos^{-1}\left(cos\left(\frac{-7\pi}{3}\right)\right)$ is (a) $\frac{7\pi}{3}$ (b) $\frac{\pi}{3}$ (iii) $\frac{-\pi}{3}$ (d) $\frac{-7\pi}{3}$ **Solution:** We have $\cos(-x) = cosx$ $\therefore \cos^{-1}\cos\left(\frac{-7\pi}{3}\right) = \cos^{-1}\cos\left(\frac{7\pi}{3}\right)$ $= \cos^{-1}\cos\left(2\pi + \frac{\pi}{3}\right)$ $= \cos^{-1}\cos\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$ Ans: (b)
6	The value of $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$ is (a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$ **Solution:** $\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$ = $\pi - \frac{\pi}{6} = 5\frac{\pi}{6}$ $\therefore \tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ = $\frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$ **Ans: (b)**
7	The value of x if $tan^{-1}\sqrt{3} + cot^{-1}x = \frac{\pi}{2}$ (a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\pi}{6}$ **Solution:** $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}\sqrt{3}$ = $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ $\therefore x = \cot\frac{\pi}{6} = \sqrt{3}$ **Ans: (a)**

8	The value of x if $sec^{-1}2 + cos\,e\,c^{-1}x = \frac{\pi}{2}$ (a) $\sqrt{3}$ (b) 2 (c) $\frac{\sqrt{3}}{2}$ (d) -2 **Solution:** $\cos ec^{-1}x = \frac{\pi}{2} - \sec^{-1}\sqrt{3}$ = $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ $\therefore x = \cos ec\frac{\pi}{6} = 2$ **Ans: (b)**
9	If $sin^{-1}x = y$ then the principal value of y is: (a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 < y < \pi$ **Ans: (b)**
10	If $tan^{-1}x = y$ then the principal value of y is: (a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 < y < \pi$ **Ans: (c)**

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
INVERSE TRIGONOMETRIC FUNCTIONS	https://youtu.be/wGsA6WJXvP8	

EXERCISE

1	The value of $cos\left(tan^{-1}\frac{3}{4}\right)$ is : (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{3}{7}$

	Answer: (b) $\frac{4}{5}$
2	The principal value of : $tan^{-1}\left[tan\left(\frac{5\pi}{4}\right)\right]$\` (a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{4}$ (iii) $\frac{-\pi}{4}$ (d) 1 Answer: (b) $\frac{\pi}{4}$
3	The value of $cot\ \left(cos^{-1}\frac{7}{25}\right)$ is : (a) $\frac{7}{24}$ (b) $\frac{24}{25}$ (c) $\frac{7}{25}$ (d) $\frac{25}{7}$ Answer: (a) $\frac{7}{24}$
4	The value of $cos^{-1}\left(cos\left(\frac{14\pi}{3}\right)\right)$ is : (a) $\frac{14\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{4\pi}{3}$ Answer: (c) $\frac{2\pi}{3}$
5	The value of $2\,sin^{-1}\left(\frac{1}{2}\right) + cot^{-1}(1)$ (a) $\frac{7\pi}{12}$ (b) $\frac{3\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$ Answer:(a) $\frac{7\pi}{12}$

ASSERTION-REASON BASED QUESTIONS

	In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.
1	ASSERTION (A): Principal value of $cos^{-1} cos\left(\frac{7\pi}{6}\right)$ is $\frac{5\pi}{6}$ REASON (R): Range of principal branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$. Ans: (a)
2	ASSERTION (A): Principal value of $sin^{-1} sin\left(\frac{13\pi}{6}\right)$ is $\frac{\pi}{6}$ REASON (R): $\sin^{-1}(-x) = -\sin^{-1}(x)$ Ans:(b)
3	ASSERTION (A): Principal value of $sin^{-1}(-1) = \frac{-\pi}{2}$ REASON (R): $\sin^{-1}(-x) = -\sin^{-1}(x)$ Ans: (a)

4	ASSERTION (A): Principal value of $sin^{-1} sin\left(\frac{3\pi}{5}\right) = \frac{3\pi}{5}$ REASON (R): $\sin^{-1}\sin(x) = x, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ Ans: (d)
5	ASSERTION (A): The principal value of $cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$` REASON (R): cosine function is an even function, therefore $\cos(-x) = cosx$. Ans: (d)
6	ASSERTION (A): The principal value of $cos^{-1}\left(\frac{-1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right)$` REASON (R): Range of $\cos^{-1}x$ is $[0, \pi]$ Ans: (b)
7	ASSERTION (A): The principal value of $tan^{-1}\left[sin\left(\frac{-\pi}{2}\right)\right] = \frac{-\pi}{2}$` REASON (R): $\tan^{-1}(-x) = \tan^{-1}(x)$ Ans: (d)
8	ASSERTION (A): The principal value of $tan^{-1} tan\left(\frac{-\pi}{4}\right) = \frac{-\pi}{4}$` REASON (R): Range of $\tan^{-1} x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\tan^{-1}(\tan x) = x$ if $x \epsilon \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ Ans: (a)
9	ASSERTION(A): One branch of $cos^{-1}x$ other than the principal value branch is $[\pi, 2\pi]$ REASON (R): $cos\left(\frac{-\pi}{2}\right) = -1$ Ans: (c)
10	ASSERTION (A): One branch of $sin^{-1}x$ other than the principal value branch is $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ REASON (R): $sin\left(\frac{3\pi}{2}\right) = sin\left(\frac{\pi}{2}\right) = 1$ Ans: (c)

EXERCISE

1	ASSERTION (A): $sin(cos^{-1} x) = cos(sin^{-1} x) = \sqrt{1 - x^2}, \quad \|x\| \leq 1$ REASON (R): Because $sin^2 \theta + cos^2 \theta = 1$ Answer: (b)
2	ASSERTION (A): The principal value of $cos^{-1}\left[cos\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$ REASON (R): Range of $\cos^{-1} x$ is $[0, \pi]$ $\cos^{-1}(cosx) = x$ if $x \epsilon [0, \pi]$ Answer: (d)
3	ASSERTION (A): The principal value of $sin[cot^{-1}(cos(tan^{-1} 1))] = \sqrt{\frac{2}{3}}$

	REASON (R): Range of $\tan^{-1} x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\tan^{-1}(tanx) = x$ if $x\epsilon \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ Answer: (b)
4	ASSERTION (A): The principal value of $cos^{-1} cos\left(\frac{-\pi}{4}\right) = \frac{\pi}{4}$ REASON (R): Cosine function is an even function, therefore $\cos(-x) = cosx$. and $\cos^{-1}(cosx) = x$ if $x\epsilon\,[0,\pi]$ Answer: (a)
5	ASSERTION (A): The value of $sin\left[2\,sin^{-1}\left(\frac{3}{4}\right)\right] = \frac{3}{4}$ REASON (R): $sin(sin^{-1} x) = x,\quad x \in [-1,,1]$ Answer: (d)

2 MARK QUESTIONS

1	Find the value of $sin^{-1}\left[sin\left(\frac{2\pi}{3}\right)\right] + cos^{-1}\left[cos\left(\frac{2\pi}{3}\right)\right]$. **Solution:** We have $sin^{-1} sin\left(\frac{2\pi}{3}\right) = sin^{-1} sin\left(\pi - \frac{\pi}{3}\right)$ $= sin^{-1} sin\left(\frac{\pi}{3}\right)$ $= \frac{\pi}{3}$ $Value\quad of\quad cos^{-1} cos\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3}$ $\therefore sin^{-1} sin\left(\frac{2\pi}{3}\right) + cos^{-1} cos\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} + \frac{2\pi}{3} = \pi$
2	Find the value of: $tan^{-1}\left[2\,cos(\,sin^{-1}\left(\frac{1}{2}\right))\right]$ **Solution:** $tan^{-1}\left[2\,cos(\,sin^{-1}\left(\frac{1}{2}\right))\right]=tan^{-1}\left[2\,cos(\frac{\pi}{6})\right]=tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=tan^{-1}\sqrt{3}=\frac{\pi}{3}$
3	Find the value of: $tan^{-1}\left[2\,sin(\,2\,cos^{-1}\left(\frac{\sqrt{3}}{2}\right))\right]$ **Solution:** $tan^{-1}\left[2\,sin(\,2\,cos^{-1}\left(\frac{\sqrt{3}}{2}\right))\right]=tan^{-1}\left[2\,sin(\,2\times\frac{\pi}{6})\right]$ $= tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=tan^{-1}\sqrt{3}=\frac{\pi}{3}$
4	If $cot^{-1}\left(\frac{1}{5}\right) = x, then\ find\ the\ value\ of\ \sin x$

	Solution: $cotx = \frac{1}{5}$ $\therefore sin\, x = \frac{5}{\sqrt{26}}$
5	Find the value of $sin^{-1}\left(\frac{-1}{2}\right) + 2\,cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ **Solution:** $sin^{-1}\left(\frac{-1}{2}\right) = -\,sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$ $cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\therefore sin^{-1}\left(\frac{-1}{2}\right) + 2\,cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{6} + 2 \times \frac{5\pi}{6} = \frac{3\pi}{2}$
6	Show that for $\lvert x \rvert < 1$, $sin(tan^{-1}\, x) = \frac{x}{\sqrt{1+x^2}}$ **Solution:** Let $tan^{-1}\, x = y$ $\therefore \quad tany = x$ $\therefore$ L H S = $sin\, y = \frac{x}{\sqrt{1+x^2}} \quad = R\ H\ S$
7	Prove that: $tan\left(\frac{1}{2} sin^{-1} \frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ **Solution**: Let $sin^{-1} \frac{3}{4} = x$ $\therefore \; sin\, x = \frac{3}{4}$ $\therefore \; cos\, x = \frac{\sqrt{7}}{4}$ L H S = $tan \frac{x}{2} = \sqrt{\frac{1-cos\, x}{1+cos\, x}} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}} = \frac{4-\sqrt{7}}{3} = RHS$
8	Find the value of $tan\left(2\, tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right)$ **Solution:** Let $tan^{-1} \frac{1}{5} = x$ $\therefore \; tan\, x = \frac{1}{5}$ $\therefore \quad tan\left(2\, tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right) = tan\left(2x - \frac{\pi}{4}\right) = \frac{tan\, 2x - tan\frac{\pi}{4}}{1 + tan\, 2x\, tan\frac{\pi}{4}} = \frac{-7}{17}$ where, $tan\, 2\, x = \frac{2\, tan\, x}{1 - tan^2\, x} = \frac{2 \times \frac{1}{5}}{1 - \left(\frac{1}{5}\right)^2} = \frac{5}{12}$

9	Find the value of $sin^{-1}\left(\frac{-1}{2}\right) + 2\, cos^{-1}\left(\frac{-1}{2}\right) + tan^{-1}(1)$ **Solution:** $sin^{-1}\left(\frac{-1}{2}\right) = -\, sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$ $cos^{-1}\left(\frac{-1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $tan^{-1}(1) == \frac{\pi}{4}$ $\therefore sin^{-1}\left(\frac{-1}{2}\right) + 2\, cos^{-1}\left(\frac{-1}{2}\right) + tan^{-1}(1) = \frac{-\pi}{6} + \frac{2\pi}{3} + \frac{\pi}{4} = \frac{3\pi}{4}$
10	Find the value of $sin\left(2\, sin^{-1}\frac{3}{5}\right)$ **Solution:** Let $sin^{-1}\left(\frac{3}{5}\right) = \theta$ $\therefore\ sin\,\theta = \frac{3}{5}$ $\therefore\ sin\left(2\, sin^{-1}\frac{3}{5}\right) = sin\, 2\,\theta = 2\, sin\,\theta\, cos\,\theta$ $= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

2 MARKS

1	Find the value of $\sin\left[2\cot^{-1}\left(-\frac{5}{12}\right)\right]$ Answer: $-\frac{120}{169}$
2	Find the value of $tan\left[\frac{\pi}{6} - tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ Answer: 0
3	Find the value of $sin\ \ (cot^{-1} x)$ in terms of x Answer: $\frac{1}{\sqrt{1+x^2}}$
4	Find the value of $sin\left[cot^{-1}\left(\frac{4}{3}\right)\right]$ Answer: $\frac{3}{5}$
5	Find the principal of $tan^{-1}\, tan\left(\frac{7\pi}{6}\right) + cot^{-1}\, cot\left(\frac{7\pi}{6}\right)$ Answer: $\frac{\pi}{3}$

CHAPTER: MATRICES

SYLLABUS: *Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).*

Definitions and Formulae:

- Matrix representation and order of matrix
- Types of Matrices
- Operations on Matrices
- Transpose of a Matrix
- Symmetric and Skew Symmetric Matrices
- Invertible Matrices

Order of Matrix:

If a matrix has m *rows and* n *columns*, then it is known as the matrix of order $m \times n$.

Representation of matrix

A general matrix of order $m \times n$ can be written as

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{pmatrix}$$

$$= [a_{ij}]_{m\times n}, where\ i = 1,2, \dots m\ and\ j = 1,2, \dots n$$

Types of Matrices:

Depending upon the order and elements, matrices are classified as:

- Column matrix
- Row matrix
- Square matrix
- Diagonal matrix
- Scalar matrix
- Identity matrix
- Zero matrix

Type of Matrix	Definition	Example
COLUMN MATRIX	A matrix is said to be a column matrix if it has only one column	$\begin{bmatrix} 2 \\ -9 \end{bmatrix}$ $0rder 2 \times 1$ $\begin{bmatrix} -\sqrt{5} \\ 0 \\ -12 \end{bmatrix}$ $order\ 3 \times 1$
ROW MATRIX	A matrix is said to be a row matrix if it has only one row	$\begin{bmatrix} 14 & 26 \end{bmatrix}$order1×2 $\begin{bmatrix} 0 & \sqrt{7} & 12 \end{bmatrix}$ order1×3
SQUARE MATRIX	A matrix in which the number of rows is equal to the number of columns, is said to be a square matrix.	$\begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}$ $\begin{bmatrix} 5 & 0 & -8 \\ 0 & 1 & 14 \\ 7 & -8 & 4 \end{bmatrix}$
DIAGONAL MATRIX	A square matrix A is said to be a diagonal matrix if all its non-diagonal elements are zero	$\begin{bmatrix} 6 & 0 & 0 \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & 9 \end{bmatrix}$
SCALAR MATRIX	A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal	$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
IDENTITY MATRIX	A square matrix in which all the elements in the diagonal are all equal to one and rest are all zero is called an identity matrix. And generally it is denoted by I.	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
ZERO MATRIX	A matrix is said to be zero matrix or null matrix if all its elements are zero.	A=$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B=$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- **OPERATION OF MATRICES:**

ADDITION OF MATRICES: Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two matrices of the same order. Then $A + B$ is defined to be the matrix of order of $m \times n$ obtained by adding corresponding elements of A and B

i.e $A + B = [a_{ij} + b_{ij}]_{m\times n}$

DIFFERENCE OF MATRICES: Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two matrices of the same order. Then $A - B$ is defined to be the matrix of order of $m \times n$ obtained by subtracting corresponding elements of A and B

i.e $A - B = [a_{ij} - b_{ij}]_{m\times n}$

MULTIPLICATION OF MATRICES: The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.

Let $= [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$. Then the product of the matrices A and B is the matrix C of order $m \times p$

Example: Let $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 6 & 9 \\ 5 & 8 \end{bmatrix}$

$$AB = \begin{bmatrix} 2\times 4 + 3\times 6 + 5\times 5 & 2\times 3 + 3\times 9 + 5\times 8 \\ 1\times 4 + 6\times 6 + 8\times 5 & 1\times 3 + 6\times 9 + 8\times 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 + 18 + 25 & 6 + 27 + 40 \\ 4 + 36 + 40 & 3 + 54 + 64 \end{bmatrix} = \begin{bmatrix} 51 & 73 \\ 80 & 121 \end{bmatrix}$$

MULTIPLICATION OF A MATRIX BY A SCALAR:

Let $A = [a_{ij}]_{m\times n}$ and k is a scalar, then $kA = k[a_{ij}]_{m\times n} = [k.a_{ij}]_{m\times n}$

Example: $\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{4} & \mathbf{-5} \\ \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{x} \end{bmatrix} \Rightarrow \mathbf{3}\boldsymbol{A} = \begin{bmatrix} \mathbf{3(2)} & \mathbf{3(4)} & \mathbf{3(-5)} \\ \mathbf{3}\boldsymbol{y} & \mathbf{3}\boldsymbol{z} & \mathbf{3}\boldsymbol{x} \end{bmatrix} =$
$\begin{bmatrix} \mathbf{6} & \mathbf{12} & \mathbf{-15} \\ \mathbf{3}\boldsymbol{y} & \mathbf{3}\boldsymbol{z} & \mathbf{3}\boldsymbol{x} \end{bmatrix}$

- **TRANSPOSE OF A MATRIX:** If $A = [a_{ij}]_{m\times n}$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or AT.

  If $A = [a_{ij}]_{m\times n}$, then $A' = [a_{ji}]_{n\times m}$

  Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 7 & 9 \\ 5 & 1 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 7 & 1 \\ 3 & 9 & 0 \end{bmatrix}$

- **SYMMETRIC MATRIX:** A square matrix If $A = [a_{ij}]$ is said to be symmetric if $A^T = A$

Example: $A = \begin{bmatrix} 2 & 5 & 12 \\ 5 & 7 & 3 \\ 12 & 3 & 6 \end{bmatrix}$

- **SKEW-SYMMETRIC MATRIX:** A square matrix $A = [a_{ij}]$ is said to be skew symmetric matrix if $A^T = -A$.

  Example: $A = \begin{bmatrix} 0 & 5 & -12 \\ -5 & 0 & -3 \\ 12 & 3 & 0 \end{bmatrix}$

- **INVERTIBLE MATRICES:** If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible.

- **PROPERTIES OF MATRICES:**
  - $A + B = B + A$
  - $A - B \neq B - A$
  - $AB \neq BA$
  - $(AB)C = A(BC)$
  - $(A')' = A$
  - $AI = IA = A$
  - $AB = BA = I, then\ A^{-1} = B\ and\ B^{-1} = A$
  - $AB = 0 \Longrightarrow$ $it\ is\ not\ necessary\ that\ one$ of $the\ matrix\ is\ zero.$
  - $A(B + C) = AB + AC$
  - Every square matrix can possible to express as the sum of symmetric and skew-symmetric matrices.

    $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where $\frac{1}{2}(A + A')$ is symmetric matrix and $\frac{1}{2}(A - A')$ is skew-symmetric matrices.

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND SOLUTIONS
1	A is 2×2 matrix and A=$[a_{ij}]$ where $a_{ij} = (i+j)^2$, then A is (a) $\begin{bmatrix}4 & 9\\9 & 16\end{bmatrix}$ (b) $\begin{bmatrix}9 & 4\\16 & 9\end{bmatrix}$ (c) $\begin{bmatrix}2 & 3\\3 & 2\end{bmatrix}$ (d) *none of the above* **Solution:** $a_{11} = 4$, $a_{12} = 9$, $a_{21} = 9$, $a_{22} = 16$ Option (a) $\begin{bmatrix}4 & 9\\9 & 16\end{bmatrix}$
2	A and B are two matrices such that $AB = A$ *and* $BA = B$ then B^2 is (a) A (b) B (c) 0 (d) I **Solution:** $B^2 = BB = (BA)B = B(AB) = BA = B$ Option: (b) B
3	If A = $\begin{bmatrix}x & 1\\1 & 0\end{bmatrix}$ and A^2 is unit matrix, then what is value of x? (a) 1 (b) 2 (c) 0 (d) -1 **Solution:** $A^2 = \begin{bmatrix}x & 1\\1 & 0\end{bmatrix}\begin{bmatrix}x & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}x^2+1 & x\\x & 1\end{bmatrix}$ $x^2 + 1 = 1$ .So $x = 0$ Option (c)0
4	A = $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, then A^{10} is (a) 10A (b)9A (c) 2^9A (d)2^{10}A **Solution:** $A^2 = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$ =2I, A^3 =4I=2^2I….. and so $A^{10} = 2^9$I Option: (c) 2^9A
5	A is a 3 × 4 matrix. A matrix B is such that A'B and BA' are defined. Then the order of B is (a) 4 × 3 (b) 3 × 3 (c) 4 × 4 (d) 3 × 4 **Solution:** Let $O(B) = m \times n$. A'B is defined. So $m = 3$. BA' is defined. So $n = 4$. Option: (d) 3 × 4
6	If $A = [a_{IJ}] = \begin{bmatrix}2 & 3 & -5\\1 & 4 & 9\\0 & 7 & -2\end{bmatrix}$, then find $a_{12}a_{21} + a_{22}$ (a) 4 (b)12 (c) -4 (d) 7 **Solution:** $a_{12}a_{21} + a_{22} = 3 \times 1 + 4 = 7$ Option: (d) 7
7	The matrix A= $\begin{bmatrix}0 & 0 & 4\\0 & 4 & 0\\4 & 0 & 0\end{bmatrix}$ is a (a) diagonal matrix (b)square matrix

	(c) unit matrix (d) None of these **Solution:** Ans: (b) square marix
8	A and B are symmetric matrices of same order , then $\mathrm{AB^T - BA^T}$ is always (a) symmetric matrix (b) skew symmetric matrix (c) zero matrix (d) unit matrix **Solution:** $(\mathrm{AB^T - BA^T})^\mathrm{T} = (\mathrm{B^T})^\mathrm{T}\mathrm{A^T} - (\mathrm{A^T})^\mathrm{T}\mathrm{B^T} = \mathrm{BA^T - AB^T}$ Option: (b)skew symmetric matrix
9	If A=$\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$, then $\mathrm{A^2 - 3I}$ is (a) 2A (b)3A (c) zero matrix (d) 2I **Solution:** $\mathrm{A}^2 = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} - \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix} = \begin{bmatrix}2 & 4\\4 & 2\end{bmatrix}$ Option: (a)2A
10	If $A = \begin{bmatrix}5 & \mathrm{x}\\\mathrm{y} & 0\end{bmatrix}$ and $A = A^T$, then (a) $x = 0, y = 5$ (b) $\mathrm{x + y = 5}$ (c) $x = y$ (d) none of these **Solution:** $\begin{bmatrix}5 & \mathrm{x}\\\mathrm{y} & 0\end{bmatrix} = \begin{bmatrix}5 & \mathrm{y}\\\mathrm{x} & 0\end{bmatrix}$ Option: (c) $x = y$
11	The number of all possible matrices of order 3×3 with each entry 0 or 1 is (a) 32 (b) 64 (c) 512 (d) none of these **Solution:** There are nine places. Each can be filled in two ways. 2^9ways Option: (c) 512
12	A is a 2×2 matrix whose elements are given by $$\mathrm{a_{ij}} = \begin{cases}1 & \mathrm{if\ i \neq j}\\0 & \mathrm{, if\ i = j}\end{cases}$$ Then value of $\mathrm{A^2}$is (a)$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}$ **Solution:** In A=$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, $\mathrm{A}^2 = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ Option: (c) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
13	P and Q are two different matrices order $3 \times n$ and $n \times p$, then the order of matrix P×Q is (a) $3 \times p$ (b) $p \times 3$ (c) $n \times n$ (d) 3×3 **Solution:** By property of order of matrix, order of matrix P×Q is $3 \times p$ Option: (a) $3 \times p$
14	If $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathrm{x}\\-\mathrm{y}\\\mathrm{z}\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix}$, find value of $x + 2y + 3z$. (a) 1 (b) 2 (c) 0 (d) -1 **Solution:**

	$\begin{bmatrix} x \\ -y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $\quad x = 1. y = -2, z = 1$ Option: (c) 0
15	A and B are two matrices such that AB exists, then which is true (a) $AB = BA$ (b) BA does not exist. (c) BA may or may not exists. (d) none of these **Solution:** If O(A)=3×2 and O(B) is 2×3, ,then both AB and BA possible If O(A)=1×2 and O(B) is 2×3, ,then AB possible ,but not BA Option: (c) BA may or may not exist.
16	For a 3×3 matrix $A = [a_{ij}]$ whose elements defined by $a_{ij} = \frac{i}{j}$, then write $a_{12} + a_{21}$ (a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{5}{2}$ **Solution:** $a_{11} = \frac{1}{2}$ $a_{21} = \frac{2}{1}$. $A_{12} + a_{21} = \frac{5}{2}$ Option: (d) $\frac{5}{2}$
17	A is a matrix of order 2×3 and B is a matrix of order 3×2. $C = AB$ *and* $D = BA$, then order of CD is (a) 3×3 (b) 2×2 (c) 3×2 (d) CD not defined **Solution:** O(C)=2×2 and O(D)=3×3. The number of columns of A not equal to number of rows of B. Therefor CD not defined Option: (d) CD not defined
18	$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = I$, then which of the following is correct? (a) $1 + \alpha^2 + \beta\gamma=0$ (b) $1 - \alpha^2 + \beta\gamma=0$ (c) $1 - \alpha^2 - \beta\gamma=0$ (d) $1 + \alpha^2 - \beta\gamma=0$ **Solution:** $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \alpha^2 + \beta\gamma=1$ Ans: (c) $1 - \alpha^2 - \beta\gamma=0$

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
MATRICES	https://youtu.be/DuFAHDTsDms	

EXERCISE

1	Given that A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, the order of matrix $C = 5A + 3B$ is (a) 3×5 (b) 5×3 (c)3×3 (d) 5×5
2	If A= $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is identity then value of α (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{3}$
3	The elements of 3×3 matrix are given by $$a_{ij} = \frac{1}{2}(-3i + j)$$ Write the element a_{32} (a) -7/2 (b) 18 (c)27 (d) 3
4	If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ and B=$\begin{bmatrix} -4 \\ 3 \end{bmatrix}$, AB= (a)$\begin{bmatrix} -7 \\ 2 \end{bmatrix}$ (b) (a)$\begin{bmatrix} 2 \\ 7 \end{bmatrix}$ (c) (a)$\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ (dα) (a)$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$
5	A=$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A^T = I$ .find value of α a) π/6 (b) π/3 (c) π (d) 3π/2
Answers: 1. (a) 3×5 2. (c) 0 . 3. (a) -7/2 4. (a)$\begin{bmatrix} -7 \\ 2 \end{bmatrix}$ 5. (b) π/3	

ASSERTION - REASON BASED QUESTIONS

Q.NO	QUESTION AND ANSWER
	In the following questions from 1 to 20 , a statement of Assertion(A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both(A) and (R) are true and (R) is the correct explanation of (A). (b) Both(A) and (R) are true and (R) is not the correct explanation of (A) (c) (A) is true and (R) is false (d) (A) is false and (R) is true
1	**Assertion (A):** matrix $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ is a column matrix **Reason (R):** Any matrix of order $n \times 1$ is called column matrix **Solution:** Matrix$\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ is of order 3×1. Both are correct and second is explanation for first option (a)

2	**Assertion (A):** The sum of two square matrices is always commutative. **Reason (R):** If A and B are two $m \times n$ matrices, then $A + B = B + A$ **Solution:** $A + B = B + A$. Both are correct and second is correct explanation for first. option (a)
3	**Assertion (A):** For two matrices A and B , $AB \neq BA$ **Reason (R):** Matrix multiplication follows the commutative property. **Solution:** AB and BA are different.. e. g [1 2] $\begin{bmatrix} 3 \\ 4 \end{bmatrix} = [8]$ and$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$[1 2], So first statement is correct and second is wrong option (c)
4	**Assertion (A):** For any two matrices of the same order, $(A + B)^T = A^T + B^T$. **Reason (R):** For any two matrices such that AB is defined, then $(A.B)^T = A^T.B^T$ **Solution:** By definition assertion is true and reason is false. Option: (c)
5	**Assertion (A):** $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then A^{10}=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. **Reason (R):** A is unit matrix and A$\times A = A$ **Solution:** Given matrix is unit matrix. By property of unit matrx, $I \times I = I$. Both are correct. and second is correct explanation for first Option (a)
6	**Assertion (A):** If A is a matrix of order 3×3 and B is a matrix of order 2×3, Then order of AB is 3×3. **Reason (R):** Product AB is possible if number of columns of A and number of rows of B are equal . **Solution:** Here number of column of matrix is 3 and number of rows of B is 2. So AB not possible. First statement is not correct. Second statement is correct Option: (d)
7	**Assertion (A):**If A is a square matrix and $\mathrm{A}^2 = \mathrm{A}$, then$(\mathrm{A} + \mathrm{I})^3 - 7\mathrm{A} = \mathrm{I}$ **Reason (R):** $AI = I = AI$ where I is unit matrix. **Solution:** $(I + A)^3 - 7\mathrm{A} = \mathrm{I}^3 + 3\mathrm{A}^2 + 3\mathrm{A} + \mathrm{A}^3 - 7\mathrm{A} = \mathrm{I} + 3\mathrm{A} + 3\mathrm{A} + \mathrm{A} - 7\mathrm{A} = \mathrm{I}$ Since $\mathrm{A}^2 = \mathrm{A}$ and $\mathrm{A}^3 = \mathrm{A}$ So both the statements are correct. Second statement is reason for first. Option (a)
8	**Assertion (A):** If a matrix is skew symmetric, then its diagonal elements must be zero. **Reason (R):** A matrix A is skew symmetric if $\mathrm{A}^\mathrm{T} = \mathrm{A}$ **Solution:** By definition of skew symmetric matrix $a_{ij} = -a_{ji}$. So $a_{jj} = -a_{jj}. hence\ a_{jj} = 0$ Both the statements are true and second is not the reason for first Option (c)

9	**Assertion (A):** The matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & -2 & 6 \\ 5 & 6 & 0 \end{bmatrix}$ is symmetric matrix **Reason (R):** A matrix A is symmetric if $A^T = A$ **Solution:** Here transpose of the given matrix is same matrix, So symmetric. Both the statements are correct. Option (a)
10	**Assertion (A):** The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 5 & -7 & 8 \\ 0 & -1 & 9 \end{bmatrix}$ can be expressed as sum of a symmetric and a skew symmetric matrices **Reason (R):** If A and B, are skew symmetric matrices of same order, then AB is symmetric if $AB = BA$ **Solution:** Assertion is correct, since any matrix A can be written as $A = P + Q$ where $\frac{A+A^T}{2}$ $and\ Q = \frac{A-A^T}{2}$ where P symmetric and Q skew symmetrices. Reason is also correct, since $(AB)^T = B^T A^T = -B \times -A = BA.$ AB is symmetric if $AB = BA$ But second statement is not correct reason for first Option: (b)
11	**Assertion (A):** The product of a matrix and the identity matrix is always the original matrix. **Reason (R):** Identity matrix is a square matrix in which all diagonal elements are zeros and all other elements are unity **Solution:** Assertion(A) is true by property of unit matrix. Reason(R) is wrong. Identity matrix is a square matrix in which all diagonal elements are unity and all other elements are zeros Option (c)
12	**Assertion (A):** If two matrices have the same order, their addition is always defined. **Reason (R):** Matrix multiplication is defined only for matrices with the same order.. **Solution:** Assertion (A) is true, by condition for addition. Reason (R) is not correct since $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ not possible Option (c)
13	**Assertion (A):** The product of two non-square matrices can not be a square matrix.

	Reason (R): Matrix multiplication is defined for matrices if the number of columns of first matrix is equal to number of rows of second matrix **Solution:** Assertion (A) not correct. Let A and B are matrices of orders 2×3 and 3×1 respectively, then AB is of order 2×1 which is not square matrix. Reason (R) is correct. Option: (d)
14	**Assertion (A):**If A and B are symmetric matrices of same order, $AB - BA$ is skew symmetric matrix **Reason (R):** If A and B are symmetric matrices of same order, $AB + BA$ is symmetric matrix **Solution:** Both the statements are correct. . But second statement is not the reason for first statement. $(AB + BA)^T$=$(AB)^T + (BA)^T = B^TA^T + A^TB^T = BA + AB$ and $(AB - BA)^T$=$(AB)^T - (BA)^T = B^TA^T - A^TB^T = BA - AB = -(AB - BA)$ Option: (b)
15	**Assertion (A):** A= $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and B= $\begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$, Then $(A + B)^2 = A^2 + 2AB + B^2$ **Reason (R):** A and B for two matrices$(\mathrm{A} + \mathrm{B})^2 = \mathrm{A}^2 + 2\mathrm{AB} + \mathrm{B}^2$ if AB=BA **Solution:** $(\mathrm{A} + \mathrm{B})^2 = (\mathrm{A} + \mathrm{B})(\mathrm{A} + \mathrm{B}) = \mathrm{A}^2 + \mathrm{AB} + \mathrm{BA} + \mathrm{B}^2$ $= \mathrm{A}^2 + 2\mathrm{AB} + \mathrm{B}^2$ only if $\mathrm{AB} = \mathrm{BA}$ both the statements are true and second statement is reason for first. Option: (a)

EXERCISE

1	Assertion (A): product of a matrix and the identity matrix is always the original matrix. Reason (R): The identity matrix serves as the multiplicative identity for matrices.
2	Assertion (A): Two matrices A and B are of order 2×3 and 3×2 respectively. Then order of AB is 2×2 and order of BA is 3×3. Reason (R): Order of a matrix is $m \times n$ where m is the number of rows and m is the number of columns
3	Assertion (A): Transpose of the matrix $[1 \quad -4 \quad 5]$ is a column matrix Reason (R): Row matrix is of order $1 \times m$.
4	Assertion (A): The element a_{12} in the matrix $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 2 & -1 \\ 2 & 2 & 3 \end{bmatrix}$ is 1 Reason (R): a_{ij} is the element in ith row and jth column.
5	Assertion (A): $\begin{bmatrix} y & 4 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & x \end{bmatrix}$, then $x = 8$ *and* $y = 1$

	Reason (R): Two matrices are equal , if they are of same order and elements in same places are equal.
Answers 1. Option (a) 2.Option (b) 3.Option (b) 4. Option (d) 5.Option (a)	

2 MARK QUESTIONS

1	A=$\begin{bmatrix}1\\2\\3\end{bmatrix}$ and B=$\begin{bmatrix}2 & 2 & 0\end{bmatrix}$ Find AB and BA **Solution:** AB=$\begin{bmatrix}2 & 2 & 0\\4 & 4 & 0\\6 & 6 & 0\end{bmatrix}$ and BA=[6]
2	If $\begin{bmatrix}x+3 & 4\\y-4 & x+y\end{bmatrix}=\begin{bmatrix}5 & 4\\3 & 9\end{bmatrix}$ Find x *and* y **Solution:** $x+3=5,\quad y-4=3,\quad x+y=9$ $x=2$ *and* $y=7$
3	Construct a matrix of order 3×3 whose elements are given by $a_{ij}=1$ if $i\neq j$ and $a_{ij}=0$ if $I=j$. **Solution:** $a_{11}=a_{22}=a_{33}=0$. and other elements zero $\begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$
4	Matrix $\begin{bmatrix}0 & 2b & -2\\3 & 1 & 3\\3a & 3 & -1\end{bmatrix}$ is symmetric, then find a and b. **Solution:** $\begin{bmatrix}0 & 2b & -2\\3 & 1 & 3\\3a & 3 & -1\end{bmatrix}=\begin{bmatrix}0 & 3 & 3a\\2b & 1 & 3\\-2 & 3 & -1\end{bmatrix}$ A=A^T . $b=\frac{3}{2}$ *and* $a=-\frac{2}{3}$
5	If A=$\begin{bmatrix}\cos\propto & -\sin\propto\\\sin\propto & \cos\propto\end{bmatrix}$ and A+A^T=I. then find $\propto$ **Solution:** A+A^T=I $\begin{bmatrix}\cos\propto & -\sin\propto\\\sin\propto & \cos\propto\end{bmatrix}+\begin{bmatrix}\cos\propto & \sin\propto\\-\sin\propto & \cos\propto\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\propto=\frac{\pi}{3}$
6	Let $A=\begin{bmatrix}p & 0\\1 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$. Find the value of p, if $A^2=B$. **Solution:** $\begin{bmatrix}p & 0\\1 & 1\end{bmatrix}\begin{bmatrix}p & 0\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$ $\begin{bmatrix}p^2 & 0\\p+1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$. $p^2=1$ and $p+1=5$. So p has no common value.

7	Evaluate $\begin{bmatrix}7 & 1 & 2\\9 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}+2\begin{bmatrix}4\\2\end{bmatrix}$ Solution: $\begin{bmatrix}7 & 1 & 2\\9 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}=\begin{bmatrix}35\\40\end{bmatrix}$ and $2\begin{bmatrix}4\\2\end{bmatrix}=\begin{bmatrix}8\\4\end{bmatrix}$ Therefore $\begin{bmatrix}7 & 1 & 2\\9 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}+2\begin{bmatrix}4\\2\end{bmatrix}=\begin{bmatrix}43\\44\end{bmatrix}$
8	A=$\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$ and $A^2=kA$. Find the value of k. **Solution:** $\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}=\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}=\begin{bmatrix}k & -k\\-k & k\end{bmatrix}$ $k=2$
9	A=$\begin{bmatrix}2\\3\\6\end{bmatrix}$ and B=$[1\quad 2\quad 6]$ Find A^TB^T. **Solution:** $[2\quad 3\quad 6]\begin{bmatrix}1\\2\\6\end{bmatrix}$ =[2+6+36]=[44]
10	Construct a 2×3 matrix whose elements in i^{th}row and j^{th}column are given by $a_{ij}=\begin{cases}i-j & ,\text{if } i\geq j\\ i+j, & \text{if } i<j\end{cases}$ **Solution:** $a_{11}=0,\quad a_{12=3,}\quad ....\quad a_{32}=5$ Ans:$\begin{bmatrix}0 & 3 & 4\\1 & 0 & 5\end{bmatrix}$
11	A and B are matrices of order 3×4 and 4×3 respectively. Write the order of A^TB^T. **Solution:** 1 order of A^T is 4×3 order of B^T is 3×4 Ans: Order of $A^TB^T\ is\ 4\times 4$
12	If A and B is symmetric matrices of same order, show that AB is symmetric iff $AB=BA$ **Solution:** $(AB)^T=B^TA^T$ = BA $(AB)^T=AB$ iff $AB=BA$
13	If A=$\begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and B=$\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and AB is identity matrix of order 3×3,then find $x+y$. **Solution:** $\begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0 & x+y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $x+y=0$

14	$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$ find $a+b-c+2d$ **Solution:** $2a+b=4,$ $a-2b=-3$ $5c-d=11$ $4c+3d=24$ $a=1, b=2, c=3, d=4.$ Ans: $a+b-c+2d=8$
15	A=$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 6 \\ 3 & 4 & 1 \end{bmatrix}$ Find $a_{12}a_{21}+a_{13}a_{31}$ **Solution:** $a_{12}a_{21}+a_{13}a_{31}=2\times 0 \pm 2\times 3=-6$

EXERCISE

1	P=$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ -1 & 1 & 2 \end{bmatrix}$ and Q=$\begin{bmatrix} 0 & 2 & -1 \\ 1 & 3 & 4 \\ 0 & -2 & -3 \end{bmatrix}$ Find PQ and QP
2	A=$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, Find $A + A^T$ and check whether $A + A^T$ is symmetric or not
3	Given A=$\begin{bmatrix} \alpha & \beta \\ \gamma & \alpha \end{bmatrix}$ and $A^2 = 3I$. Find the value of $3-\alpha^2-\beta\gamma$
4	Find a matrix B such that $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$ B=$\begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$
5	Evaluate $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Answers:

(1) PQ=$\begin{bmatrix} 2 & 2 & -2 \\ 3 & 5 & -2 \\ 1 & -3 & -1 \end{bmatrix}$ and QP = $\begin{bmatrix} 5 & 5 & 6 \\ 3 & 15 & 23 \\ -1 & -9 & -14 \end{bmatrix}$

(2) $A+A^T = \begin{bmatrix} 4 & 7 \\ 7 & 10 \end{bmatrix}$. Yes, it is symmetric. (3) zero

(4) B=$\begin{bmatrix} 6 & -5 \\ -5 & 6 \end{bmatrix}$ (5) $[ac+bd+a^2+b^2+c^2]$

3 MARK QUESTIONS

1	A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ and AA^T=9I. Find x **Solution:** $AA^T=9I$ $\begin{bmatrix} 9 & 4+2x & 0 \\ 4+2x & 5+x^2 & -2-x \\ 0 & -2-x & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ $x=-2$

2	$A=\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ is a skew symmetric matrix, find a and b **Solution:** $A^T = -A$ $\begin{bmatrix}0 & 2 & b\\ a & 0 & 1\\ -3 & -1 & 0\end{bmatrix} = -\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ $a = -2 \text{ and } b = 3$
3	Express $\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix **Solution:** $P=\frac{A+A^T}{2}=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}$ $Q=\frac{A-A^T}{2}=\begin{bmatrix}0 & -\frac{5}{2} & -4\\ \frac{5}{2} & 0 & -3\\ -1 & 3 & 0\end{bmatrix}$ $\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix} + \begin{bmatrix}0 & -\frac{5}{2} & -\frac{3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{bmatrix} = \begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$ P+Q=A
4	Show that $A = \begin{bmatrix}2 & -3\\ 3 & 4\end{bmatrix}$ satisfies the equation $x^2 - 6x + 17I = 0$ **Solution:** $A^2 = \begin{bmatrix}-5 & -18\\ 18 & 7\end{bmatrix}$ $6A=\begin{bmatrix}12 & -18\\ 18 & 24\end{bmatrix}$ and $17I=\begin{bmatrix}17 & 0\\ 0 & 17\end{bmatrix}$ $X^2 - 6x + 17 = \begin{bmatrix}-5 & -18\\ 18 & 7\end{bmatrix} - \begin{bmatrix}12 & -18\\ 18 & 24\end{bmatrix} + \begin{bmatrix}17 & 0\\ 0 & 17\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = \mathbf{0}$
5	A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds,if the trust fund must obtain an annual total interest of Rs 1,800. **Solution:** Let Investment in 1st bond = Rs x So, Investment in 2nd bond = Rs 30,000−x $[x \quad 30000 - x]\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix} = 1800$ x=15000

6	$X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$ Find matrices X and Y. **Solution:** Add both, we get 2X=$\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$ X= $\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$ Y= $\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$
7	A $=\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}$, B $=\begin{bmatrix}5 & 4\\ 3 & 2\end{bmatrix}$. Verify $(\mathrm{AB})^{\mathrm{T}}=\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}$ **Solution:** AB=$\begin{bmatrix}22 & 16\\ 30 & 22\end{bmatrix}$, $(\mathrm{AB})^{\mathrm{T}}$=$\begin{bmatrix}22 & 30\\ 16 & 22\end{bmatrix}$ $\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}=\begin{bmatrix}5 & 3\\ 4 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}=\begin{bmatrix}22 & 30\\ 16 & 22\end{bmatrix}$
8	A= $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$, then show that $(A-2I)(A-3I)=0$ Solution: $A-2I=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}$, $A-3I=\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}$ $(A-2I)(A-3I)=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$
9	Let A and B be symmetric matrices of the same order,then show that (i)$A+B$ is symmetric (ii)$AB-BA$ is skew symmetric (iii) $AB+BA$ is symmetric **Solution:** $(\mathrm{A}+\mathrm{B})^{\mathrm{T}}=\mathrm{A}^{\mathrm{T}}+\mathrm{B}^{\mathrm{T}}=A+B$ $(\mathrm{AB}-\mathrm{BA})^{\mathrm{T}}=(\mathrm{AB})^{\mathrm{T}}-(\mathrm{BA})^{\mathrm{T}}=\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}-\mathrm{A}^{\mathrm{T}}\mathrm{B}^{\mathrm{T}}=\mathrm{BA}-\mathrm{AB}=-(\mathrm{AB}-\mathrm{BA})$ $(\mathrm{AB}+\mathrm{BA})^{\mathrm{T}}=(\mathrm{AB})^{\mathrm{T}}+(\mathrm{BA})^{\mathrm{T}}=\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}+\mathrm{A}^{\mathrm{T}}\mathrm{B}^{\mathrm{T}}=\mathrm{BA}+\mathrm{AB}=(\mathrm{AB}+\mathrm{BA})$
10.	Find the matrix X such that: X$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$ **Solution:** $\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$ $a+4b=-7$, $2a+5b=-8, c+4d=2, 2c+5d=4.$ Solving $a=1, b=-2, c=2$ and $d=0$ $X=\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$

EXERCISE

1	Find a matrix A such that $2A-3B+5C=0$ where $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 2\end{bmatrix}$ and $C=\begin{bmatrix}2 & 0 & 1\\ 7 & 1 & 6\end{bmatrix}$

2	Find the values of x, y, a and b when $\begin{bmatrix}2x+3y & a-2b\\ 2a+b & 3x-2y\end{bmatrix}=\begin{bmatrix}3&8\\6&11\end{bmatrix}$
3	If A=$\begin{bmatrix}1&-1\\2&-1\end{bmatrix}$ and B $=\begin{bmatrix}x&1\\4&-1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$. Find x
4	If A=$\begin{bmatrix}0\\1\\2\end{bmatrix}$, B $=\begin{bmatrix}1&5&7\end{bmatrix}$, then verify $(AB)^T=B^TA^T$
5	$\begin{bmatrix}x&2&-3\\5&y&2\\1&-1&1\end{bmatrix}\begin{bmatrix}3&-1&2\\4&2&5\\2&0&3\end{bmatrix}=\begin{bmatrix}5&3&3\\19&-5&16\\1&-3&0\end{bmatrix}$. Find x and y
Answers: (1) A=$\begin{bmatrix}-8&3&-\frac{5}{2}\\-13&-1&-12\end{bmatrix}$ (2) $a=4, x=3, y=-1$ and $b=-2$ (3) $x=1$ (5) $x=1$ $y=0$	

5 MARK QUESTIONS

1	1.If A=$\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ and $A^3-6A^2+7A+kI=0$. Find k. Solution $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ and $A^2=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$ Sub in eqn $A^3-6A^2+7A+kI$ $=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ $\begin{bmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{bmatrix}+\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}=[0]$ $k=2$
2	Find the value of x, . if $\begin{bmatrix}x&-5&-1\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=0$. Hence find$\begin{bmatrix}x&-5&-1\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}$ **Solution:** $\begin{bmatrix}x&-5&-1\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}x-2&-10&2x-8\end{bmatrix}$ $\begin{bmatrix}x-2&-10&2x-8\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=[x^2-48]$ $[x^2-48]=0$ $x=\pm 4\sqrt{3}$ $\begin{bmatrix}x&-5&-1\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=[48-20-1]=[27]$

3	Find a matrix A such that $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix} A = \begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$ **Solution:** Let A=$\begin{bmatrix}x & y & z\\ a & b & c\end{bmatrix}$ Getting $\begin{bmatrix}2x-a & 2y-b & 2z-c\\ x & y & z\\ -3x+4a & -3y+4b & -3z+4c\end{bmatrix} = \begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$ Equating corresponding elements, getting equations in variable. $2x-a=-1, x=1, -3x+4a=9, 2y-b$=-8, y=-2, $-3y+4b=22$ $2z-c=-10$, z=-5, $-3z+4c=15$ A=$\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$
4	If A=$\begin{bmatrix}0 & -tan\frac{\alpha}{2}\\ tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of matrix of order2, show that $I+A=(I-A)\begin{bmatrix}cos\alpha & -sin\alpha\\ sin\alpha & cos\alpha\end{bmatrix}$ **Solution:** Let $tan\frac{\alpha}{2}$ =t, then $cos\alpha = \frac{1-t^2}{1+t^2}$ and $sin\alpha = \frac{2t}{1+t^2}$ I+A =$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$+$\begin{bmatrix}0 & -t\\ t & 0\end{bmatrix}$=$\begin{bmatrix}1 & -t\\ t & 1\end{bmatrix}$ RHS= $\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}0 & -t\\ t & 0\end{bmatrix}\right)\begin{bmatrix}\frac{1-t^2}{1+t^2} & -\frac{2t}{1+t^2}\\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2}\end{bmatrix}$ =$\begin{bmatrix}1 & t\\ -t & 1\end{bmatrix}\begin{bmatrix}\frac{1-t^2}{1+t^2} & -\frac{2t}{1+t^2}\\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2}\end{bmatrix}$ $= \begin{bmatrix}1 & -t\\ t & 1\end{bmatrix}$

EXERCISE

1	If A=$\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and I=$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k such that $A^2 = KA - 2I$
2	$2X+3Y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ $and\ 3X+2Y = \begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$. Find the matrices X and Y
3	A=[2 1] B=$\begin{bmatrix}5 & 3 & 4\\ 8 & 7 & 6\end{bmatrix}$ C=$\begin{bmatrix}-1 & 2 & 1\\ 1 & 0 & 2\end{bmatrix}$ Prove that A(B+C)=AB+AC
Answers: (1) $k=1$ (2) $X = \begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$ $and\ Y = \begin{bmatrix}\frac{2}{3} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$	

CASE BASED QUESTIONS

1 A manufacture produces three types of emergency lamps which he sells in two markets. Their Monthly sales are indicated below

	Type 1	Type 2	Type 3
Market A	100	100	50
Market B	80	100	100

If the unit Sale price of the three types of emergency lights are 2000, 3000 and 2500 respectively, and unit cost of the above three commodities are Rs. 1500,2200, and Rs. 2000 respectively, then **based on the above information answer the following.**

(i) Which of the following gives the total revenue of market A

(a) $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}2000\\3000\\2500\end{bmatrix}$ (b) $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}1500\\2200\\2000\end{bmatrix}$

(c) $\begin{bmatrix}100\\100\\50\end{bmatrix}\begin{bmatrix}2000 & 3000 & 2500\end{bmatrix}$ (d) $\begin{bmatrix}100\\100\\50\end{bmatrix}\begin{bmatrix}1500 & 2200 & 2000\end{bmatrix}$

Solution:

total revenue of market A

$$=100 \times 2000 + 100 \times 3000 + 50 \times 2500$$

Which is in matrix form $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}2000\\3000\\2500\end{bmatrix}$

option: (a) $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}2000\\3000\\2500\end{bmatrix}$

(ii) Total revenue of market B

(a) ₹470,000 (b) ₹155,000

(c) ₹520,000 (d) ₹ 710,000

Solution:

Total revenue of market B= $80\times 2000 + 100 \times 3000 + 100 \times 2500$

= 710000

Option :(d) ₹ 7,10,000

(iii)Total profit of market B is …

(a) 470,000 (b) 155,000

(c) 625000 (d) 170,000

Solution:

Total profit of market B is $80\times 500 + 100 \times 800 + 100 \times 500$ =170000

Option :(d) ₹170,000

(iv) Which of the following gives the total profit of market A.

(a) $\begin{bmatrix}100\\100\\50\end{bmatrix}\begin{bmatrix}500 & 800 & 500\end{bmatrix}$ (b) $\begin{bmatrix}80\\100\\100\end{bmatrix}\begin{bmatrix}1500 & 2200 & 2000\end{bmatrix}$

(c) $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}500\\800\\500\end{bmatrix}$ (d) $\begin{bmatrix}80 & 100 & 100\end{bmatrix}\begin{bmatrix}500\\800\\500\end{bmatrix}$

Solution:

	Total profit of market A =100$\times 500+100\times 800+50\times 500$ which can be represented in matrix form as $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}500\\800\\500\end{bmatrix}$ Option :(c) $\begin{bmatrix}100 & 100 & 50\end{bmatrix}\begin{bmatrix}500\\800\\500\end{bmatrix}$ OR Find gross profit in both market (a) Rs.325,000 (b) Rs. 90,000 (c) Rs. 696000 (d) None of the above Profit of market A=155000 Profit of market B=170000 Obtion:(a) ₹325,000
2	Ashish wants to purchase a rectangular plot from his neighbour to construct a house. He asked about the dimensions of the plot , his neighbour told that if the length is decreased by 50m and the breadth is increased by 50m, then the area will remain the same, but If the length is decreased by 20m and breadth is increased by 30 m ,the area will increase by 1400m^2. **Based on the information given above, answer the following questions** (i) Let $x\ and\ y$ denote the length and breadth of the plot, then equations in terms of $x\ and\ y$ are. (a) $x+y=50; 3x+2y=200$ (b) $x-y=50;\ 3x-2y=200$ (c) $x+y=50;\ 3x-2y=200$ (d) $x-y=50;\ 3x+2y=200$ **Solution:** $(x-50)(y+50)=xy\ and\ (x-20)(y+30)=xy+1400$ on simplification we get $x-y=50\ and;\ 3x-2y=200$ Option :(b) $x-y=50;\ 3x-2y=200$ (ii) Which of the following matrix equations is represented by the given information? (a)$\begin{bmatrix}1 & -1\\3 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}200\\50\end{bmatrix}$ (b)$\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\200\end{bmatrix}$ (c)$\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}y\\x\end{bmatrix}=\begin{bmatrix}200\\50\end{bmatrix}$ (d)$\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}200\\50\end{bmatrix}$ **Solution:** $3x-2y=200\ and\ x-y=50$ So $\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}200\\50\end{bmatrix}$ is matrix form Option :(d)$\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}200\\50\end{bmatrix}$ (iii) If A=$\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}$, Find AA^T **Solution:** AA^T= $\begin{bmatrix}3 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}3 & 1\\-2 & -1\end{bmatrix}=\begin{bmatrix}13 & 5\\5 & 2\end{bmatrix}$ Option : $\begin{bmatrix}13 & 5\\5 & 2\end{bmatrix}$ (iv) If P=$\begin{bmatrix}1 & -1\\3 & -2\end{bmatrix}$ and Q=$\begin{bmatrix}200\\50\end{bmatrix}$ Find PQ and QP **Solution:** PQ=$\begin{bmatrix}1 & -1\\3 & -2\end{bmatrix}\begin{bmatrix}200\\50\end{bmatrix}=\begin{bmatrix}150\\500\end{bmatrix}$.

Option : PQ= $\begin{bmatrix}150\\500\end{bmatrix}$

Number of columns in Q is not equal to number rows in P. So QP not possible

3 Number of girls and boys of three sections of XI are given below

Section	No of Boys	No of Girls
11A	20	25
11B	25	23
11C	23	24

Fee for girls per month is ₹1000 and for boys ₹1200.

Based on the above the information answer the following questions

(i) Which of the following gives the total amount of fee of paid by class 11A?

(a)$\begin{bmatrix}20 & 25\end{bmatrix}\begin{bmatrix}1200\\1000\end{bmatrix}$ (b)$\begin{bmatrix}25 & 23\end{bmatrix}\begin{bmatrix}1200\\1000\end{bmatrix}$

(c)$\begin{bmatrix}23 & 24\end{bmatrix}\begin{bmatrix}1000\\1200\end{bmatrix}$ (d)$\begin{bmatrix}20\\25\end{bmatrix}\begin{bmatrix}1000 & 1200\end{bmatrix}$

Solution

Total amount of fee paid by class 11A= $20\times 1200 + 25 \times 1000.$

Matrix form is$\begin{bmatrix}20 & 25\end{bmatrix}\begin{bmatrix}1200\\1000\end{bmatrix}$

Option : (a)$\begin{bmatrix}20 & 25\end{bmatrix}\begin{bmatrix}1200\\1000\end{bmatrix}$

(ii) Which of the following will not give the total fee paid by sections A and B together is

(a)$\begin{bmatrix}1200 & 1000\end{bmatrix}\begin{bmatrix}45\\48\end{bmatrix}$ (b)$\begin{bmatrix}1000 & 1200\end{bmatrix}\begin{bmatrix}48\\45\end{bmatrix}$

(c)$\begin{bmatrix}45 & 48\end{bmatrix}\begin{bmatrix}1200\\1000\end{bmatrix}$ (d)$\begin{bmatrix}45 & 48\end{bmatrix}\begin{bmatrix}1000\\1200\end{bmatrix}$

Solution:

The total fee paid by sections A and B together is $45\times 1200 + 48 \times 1000$

Option :(b)$\begin{bmatrix}45 & 48\end{bmatrix}\begin{bmatrix}1000\\1200\end{bmatrix}$

(iii) What is the total fee paid by all boys of class11?

OR

What is the total fee paid by students of class 11C?

Solution

(iii)$20\times 1200 + 25 \times 1200 + 23 \times 1200$ = 24000+30000+27600 = ₹81600

OR

$23 \times 1200 + 24 \times 1000 =$ ₹51600

4 In farewell party ,students of class12 A,B and C decided to have some dress code. Girls of 12 A ,B and C selected red ,green and blue saries of rate 2000,2500 and 2500 respectively from a shop and boys of each section purchased the dress of same colour matching with girls of the same section and of rate 3000,2800 and3000 respectively. Number of girls and boys of three sections of XI are given below

Section	No of Boys	No of Girls
A	20	20
B	25	20
C	20	25

Based on the above data, answer the following questions

(i) Which of the following product gives the total amount paid by girls of all sections

	(a) $[20 \quad 25 \quad 20]\begin{bmatrix}3000\\2500\\2500\end{bmatrix}$ (b)$[20 \quad 20 \quad 25]\begin{bmatrix}2000\\2500\\2500\end{bmatrix}$ (c) $[20 \quad 25 \quad 20]\begin{bmatrix}2000\\2500\\3000\end{bmatrix}$ (d)$[20 \quad 20 \quad 25]\begin{bmatrix}3000\\2500\\2500\end{bmatrix}$ **Solution:** the total amount paid by girls of all sections= $20 \times 2000 + 20 \times 2500 + 25 \times 2500$ which is same as $[20 \quad 20 \quad 25]\begin{bmatrix}2000\\2500\\2500\end{bmatrix}$ Option : (b) $[20 \quad 20 \quad 25]\begin{bmatrix}3000\\2500\\2500\end{bmatrix}$ (ii) Which of the following product gives the total amount paid by boys of all sections (a) $[20 \quad 25 \quad 20]\begin{bmatrix}3000\\2800\\3000\end{bmatrix}$ (b)$[20 \quad 20 \quad 25]\begin{bmatrix}2000\\2800\\3000\end{bmatrix}$ (c) $[20 \quad 25 \quad 20]\begin{bmatrix}2000\\2800\\3000\end{bmatrix}$ (d)$[20 \quad 20 \quad 25]\begin{bmatrix}3000\\2800\\2500\end{bmatrix}$ **Solution:** the total amount paid by boys of all sections= $20 \times 3000 + 25 \times 2800 + 20 \times 3000$ which is obtained by $$[20 \quad 25 \quad 20]\begin{bmatrix}3000\\2800\\3000\end{bmatrix}$$ Option (a) $[20 \quad 25 \quad 20]\begin{bmatrix}3000\\2800\\3000\end{bmatrix}$ (iii) Amount paid by students of A (a)$[20 \quad 20]\begin{bmatrix}3000\\2000\end{bmatrix}$ (b)$[25 \quad 20]\begin{bmatrix}2500\\2500\end{bmatrix}$ (c)$[20 \quad 25]\begin{bmatrix}2500\\3000\end{bmatrix}$ (b)$[2500 \quad 2500]\begin{bmatrix}20\\25\end{bmatrix}$ **Solution:** Amount paid by students of A=$20 \times 2000 + 20 \times 3000$ Which is represented by$[20 \quad 20]\begin{bmatrix}3000\\2000\end{bmatrix}$ Option : (a)$[20 \quad 20]\begin{bmatrix}3000\\2000\end{bmatrix}$ (iv) A=$\begin{bmatrix}3000 & 2000\\2500 & 2500\\2500 & 3000\end{bmatrix}$ Find $a_{11} + a_{32}$ (a) 4500 (b) 5000 (c) 5500 (d) 6000 **Solution:** $$a_{11} + a_{32} = 3000 + 3000$$ Option -:(d) 6000
5	Sushama owns a P.G for girls. One day she went to market purchase the food items. She bought 4kg onion,3 kg wheat, and 2kg rice for Rs 560.Next day she bought 2kg onion,4kgwheat and 6 kg rice. It cost her Rs:780. Another day she bought 6 kg onion,2kg wheat and3 kg rice which cost Rs: 640.

(i) Convert the given condition above in matrix equation of the form AX=B

Solution:

$$\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}560\\780\\640\end{bmatrix}$$

(ii) Find A+A^T. Is it symmetric?

Solution:

$$\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix}+\begin{bmatrix}4 & 2 & 6\\3 & 4 & 2\\2 & 6 & 3\end{bmatrix}=\begin{bmatrix}8 & 5 & 8\\5 & 8 & 8\\8 & 8 & 6\end{bmatrix}$$ .It is symmetric

Find a matrix P such that $P = A^2 - 5A$

OR

(iii) Find A^3

Solution:

$$A^2 - 5A=\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix}-5\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}34 & 28 & 32\\52 & 34 & 46\\46 & 32 & 33\end{bmatrix}-\begin{bmatrix}20 & 15 & 10\\10 & 20 & 30\\30 & 10 & 15\end{bmatrix}=\begin{bmatrix}14 & 13 & 22\\42 & 14 & 16\\16 & 22 & 18\end{bmatrix}$$

OR

$$A^3 = \begin{bmatrix}34 & 26 & 32\\52 & 34 & 46\\46 & 32 & 33\end{bmatrix}\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix} = \begin{bmatrix}348 & 278 & 332\\552 & 384 & 446\\446 & 332 & 383\end{bmatrix}$$

EXERCISE

1 Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommend daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

(i) Requirement of calories of family A is

(a) 24000 (b) 24400 (c) 15000 (d) 15800

(ii) Requirement of proteins for family B is

(a) 560 grams (b) 332 grams

(c) 266 grams (d) 300 grams

(iii) If A=$\begin{bmatrix}4 & 4 & 4\\2 & 2 & 2\end{bmatrix}$, B=$\begin{bmatrix}2400\\1900\\1800\end{bmatrix}$ and C=$\begin{bmatrix}45\\55\\33\end{bmatrix}$ Then Find A(B+C)

(iv) Which of the following product gives the total calories required for children in both the families

(a) $\begin{bmatrix} 4 & 2 \end{bmatrix}\begin{bmatrix} 1800 \\ 1800 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 \end{bmatrix}\begin{bmatrix} 1900 \\ 1800 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 4 \end{bmatrix}\begin{bmatrix} 1900 \\ 1800 \end{bmatrix}$ (d) none of the above.

2 Three schools DPS, CVC and KVS have decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs.100 and Rs. 50 each respectively. The numbers of articles sold are given

School /Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the information given above, answer the following questions

(i) What is the total money (in Rupees) collected by the school DPS?

(ii) If A= $\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}$ and B= $\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$ Find AB.

OR

Find AA^T

(iii) What is the total amount of money collected by all three schools DPS, CVC and KVS?

Answers:

1. (i) (b) 24400 (ii). (c) 266 grams (iii). $\begin{bmatrix} 24932 \\ 12466 \end{bmatrix}$ (iv) .$(a)\begin{bmatrix} 4 & 2 \end{bmatrix}\begin{bmatrix} 1800 \\ 1800 \end{bmatrix}$

2. (i) ₹7000 (ii)$AB = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$ OR $AA^T = \begin{bmatrix} 4500 & 3600 & 4700 \\ 3600 & 3125 & 4075 \\ 4700 & 4075 & 5325 \end{bmatrix}$

(iii) ₹21000

CHAPTER: DETERMINANTS

SYLLABUS: *Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.*

Definitions and Formulae:

To every square matrix we can assign a number called determinant

- **Determinant:**

  - $Let\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, then\ det(A) = |A| = ad - bc$
  - $Let\ A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}, then\ |A| = a\begin{vmatrix} e & f \\ h & k \end{vmatrix} - b\begin{vmatrix} d & f \\ g & k \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
  - For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeros
  - The area of a triangle whose vertices are (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3), is

    $$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

    Since area is a positive quantity, we always take the absolute value of the determinant
  - The area of the triangle formed by three collinear points is zero.
  - Equation of line joining the points (x_1 , y_1) and (x_2 , y_2) is

    $$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

- **Minors:** Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and jth column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

- **Co-Factors:** Cofactor of an element a_{ij} , denoted by A_{ij} is defined by

  $A_{ij} = (-1)^{i+j}.M_{ij}$, where M_{ij} is minor of a_{ij}

- The value of a determinant Δ = sum of the product of elements of any row (or column) with their corresponding cofactors.

- If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero. For example $a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}=0$

- **Adjoint of a Matrix:** The adjoint of a square matrix $A = [a_{ij}]$ is defined as the transpose of the matrix $[A_{ij}]$, where A_{ij} is the cofactor of the element a_{ij} .

  Adjoint of the matrix A is denoted by adj A.

- To find adjoint of a 2×2 matrix interchange the diagonal elements and change the sign of non – diagonal elements.

- **Inverse of a Matrix**: Let A be a square matrix.

$$A^{-1} = \frac{1}{|A|} adjA$$

- **Solution of system of linear equations by using matrix method:**

  Let the system of linear equations be

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

These equations can be written as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

  - $A^{-1} exists, if\ |A| \neq 0\ \ i.e\ the\ solution\ exists\ and\ it\ is\ unique.$
  - $The\ system\ of\ equations\ is\ said\ to\ be\ consistent\ if\ the\ solution\ exists.$
  - $if\ |A| = 0\ , then\ we\ calculate\ (adjA)B.$
  - $If\ |A| = 0\ and\ (adjA)B \neq O$, (O being zero matrix), then solution does not exist and the system of equations is called inconsistent.
  - $If\ |A| = 0\ and\ (adjA)B = O$, then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

- **Important notes:**

  - The matrix A is singular if $|A| = 0$

- A square matrix A is said to be non-singular if $|A| \neq 0$
- If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order
- If A is an invertible matrix, then $|A| \neq 0$ and $(A^{-1})^T = (A^T)^{-1}$
- $|\lambda A| = \lambda^n |A|, where\ n = order\ of\ matrix\ A$
- $A(adjA) = (adjA)A = |A|I$
- $|adjA| = |A|^{n-1}, where\ n = order\ of\ matrix\ A$
- $|A(adjA)| = |A|^n, where\ n = order\ of\ matrix\ A$
- $|AB| = |A||B|$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $|A^{-1}| = |A|^{-1}$
- $|A^T| = |A|$
- If A and B are square matrices of the same order, then $adj(AB) = (adjB).(\,adj\ A)$

MULTIPLE CHOICE QUESTIONS

Q.No	QUESTIONS WITH SOLUTIONS
1	If A is a square matrix of order 3 such that $|A| = -5$, then value of $|-A|$ is (a) 125 (b) - 125 (c) 5 (d) – 5 Solution:- $|-A| = (-1)^3|A|$ $= -(-5)$ $= 5$ Correct option: c
2	Evaluate $\begin{vmatrix} cos15 & sin15 \\ sin75 & cos75 \end{vmatrix}$ (a) 1 (b) 0 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$ Solution:- $\begin{vmatrix} cos15 & sin15 \\ sin75 & cos75 \end{vmatrix}$=cos15cos75-sin15sin75 =cos(15+75) $=\cos 90^0$ =0 Correct option: b
3	What positive value of x makes the following pair of determinants equal $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

	(a) 4 (b) 8 (c) 2 (d) ±4 <u>Solution:-</u> $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$ $2x^2-15 = 32-15$ $=17$ $2x^2=32$ $x^2=16$ $x=\pm4$ The positive value of x is 4 Correct option: a
4	If A is a square matrix of order 3 such that $\lvert adjA \rvert$=64, then what is the value of $\lvert A \rvert$ (a)64 (b) 8 (c) -8 (d) ± 8 <u>Solution</u>:-$\lvert adjA \rvert = \lvert A \rvert^2 = 64$ $\lvert A \rvert = \pm 8$ Correct option : d
5	If for a square matrix A, $A^2 - 3A + I = 0$ and $A^{-1} = xA + yI$, then the value of $x + y$ is (a) -2 (b) 2 (c) 3 (d) -3 <u>Solution:-</u> $A^2-3A+I=0$ Multiply by A^{-1} We get $A-3I+A^{-1}=0$ $A^{-1}= -A+3I$ Hence x= -1,y=3 x+y=2 Correct option : b
6	If $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$ =0, then its roots are a) 0 (b)1 (c) 0,1 (d) $0,1,-1$ <u>Solution:-</u> $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}=0$ On expanding the determinant, we get $(1-x)x^2=0$ Hence x=0,1 Correct option: c
7	If $A\ (3,4), B(-7,2), C(x,y)$ are collinear, then which of the following is true? (a) $x + 5y + 17 = 0$ (b) $x + 5y + 13 = 0$ (c) $x - 5y + 17 = 0$ (d) $x - 5y - 17 = 0$ <u>Solution:-</u> If A (3,4), B(-7,2), C(x, y) are collinear, then area of triangle ABC=0 i.e $\frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ -7 & 2 & 1 \\ x & y & 1 \end{vmatrix}=0$

	i.e $\begin{vmatrix}3&4&1\\-7&2&1\\x&y&1\end{vmatrix}=0$ $i.e\ 3(2-y)-4(-7-x)+1(-7y-2x)=0$ i.e x-5y+17=0 Correct option: c
8	If A is an invertible matrix of order 2, then det(A^{-1})= a) $\frac{1}{detA}$ (b) 0 (c) 1 (d) det(A) Solution:- det$(A^{-1})=\left\|\frac{adj(A)}{IAI}\right\|$ $=\frac{1}{\|A\|^2}\|adj(A)\|$ $=\frac{1}{\|A\|^2}\|A\|^{2-1}$ $=\frac{1}{\|A\|}$ $=\frac{1}{detA}$ Correct option: a
9	If A is a square matrix such that $A^2=I$, then A^{-1} is equal to: a) 2A (b) O (c) A (d) A+I Solution:- $A^2=I$ Multiply by A^{-1} on both sides $A^{-1}A^2=A^{-1}I$ $A=A^{-1}$ $A^{-1}= A$ Correct option:c
10	The value of $\begin{vmatrix}x+y&y+z&z+x\\z&x&y\\1&1&1\end{vmatrix}$ is equal to (a) 0 (b) 1 (c) $x+y+z$ (d) $2(x+y+z)$ Solution:- $\begin{vmatrix}x+y&y+z&z+x\\z&x&y\\1&1&1\end{vmatrix}$=(x+y)(x-y)-(y+z)(z-y)+(z+x)(z-x) $=(x^2-y^2)-(z^2-y^2)+(z^2-x^2)$ =0 Correct option: a

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
DETERMINANTS	https://youtu.be/nrquFxCgHsE	

EXERCISE

1	If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is a non-singular matrix and $a \in A$, then set A is a) R (b){0} (c) {4} (d) R-{4} Correct option: d
2	If $\|A\|=\|kA\|$, where A is a square matrix of order 2, then sum of all possible values of k is a) 1 (b) -1 (c) 2 (d) 0 Correct option: d
3	If A and B are square matrix of order of 3 such that $\|A\| = -1$ and $\|B\| = 3$ then what is the value of $\|3AB\|$? a) -9 (b) -27 (c) -81 (d) 81 Correct option: c
4	The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 0 & sinx & cosx \\ 0 & cosx & sinx \end{vmatrix}$ is a) 1 (b) -1 (c) $-cos2x$ (d) $cos2x$ Correct option: c
5	If A=$\begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$ then the value of $\|adj(A)\|$ is a) x^3 (b) x^6 (c) x^9 (d) x^{27} Correct option: b

ASSERTION REASONING QUESTIONS

Two statements are given, one labelled Assertion(A) and the other labelled Reason(R).Select the correct answer from the codes (a),(b),(c) and (d) as given below

(a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A)

(b) Both Assertion (A) and Reason(R) are true but Reason(R) is *not* the correct explanation of the Assertion(A)

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

1	**Assertion(A):**The value of $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ is equal to 1 **Reason(R):**The value of the determinant of a matrix A order 2×2, where A=$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$

	Solution: $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = x\,(x) – (x + 1)\,(x – 1) = x^2 – (x^2 – 1) = x^2 – x^2 + 1 = 1$ So Assertion A is true. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ So Reason R is true. Hence Reason(R) is the correct explanation of the Assertion(A) **Correct option: a**
2	**Assertion(A):** If A is an invertible matrix of order 2, and det A= 3 then det(A^{-1})is equal to $\frac{1}{3}$ **Reason(R):** If A is an invertible matrix of order 2 then det (A^{-1}) =det(A) Solution:-Since det(A^{-1})=$\frac{1}{detA} = \frac{1}{3}$ Hence A is true but R is false. **Correct option: c**
3	**Assertion(A):** In a square matrix of order 3 the minor of an element a_{22} is 3 then cofactor of a_{22} is -3. **Reason(R):** Cofactor an element $a_{ij} = A_{ij} = (-1)^{i+j} M_{ij}$ Solution:- Cofactor an element $a_{ij} = A_{ij} = (-1)^{i+j} M_{ij}$ Cofactor an element $a_{22} = A_{22} = (-1)^{2+2}(3)$ $= 3$ Hence Assertion (A) is false but Reason (R) is true **Correct option:d**
4	**Assertion(A):**If A=$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then$\|3A\|=27\|A\|$ **Reason(R):** If A = *k*B where A and B are square matrices of order *n*, then | A| = k^n| B |, where *n* = 1, 2, 3 Solution:- Since |kA| = k^n| A | $\|3A\|=3^3\|A\|$ $=27\|A\|$ Hence Reason(R) is the correct explanation of the Assertion(A) **Correct option :a**
5	**Assertion(A):** If A=$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ then Adj(A)= $\begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix}$ **Reason(R):**If A=$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then Adjoint(A) can be obtained by interchanging a_{11} and a_{22} and by changing signs of a_{12} and a_{21} Solution:- Adj(A)= $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$. Hence A is false but R is true **Correct option:d**
6	**Assertion(A):** If A is a square natrix of order 3, then $\|2A\|=8\|A\|$ **Reason(R):** Let A be a square matrix of order n. Then | *adj* A| = $\|A\|^{n-1}$ Solution:-A is true since $\|2A\|=2^3\|A\|=8\|A\|$ R is also true, but R is not the correct explanation of A **Correct option:b**
7	**Assertion(A):** if A=$\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and B=$\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ then $(AB)^{-1} = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

	Reason(R): For any 2 matrix A and B , $(AB)^{-1}=B^{-1}A^{-1}$ Solution:- $(AB)^{-1}=\frac{adj(AB)}{\|AB\|}=-\frac{1}{2}\begin{bmatrix}61 & -87\\ -47 & 67\end{bmatrix}=B^{-1}A^{-1}$ $(AB)^{-1}=B^{-1}A^{-1}$ Hence Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A) **Correct option: a**
8	**Assertion(A):** Value of x for which the matrix$\begin{bmatrix}1 & 2\\ 2 & x\end{bmatrix}$ is singular is 4 **Reason(R):** A square matrix is singular if |A | =0 Solution:- A square matrix is singular if |A | =0 $\begin{vmatrix}1 & 2\\ 2 & x\end{vmatrix}=0$ $x-4=0$ $x=4$ **Correct option: a**
9	**Assertion(A):** The system of equations $2x+5y=1;\ 3x+2y=7$ are consistent **Reason(R):** A system of equations is said to be consistent if they have one or more solution. Solution:- The system of equations can be written in the form AX = B, where A=$\begin{bmatrix}2 & 5\\ 3 & 2\end{bmatrix}$,X=$\begin{bmatrix}x\\ y\end{bmatrix}$ B=$\begin{bmatrix}1\\ 7\end{bmatrix}$ On solving these system of equations by matrix method$\|A\|=-11\neq 0$, Hence, A is non singular matrix and so has a unique solution. Hence they are consistent. **Correct option:a**
10	**Assertion(A):** For two matrices A and B of order 3, $\|A\|=2$ $\|B\|=-3$, then $\|AB\|=6$ **Reason(R):** The determinant of the product of matrices is equal to product of their respective determinants, that is, $\|AB\|=\|A\|\|B\|$ where A and B are square matrices of the same order Solution:- $\|AB\|=\|A\|\|B\|$ $=2(-3)$ $=-6$ Hence A is false but R is true. **Correct option:d**

EXERCISE

1	Assertion(A): The value of x for which $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 4\\ 4 & 1\end{vmatrix}$ is ±4 Reason(R) : The determinant of a matrix A order 2 × 2, A=$\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $is\ \ ad-bc$ Correct option: a
2	Assertion(A): For two matrices A and B of order 3, $\|A\|=1,\|B\|=-4$ then $\|2AB\|$ is -32. Reason(R): For a square matrix A, A(adj A)=(adj A)A=|A | I Correct option: b

3	Assertion(A): The equation of the line joining (1,2) and (3,6) using determinants is $y+2x=0$. Reason(R): The area of Δ PAB is zero if P(x, y) is a point on the line joining two points A and B. Correct option: d
4	Assertion(A):The maximum value of $\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin A & 1\\ 1 & 1 & 1+\cos A\end{vmatrix}$ is $\frac{1}{2}$ Reason(R): Principal value branch of $\sin^{-1}A$ is $[-\frac{\pi}{2},\frac{\pi}{2}]$ Correct option:b
5	Assertion(A): A=$\begin{bmatrix}2 & x & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then A^{-1} exists if $x=\frac{-8}{5}$ Reason(R): A square matrix A has inverse if and only if A is non-singular. Correct option: d

2 MARK QUESTIONS

Q. No	QUESTIONS WITH SOLUTIONS
1	If A=$\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}$ and B=$\begin{bmatrix}1 & -4\\ 3 & -2\end{bmatrix}$ find $\|AB\|$ Solution:- AB=$\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}\times\begin{bmatrix}1 & -4\\ 3 & -2\end{bmatrix}$ $=\begin{bmatrix}7 & -8\\ 0 & -10\end{bmatrix}$ $\|AB\| = -70$
2	If A=$\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ write A^{-1} in terms of A Solution:- $A^{-1}=\frac{Adj(A)}{\|A\|}$ $=\frac{-1}{19}\begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix}$ $=\frac{1}{19}\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ $=\frac{1}{19}$ A
3	What positive value of x makes the following pair of determinants equal $\begin{vmatrix}2x & 3\\ 5 & x\end{vmatrix}, \begin{vmatrix}10 & 3\\ 5 & 5\end{vmatrix}$ Solution:- $2x^2-15=50-15$ $2x^2=50$ $x=\pm 5$

4	For what value of x, is the following matrix singular? $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ Solution:- A matrix is singular if $\lvert A\rvert$=0 $(3-2x)4-(x+1)2=0$ On solving we get $x=1$
5	If for any 2×2 square matrix A, A(AdjA)= $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $\lvert A\rvert$ Solution:- A (adj A) = |A| I $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}=\lvert A\rvert\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}=\begin{bmatrix} \lvert A\rvert & 0 \\ 0 & \lvert A\rvert \end{bmatrix}$ $\lvert A\rvert$=8
6	If A is a nonsingular matrix of order 3 and $\lvert A\rvert=-4$, find $\lvert A.adjA\rvert$ Solution:- $\lvert A.adjA\rvert = \lvert A\rvert\lvert adjA\rvert$ $=\lvert A\rvert\lvert A\rvert^2$ $=\lvert A\rvert^3$ $=(-4)^3$ $=-64$
7	Find the equation of the line joining A (1 , 3) and B (0, 0) using determinants Solution:- Let $p(x,y)$ be any point on the line AB Then area of $\Delta PAB=0$ $\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix}=0$ Equation of line AB is $y = 3x$
8	If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find the value of a_{32} A_{32} Solution:- a_{32} $A_{32} = 5\times-(8-30)$ $=5(22)$ $=110$
9	If A = $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the value of $\lvert A^2-2A\rvert$ Solution:- A^2-2A=$\begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix}-\begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix}$ $=\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ $\lvert A^2-2A\rvert=25$
10	If A = [a_{ij}] is a matrix of order 2 ×2 , such that $\lvert A\rvert=-15$ and A_{ij} is the cofactor of the element a_{ij} then find a_{21} A_{21} + a_{22} A_{22} Solution:- a_{21} A_{21} + a_{22} A_{22} $=\lvert A\rvert$ $=-15$

EXERCISE

1	Given A= $\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$, compute A^{-1} and show that $2A^{-1}$=9I-A
2	Find k if the matrix $\begin{bmatrix}1 & k & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of matrix A and $\|A\| = 4$ Answer: $k = 11$
3	If $\begin{vmatrix}x+1 & x-1\\ x-3 & x+2\end{vmatrix} = \begin{vmatrix}4 & -1\\ 1 & 3\end{vmatrix}$ then find the value of x Answer:- $x = 2$
4	Find the inverse of the matrix $\begin{bmatrix}1 & 0 & 0\\ 0 & cosx & sinx\\ 0 & sinx & -cosx\end{bmatrix}$ Answer:- $\begin{bmatrix}1 & 0 & 0\\ 0 & cosx & sinx\\ 0 & sinx & -cosx\end{bmatrix}$
5	Evaluate $\begin{vmatrix}cosAcosB & cosAsinB & -sinA\\ -sinB & cosB & 0\\ sinAcosB & sinAsinB & cosA\end{vmatrix}$ Answer: 1

3 MARK QUESTIONS

Q.No	QUESTIONS WITH SOLUTIONS
1	A coaching institute of Mathematics conduct classes in two batches, I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection of Rs.9000/-, where as in batch II 5 poor and 25 rich children and the monthly collection is Rs.26,000/-. Using matrix method finds the monthly fees paid by each child of the two types. Solution:- Let x *and* y be the fees paid by rich and poor children respectively. According to the question, *5x+20y=9000* *25x+5y=26000* Which can be written as $AX = B$, where A= $\begin{bmatrix}5 & 20\\ 25 & 5\end{bmatrix}$,X= $\begin{bmatrix}x\\ y\end{bmatrix}$,B= $\begin{bmatrix}9000\\ 26000\end{bmatrix}$ $\|A\| = -475 \neq 0$ adj A= $\begin{bmatrix}5 & -20\\ -25 & 5\end{bmatrix}$ $\therefore\ A^{-1} = \frac{-1}{475}\begin{bmatrix}5 & -20\\ -25 & 5\end{bmatrix}$

	X= $\begin{bmatrix} x \\ y \end{bmatrix}$, A^{-1}B=$\begin{bmatrix} 1000 \\ 200 \end{bmatrix}$ $x = 1000, \; y = 200$
2	If A=$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that A^3-23A-40I=0 <u>Solution:-</u> A^2=$\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$ A^3=$\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$ A^3-23A-40I= $\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$ - $\begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix}$ - $\begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$=0
3	Using determinants, find the area of ΔPQR with vertices P(3,1),Q(9,3)and R(5,7). Also find the equation of line PQ using determinants. <u>Solution:-</u> Area =$\frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}$ = 16 sq.units Equation of PQ is $\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix}$ =0 $-2x + 6y = 0$ OR $x - 3y = 0$
4	If A.(adjA)=$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$,then find the value of $\|A\|$ +$\|adjA\|$ <u>Solution:-</u> $\|A.adjA\|$=$\|A\|\|adjA\|$=$\|A\|\|A\|^2$ =$\|A\|^3$ 27=$\|A\|^3$ $\|A\|$=3 $\|adjA\|$=3^2=9 $\|A\|$ +$\|adjA\|$=12

5	$If\ x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$=0 then find the other 2 roots <u>Solution:-</u> $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$=0 $x^3 - 67x + 126 = 0$ $(x+9)(x-7)(x-2) = 0$ $x = -9,7,2$ Hence the other two roots are 7 and 2
6	If A=$\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and B=$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1}A^{-1}$ <u>Solution:-</u> AB=$\begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$ $\|AB\| = -11 \neq 0$, Therefore $(AB)^{-1}$ exists $(AB)^{-1}$= $\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$ A^{-1}= - $\frac{1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$ B^{-1} = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ $B^{-1}A^{-1}$ =$\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$=$(AB)^{-1}$ Hence proved
7	Show that the matrix A =$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}. <u>Solution:-</u> $A^2 - 4A + I$=$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$-$\begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix}$+$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$=0 $A^2 - 4A + I = O$ AA-4A+I=0 AA-4A= -I Multiplying by A^{-1},we get $AI - 4I = -A^{-1}$ $A^{-1} = 4I - A$

	$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$
8	If A is a skew symmetric matrix of order 3, then prove that $det\ A = 0$ <u>Solution:-</u> If A is a skew symmetric matrix of order 3,then A= - A^T $\|A\| = \|-A^T\|$ $= -\|A^T\|$ $= -\|A\|$ (Since $\|A^T\| = \|A\|$) $2\|A\| = 0$ Hence $\|A\|=0$
9	If $A = \begin{vmatrix} 1 & sinx & 1 \\ -sinx & 1 & sinx \\ -1 & -sinx & 1 \end{vmatrix}$, where $0 \le x \le 2\pi$. Then prove that $\|A\| \epsilon [2,4]$ <u>Solution:-</u> $\|A\| = 2+2\sin^2 x$ We know that $0 \le \sin^2 x \le 1$ i.e. $0 \le 2\sin^2 x \le 2$ i.e $2 \le 2+2\sin^2 x \le 4$ i.e. $2 \le \|A\| \le 4$ Hence $\|A\| \epsilon [2,4]$
10	If the points (a_1, b_1), (a_2, b_2) and $(a_1 + a_2, b_1 + b_2)$ are collinear, then prove that $a_1b_2=a_2b_1$ <u>Solution:-</u> If the points (a_1, b_1), (a_2, b_2) and $(a_1 + a_2, b_1 + b_2)$ are collinear, then $\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_1 + a_2 & b_1 + b_2 & 1 \end{vmatrix} = 0$ On expanding we get $a_2b_1 - a_1b_2 = 0$ Hence $a_2b_1 = a_1b_2$ i.e. $a_1b_2=a_2b_1$

EXERCISE

1	Show that the determinant $\begin{vmatrix} x & sinA & cosA \\ -sinA & -x & 1 \\ cosA & 1 & x \end{vmatrix}$ is independent of A
2	Show that points A (*a*, *b* + *c*), B (*b*, *c* + *a*), C (*c*, *a* + *b*) are collinear.
3	If *x*, *y*, *z* are nonzero real numbers and A=$\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ then prove that $A^{-1}=\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$
4	If A=$\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$, Show that A^2-12A+I=0, hence find A^{-1} Answer:- A^{-1}= $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$
5	If A=$\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ then prove that $A^{-1}A=I$

5 MARK QUESTIONS

Q. No	QUESTIONS WITH SOLUTIONS
1	Solve the following system of equations by matrix method $3x-2y+3z=8$ $2x+y-z=1$ $4x-3y+2z=4$ Solution:- The given system of equations can be written as AX=B $\|A\|$= -17≠0 A^{-1}= -1/17$\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ $x = A^{-1}B$ $x=1, y=2, z=3$
2	If A=$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and B^{-1}=$\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$ Solution:- $(AB)^{-1} = B^{-1}A^{-1}$ $\|A\|$= 1≠0

Adj(A)=$\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$A^{-1}=\frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Therefore $(AB)^{-1} = B^{-1}A^{-1}$

$$=\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & -103 \\ 17 & 12 & 36 \end{bmatrix}$$

3 Using the matrix method, solve the following system of linear equations :

$\frac{2}{X}+\frac{3}{Y}+\frac{10}{Z} = 4,\ \frac{4}{X}-\frac{6}{Y}+\frac{5}{Z} = 1,\ \frac{6}{X}+\frac{9}{Y}-\frac{20}{Z} = 2$

<u>Solution:-</u>

The given system of equations can be written in the form AX=B,

Where A=$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ $X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix}$ and B=$\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$|A| = 1200 \neq 0$, A^{-1} exists

Adj(A)=$\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Hence $A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Since AX=B, $X=A^{-1}B$

$$=\frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$=\frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

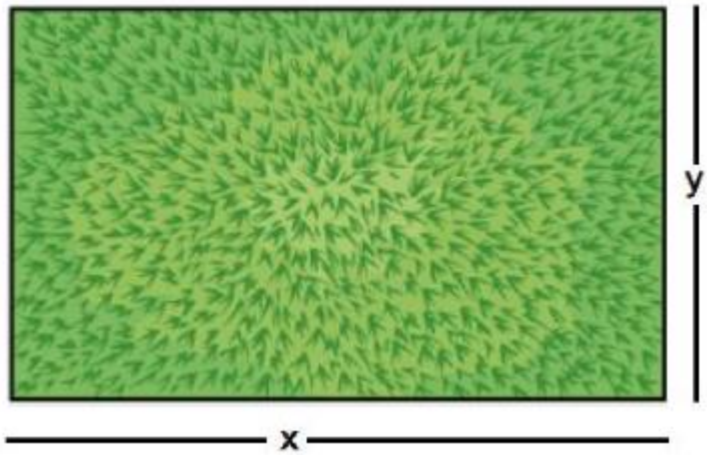
Hence $x = 2, y = 3, z = 5$

EXERCISE

1	If A=$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that A^2-5A+7I=0, hence find A^{-1} Answer:- $A^{-1}=\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$
2	Use the product $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 9 & 2 & 3 \\ 6 & 1 & 2 \end{bmatrix}$ to solve the system of equations:- $x - y + 2z = 1$ $2y - 3z = 1$ $3x - 2y + 4z = 2$ Answer:- $x = 0, y = 5, z = 3$
3	If A=$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the s ystem of equations $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$ Answer- $x = 1, \quad y = 2, z = 3$

CASE BASED QUESTIONS

Q. No	QUESTIONS WITH SOLUTIONS
1	Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2



1. Based on the information given above, form equations in terms of x and y
2. Write down matrix equation represented by the given information
3. How much is the area of rectangular field?

<u>Solution:-</u>

1) (x-50)(y+50)=xy

x-y=50--------(1)

(x-10)(y-20)=xy-5300

2x+y=550---------(2)

2) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

3) On solving

We get, x=200 m

y=150 m

Area =200 × 150 = 30, 000 sq .m

2 Ram buys 5 pens, 3 bags 1 instrument box and pays a sum of Rs. 160. From the same shop, Madhav buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of Rs.190.Also Ankit buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs.250.

Based on the above information, answer the following questions:

1) Convert the given above situation into a matrix equation of the form AX=B
2) Find $|A|$
3) Find A^{-1}

OR

Determine $P=A^2-5A$

<u>Solution:-</u>

1) Matrix equation AX=B, where A=$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & .3 \\ 1 & 2 & 4 \end{bmatrix}$ X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ B=$\begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$ where x is the number of pens bought, y the number of bags and z the number of instrument boxes.

2) $|A|$= -22

3) Adj(A)=$\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$

	$A^{-1}=\frac{1}{-22}\begin{bmatrix}-2 & -10 & 8\\ -5 & 19 & -13\\ 3 & -7 & -1\end{bmatrix}$ OR $P=A^2-5A=\begin{bmatrix}7 & 5 & 13\\ 5 & 8 & 2\\ 8 & 3 & 3\end{bmatrix}$
3	The management committee of a residential colony decided to award some of the students of their colony (say x) for honesty, some (say y) for helping others and some others (say z)for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12.Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33.If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others.  Based on the above information, answer the following questions: 1)Convert the given above situation into a matrix equation of the form AX=B 2)Find A^{-1} 3)Find the number of awardees of each category Solution:- 1) x+y+z=12----(1) 3(y+z)+2x=33-----(2) x+z=2y-------(3) i.e x+y+z=12 2x+3y+3z=33 x-2y+z=0 Matrix equation AX=B, where $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 3\\ 1 & -2 & 1\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ $B=\begin{bmatrix}12\\ 33\\ 0\end{bmatrix}$ 2) $\lvert A\rvert$ =3 $A^{-1}=\frac{1}{3}\begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ -7 & 3 & 1\end{bmatrix}$ 3) $X=A^{-1}B$ Hence x=3,y=4,z=5

EXERCISE

1

A company produces three products every day. Their production on certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product.

Using the concepts of matrices and determinants, answer the following questions.

1. If x, y and z respectively denotes the quantity (in tons) of first, second and third product produced, then convert the given above situation into a matrix equation of the form AX=B
2. Find A^{-1}
3. Find x:y:z

Answer:-

1) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -8 \\ 0 \end{bmatrix}$

2) $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 3 & 1 \\ 2 & 0 & -2 \\ 2 & -3 & 1 \end{bmatrix}$

3)11:15:19

2

Each triangular face of the Pyramid is made up of 25 smaller equilateral triangles as shown in the figure.

Using the above information and concept of determinants, answer the following questions

1) If the vertices of one of the smaller equilateral triangle are $(0, 0), (3, \sqrt{3})$ and $(3, -\sqrt{3})$ then find the area of such triangle
2) Let A (a, 0), B (0, b) and C (1, 1) be three points such that $\frac{1}{a} + \frac{1}{b} = 1$ then prove that the 3 points are collinear.

Answer:-

1) $3\sqrt{3}$

CHAPTER: CONTINUITY AND DIFFERENTIABILITY

SYLLABUS: *Continuity and differentiability, Chain rule, Derivative of inverse trigonometric functions, Derivative of implicit functions, Concept of exponential and logarithmic functions, Derivatives of logarithmic and exponential functions, Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.*

Definitions and Formulae:

- Continuous function - A real valued function f(x) is said to be continuous, if it is continuous at every point in the domain of f (x) .

Continuity of a function at a point – A real valued function f(x) is said to be continuous at

$x = a$ if

LHL= RHL= f (a)

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) =$ f (a)

- Derivative of a function- The derivative of a function f(x) is defined by

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

- Left Hand Derivative (LHD)= Lf'(a)= $\lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (RHD)= Rf'(a)= $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

- A real valued function f(x) is said to be differentiable at x= a if its LHD and RHD at x=a exist and both are equal

Standard Derivatives

Sl. No	Function	Derivative
1	x^n	nx^{n-1}
2	K (constant)	0
3	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
4	$sin\ x$	$cos\ x$
5	$cos\ x$	$-sin\ x$
6	$tan\ x$	$sec^2 x$
7	$sec\ x$	$sec\ x\ tan\ x$

8	$cosec\ x$	$-cosec\ x\ cot\ x$
9	$cot\ x$	-$cosec^2 x$
10	e^x	e^x
11	$\log_e x$	$\frac{1}{x}$
12	$sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
13	$cos^{-1}x$	$\frac{-1}{\sqrt{1-x^2}}$
14	$tan^{-1}x$	$\frac{1}{1+x^2}$
15	$sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
16	$cosec^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$
17	$cot^{-1}x$	$\frac{-1}{1+x^2}$
18	a^x	$a^x \log_e a$

- Product Rule-

  If y= u v then $\frac{dy}{dx}$ = u $\frac{dv}{dx}$ + v $\frac{du}{dx}$

- Quotient Rule-

  If y= $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- Chain Rule-

  If $y = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} . \frac{dt}{dx}$

- Derivative of implicit functions- Let $f(x, y) = 0$ be an implicit function of x, then to find $\frac{dy}{d\,x}$, first differentiate both sides of the equation w.r.t x and then take all the terms containing $\frac{dy}{dx}$ to LHS and remaining terms to the right , then find $\frac{dy}{dx}$.

- Logarithmic Differentiation-

  Used to differentiate functions of the form $u(x)^{v(x)}$

- Parametric Differentiation-

If x=f (t) and y= g (t) then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

- Second order derivative-

If y= f(x) then the second order derivative is $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND SOLUTION
1	If y= a sin mx + b cos mx ,then $\frac{d^2y}{dx^2}$ is (a) m^2y (b) $-m^2y$ (c) my (d) –my **SOLUTION: Option (b)** $\frac{dy}{dx}$ = am cos mx – bm sin mx $\frac{d^2y}{dx^2}$ = -am^2sin mx - bm^2 cos mx = -m^2 (a sin mx + b cos mx) = -m^2y
2	$\frac{d}{dx}log(x+\sqrt{x^2+1})$ is equal to (a) $\sqrt{x^2+1}$ (b) $x\sqrt{x^2+1}$ (c) $\frac{x}{\sqrt{x^2+1}}$ is (d) $\frac{1}{\sqrt{x^2+1}}$ **SOLUTION: Option (d)** $\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}[1+\frac{2X}{2\sqrt{x^2+1}}]$ $= \frac{1}{x+\sqrt{x^2+1}} . \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}$ $= \frac{1}{\sqrt{x^2+1}}$
3	If u = $\sin^{-1}\frac{2x}{1+x^2}$ and v = $tan^{-1}\frac{2x}{1-x^2}$, then $\frac{du}{dv}$ is (a) $\frac{1}{2}$ (b) x (c) $\frac{1-x^2}{1+x^2}$ (d) 1 **SOLUTION: Option (d)** Put x= tanθ u = $\sin^{-1}\frac{2tan\theta}{1+tan^{2\theta}}$ = $\sin^{-1}sin2\theta$ =2θ= 2 $\tan^{-1}x$

	v= $tan^{-1}\frac{2tan\theta}{1-tan^2\theta}$ = $tan^{-1}tan2\theta$ = 2θ = 2 $\tan^{-1}x$ $\frac{du}{dv}$ = $\frac{\frac{du}{dx}}{\frac{dv}{dx}}$ = 1
4	The points of discontinuity of the function f(x) = $\begin{cases} 2x+3, & if\ x \le 2 \\ 2x-3, & if\ x > 2 \end{cases}$ are (a) -2 (b) 2 (c) ± 2 (d) (-2,2) SOLUTION: Option (b) At x =2 LHL = 2(2)+3 =7 RHL = 2(2)-3 = 1 LHL $\neq RHL$ So f(x) is not continuous at x=2 X = 2 is the point of discontinuity.
5	Derivative of x^2 with respect to x^3 is (a) $\frac{1}{x}$ (b) $\frac{3}{2x}$ (c) $\frac{2}{3x}$ (d) $\frac{3x}{2}$ **SOLUTION: Option (c)** **u=** x^2 v = x^3 $\frac{du}{dv}$ = $\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2} = \frac{2}{3x}$
6	If f(x) = $\begin{cases} 3x-5, & x \le 4 \\ 2k, & x > 4 \end{cases}$ is continuous at x=4 then k is (a) $\frac{7}{2}$ (b) $\frac{2}{7}$ (c) $\frac{-7}{2}$ (d) $\frac{-2}{7}$ **SOLUTION: Option (a)** At x= 4 LHL = 3(4)-5 =7 RHL = 2k LHL = RHL 7=2k k= $\frac{7}{2}$

7	If x = t^2 y= t^3 then $\frac{d^2y}{dx^2}$ is equal to (a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$ **SOLUTION: Option (b)** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3t}{2}$ $\frac{d^2y}{dx^2} = \frac{3}{2} \cdot \frac{1}{2t} = \frac{3}{4t}$
8	The function f(x) =[x] is continuous at (a) 4 (b) -2 (c) 1 (d) 1.5 **SOLUTION: Option (d)** Greatest integer function is not continuous at integral values**.**
9	For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4})$ is (a) 1 (b) $\frac{1}{2}$ (c) -1 (d) none of these **SOLUTION: Option (c)** $\sqrt{x} + \sqrt{y} = 1$ $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$ $\frac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4}) = -1$
10	If y = $\frac{\log x}{x}$, then y_2 = (a) $\frac{3-2logx}{x^3}$ (b) $\frac{2logx-3}{x^3}$ (c) $\frac{2logx-3}{x^4}$ (d) $none\ of\ these$ **SOLUTION: Option (b)** y = $\frac{\log x}{x}$ $\frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right)-logx}{x^2}$ $= \frac{1-logx}{x^2}$ $y_2 = \frac{\frac{-x^2}{x} - (1-logx)2x}{x^4} = \frac{-x-2x+2xlogx}{x^4} = \frac{2logx-3}{x^3}$

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
CONTINUITY AND DIFFERENTIABILITY	https://youtu.be/fIuPO6ThkfQ	

EXERCISE

1	If y = ae^{mx} + be^{-mx} , then $\frac{d^2y}{dx^2}$ is (a) m^2y (b) $-m^2y$ (c) my (d) –my Ans: (a)
	The function f(x) = $\begin{cases} \frac{sinx}{x} + cosx, & x \neq 0 \\ k \quad , & if\ x = 0 \end{cases}$, is continuous at $x = 0$, then value of k is (a) 3 (b) 2 (c) 1 (d) 1.5 Ans: (b)
3	If x= $acos^3\theta$, y= $asin^3\theta$ then $\sqrt{1 + (\frac{dy}{dx})^2}$ (a) $tan^2\theta$ (b) $sec^2\theta$ (c) secθ (d) | secθ| Ans: (c)
4	The points of discontinuity of the function f(x) = $\begin{cases} 3x + 5\ , & if\ x \geq 2 \\ x^2\ , & if\ x < 2 \end{cases}$ are (a) -2 (b) 2 (c) ± 2 (d) (-2,2) Ans: (b)
5	Find the value of k for which the given function is continuous f(x)= $\begin{cases} \frac{2^{x+2}-16}{4^x-16} & if\ x \neq 2 \\ k & if\ x = 2 \end{cases}$ (a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$ Ans: (c)

ASSERTION & REASONING QUESTIONS

	In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R true and R is the correct explanation of A. (b) Both A and R true and R is not the correct explanation of A (c) A is true but R is false (d) A is false but R is true
1	Assertion(A) : If $x = 2at$, $y = at^2$, then $\frac{d^2y}{dx^2}$ is constant for all t Reason (R) : If $x = f(t)$, $y = g(t)$, then $\frac{d^2y}{dx^2} = \frac{f'(t)g''(t) - g'(t)f''(t)}{(f'(t))^2}$ **SOLUTION: Option (c)** **Explanation:** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2at}{2a}$ = t $\frac{d^2y}{dx^2}$ = 1($\frac{1}{2a}$) which is a constant. So A is true $\frac{d^2y}{dx^2} = \frac{d}{dt}(\frac{dy}{dx})\frac{dt}{dx}$ so R is false A is true but R is false
2	Assertion(A) : If the function f(x) = $\begin{cases} a\sqrt{x+1}, & 0 \leq x \leq 3 \\ bx+2, & 3 < x \leq 5 \end{cases}$ is differentiable, then $2a = 3b + 2$ Reason (R): Every continouou function is differentiable. **SOLUTION: Option (a)** Explanation: LHL = $\lim_{h \to 0} f(3-h)$ = $\lim_{h \to 0} a\sqrt{3-h+1}$ = 2a RHL =$\lim_{h \to 0} f(3+h)$= $\lim_{h \to 0} b(3+h) + 2$ = 3b+2 Since f is differentiable, it is continuous so LHL = RHL $2a = 3b + 2$ So A is true & R is also not true
3	Assertion (A): The function f(x) = |x| is everywhere continuous. Reason (R): Every differentiable function is continuous. **SOLUTION: Option (b)**

	Explanation: LHL = $\lim_{h\to 0} f(a-h)$ = $\lim_{h\to 0} \|a-h\|$=a RHL =$\lim_{h\to 0} f(a+h)$= $\lim_{h\to 0}\|a+h\|$ = a f(a) = |a| =a LHL= RHL= f (a) so f is continuous. So A is true & R is true Both A and R true and R is not the correct explanation of A
4	Assertion(A) : If $f(x)$ and $g(x)$ are two continuous functions such that $f(0) = 3, g(0) = 2,$ then $\lim_{x\to 0}\{f(x)+g(x)\}$ =5 Reason (R) : If f(x) and g(x) are two continuous functions at x=a then $\lim_{x\to a}\{f(x)+g(x)\}$ = $\lim_{x\to a} f(x)$ + $\lim_{x\to a} g(x)$ **SOLUTION: Option (a)** **Explanation:** **By using algebra of limits** $\lim_{x\to 0}\{f(x)+g(x)\}$ = $\lim_{x\to 0} f(x)$ + $\lim_{x\to 0} g(x) = f(0) + g(0) = 3+2 = 5$ So A is true and R is also true Both A and R true and R is the correct explanation of A.
5	Assertion (A): If $y = \sin x$, then $\frac{d^3y}{dx^3} = -1\ at\ x = 0$ Reason (R) : If $y = f(x).g(x)$ then $\frac{dy}{dx} = f(x)\, g'(x) + g(x)\, f'(x)$ **SOLUTION: Option (b)** **Explanation:** y= sin x $\frac{dy}{dx}$ **= cos x** $\frac{d^3y}{dx^3}$ = -sin x $\frac{d^3y}{dx^3}$ = - cos x $\frac{d^3y}{dx^3}$ =-1 at x=0

	So A is true & R is product rule which is also true Both A and R true and R is not the correct explanation of A
6	Assertion (A): If $f(x)$ = $\sin^{-1} x$ + $\cos^{-1} x$ + 2 , then $f'(1) = 0$ Reason (R) : $\frac{d}{dx}(\sin x)$ = cos x **SOLUTION: Option (b)** **Explanation:** f(x) = $\sin^{-1} x$ + $\cos^{-1} x$ + 2 f'(x) = $\frac{1}{\sqrt{1-x^2}}$ + $\frac{-1}{\sqrt{1-x^2}}$ +0 =0 f'(1) = 0 so A is true & R is also tue. Both A and R true and R is not the correct explanation of A
7	Assertion (A): The function f(x) = $\frac{\|x\|}{x}$ is continuous at $x = 0$ Reason (R) : $\lim\limits_{x\to 0^-} \frac{\|x\|}{x}$ and $\lim\limits_{x\to 0^+} \frac{\|x\|}{x}$ are -1 *and* 1 respectively **SOLUTION: Option (d)** **Explanation:** **LHL =** $\lim\limits_{x\to 0^-} \frac{\|x\|}{x}$ = -1 , RHL = $\lim\limits_{x\to 0^+} \frac{\|x\|}{x}$ =1 LHL≠ RHL So A is false. A is false but R is true
8	Assertion (A): If f(x) = $\begin{cases} \frac{\sin 5x}{x} & if\ x \neq 0 \\ \frac{k}{3} & if\ x = 0 \end{cases}$ is continuous at $x = 0\ then\ k = 15$ Reason (R) : If $f(x)$ is continuous at a point $x = a$ in its domain , then $\lim\limits_{x\to a} f(x) = f(a)$ **SOLUTION: Option (a)** **Explanation:** LHL =$\lim\limits_{h\to 0} f(a-h)$ = $\lim\limits_{h\to 0} \frac{\sin 5(0-h)}{0-h}$ = $\lim\limits_{h\to 0} 5.\frac{\sin -5h}{-5h}$ =5 f(a)= f(0) = $\frac{k}{3}$ Since f is continuous LHL = RHL= f (a)

	$k/3 = 5$ $k = 15$ Both A and R true and R is the correct explanation of A
9	Assertion (A): If y= $\tan^{-1}(\frac{sinx+cosx}{cosx-sinx})$, then $\frac{dy}{dx}$ =1 Reason (R) : $\frac{sinx+cosx}{cosx-sinx}$ = tan $(\frac{\pi}{4}+x)$ **SOLUTION: Option (a)** **Explanation:** y= $\tan^{-1}(\frac{sinx+cosx}{cosx-sinx})$ y= $\tan^{-1}(\frac{1+\tan x}{1-tanx})$ y= $\tan^{-1}(\tan(\frac{\pi}{4}+x)) = \frac{\pi}{4}+x$ $\frac{dy}{dx}$ =1 Both A and R true and R is the correct explanation of A
10	Assertion (A): The real valued function $f(x) = 3x^2 - 2x + 7$ is continuous at $x = 2$ Reason (R) : Every polynomial function is continuous. **SOLUTION: Option (a)** **Both A & R are true** Both A and R true and R is the correct explanation of A.

EXERCISE

1	Assertion (A): Every continuous function is differentiable Reason (R) : Every differentiable function is continuous. Ans: (d)
2	Assertion (A): The function f(x) =[x], greatest integer function, is not differentiable at integer points Reason (R) : The greatest integer function is not continuous at integer points Ans: (b)
3	Assertion (A): If y = sin ax^0 , then $\frac{dy}{dx} = \frac{a\pi}{180}$ cos ax^0 Reason (R) : $\pi^c = 180^0$ Ans: (a)
4	Assertion (A): If f(x) = 2$\tan^{-1}x$ + $\sin^{-1}\frac{2x}{1+x^2}$, $f'(2) = f'(3)$

	Reason (R): $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$ for all x. Ans: (d)
5	Assertion (A): If $f(x)$ *differentiable at* $x = a$, then $\lim_{x\to a} f(x)$ = f(a) Reason (R) : Every differentiable function is continuous. Ans: (a)

2 MARKS QUESTIONS

Q.NO	QUESTIONS WITH SOLUTIONS
1	For what value of k, is the following function continuous at $x = 0$ f(x)= $\begin{cases} \frac{1-\cos 4x}{8x^2}, when, x \neq 0 \\ k \quad , when, x = 0 \end{cases}$ **SOLUTION:** $\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{1-\cos 4x}{8x^2}$ $= \lim_{x\to 0}\frac{2\sin^2 2x}{8x^2}$ $= \lim_{x\to 0}\frac{\sin 2x}{2x} \times \lim_{2x\to 0}\frac{\sin 2x}{2x}$ $= 1\times 1 = 1$ $f(0) = k$ $f(x)$ is continuous at x=0 if $\lim_{x\to 0} f(x) = f(0)$ $\therefore k = 1$,
2	If $x = a\,(\theta - \sin\theta) and\ y = a\,(1 - \cos\theta), find\ \frac{dy}{dx} at\ \theta = \frac{\pi}{2}$ **SOLUTION:** $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\frac{dy}{dx} = \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{2sin\frac{\theta}{2}cos\frac{\theta}{2}}{2sin^2\frac{\theta}{2}}$ $= \cot\frac{\theta}{2}$ $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ $= \cot\frac{\pi}{4} = 1$
3	If $x = a(cos\theta + \theta sin\theta)$, $y = a(sin\theta - \theta cos\theta)$,find $\frac{dy}{dx}$. **SOLUTION:**

	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $= \frac{a(cos\theta-(-\theta sin\theta+cos\theta)}{a(-\sin\theta+\theta cos\theta+\sin\theta)}$ $= \frac{\theta sin\theta}{\theta cos\theta}$ $= \tan\theta$
4	If y= $(\tan^{-1} x)^2$,show that $(x^2+1)^2 y_2$+2x $(x^2+1)y_1$=2 **SOLUTION:** y= $(\tan^{-1} x)^2$ $y' = \frac{2\tan^{-1} x}{1+x^2}$ $(1+x^2)y'$ = $2\tan^{-1} x$ $(1+x^2)y'' + 2xy' = \frac{2}{1+x^2}$ $(x^2+1)^2 y_2$+2x $(x^2+1)y_1$=2
5	If $cos\ y = x\ cos\ (a + y)$, Prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{\sin a}$ **SOLUTION:** $cos\ y = x\ cos\ (a + y)$ $x = \frac{\cos y}{\cos(a+y)}$ $\frac{dx}{dy} = \frac{-\cos(a+y)\ siny + cosysin(a+y)}{cos^2(a+y)}$ $\frac{dx}{dy} = \frac{\sin a}{cos^2(a+y)}$ $\frac{dy}{dx} = \frac{cos^2(a+y)}{\sin a}$
6	If $y = \tan^{-1}(\frac{cosx-sinx}{cosx+sinx})$, find $\frac{dx}{dy}$ **SOLUTION:** y= $\tan^{-1}(\frac{cosx-sinx}{cosx+sinx})$ Dividing by cos x y= $\tan^{-1}(\frac{1-tanx}{1+tanx})$

	y= $\tan^{-1}\tan(\frac{\pi}{4}-x)$ y = $\frac{\pi}{4}-x$ $\frac{dy}{dx} = -1$
7	Find all the points of discontinuity of the function $f(x) = \begin{cases} x^{10}-1\,, & if\ x \leq 1 \\ x^2 & if\ x > 1 \end{cases}$ **SOLUTION:** At x = 1 LHL= $\lim_{x\to a^-} f(x)$ = $\lim_{h\to 0} f(1-h)$ = $\lim_{h\to 0}(1-h)^{10}-1$ = 0 RHL = $\lim_{x\to a^+} f(x)$ = $\lim_{h\to 0} f(1+h)$ = $\lim_{h\to 0}(1+h)^2$ = 1 Since LHL ≠RHL $f(x)\ is\ not\ continuous\ at\ x = 1$ $So\ x = 1\ is\ the\ point\ of\ discontinuity$.
8	If y= $\cos^{-1}\frac{2x}{1+x^2}$, find $\frac{dx}{dy}$ **SOLUTION:** y= $\cos^{-1}\frac{2x}{1+x^2}$ Put x= tan θ y= $\cos^{-1}\frac{2x}{1+x^2}$ y= $\cos^{-1}\frac{2tan\theta}{1+tan^2\theta}$ y= $\cos^{-1} sin2\theta$

	$y = \cos^{-1}\cos(\frac{\pi}{2} - 2\theta)$ $y = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1}x$ $y = \frac{-2}{1+x^2}$
9	If $y = \tan^{-1}x$, then find $\frac{dy}{dx}$ in terms of y alone **SOLUTION:** $y = \tan^{-1}x$ $\frac{dy}{dx} = \frac{1}{1+x^2}$ $= \frac{1}{1+tan^2y} = \frac{1}{sec^2y} = cos^2y$
10	Differentiate log $(1+x^2)$ with respect to $\tan^{-1}x$. **SOLUTION:** Let u = log $(1+x^2)$ and v= $\tan^{-1}x$ $\frac{du}{dv} = \frac{du/dx}{dv/dx}$ $= \frac{2x}{1+x^2} .1+x^2 = 2x$

EXERCISE

1	Find the value of k for which $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & , -1 \leq x < 0 \\ \frac{2x+1}{x-1} & , 0 \leq x \leq 1 \end{cases}$ is continuous at $x = 0$. Ans: $k = -1$
2	Show that the function $f(x) = \|x + 2\|$ is continuous at every x$\in R$, but fails to be differentiable at $x = -2$
3	Check whether the function f(x) = $\begin{cases} 3x+5, & x \geq 2 \\ x^2, & x < 2 \end{cases}$ is continuous at $x = 2$. Ans: Not continuous
4	Prove that $f(x) = \begin{cases} 1+x, & x \leq 2 \\ 5-x, & x > 2 \end{cases}$ is not differentiable at $x = 2$.
5	If $y = 5cos\,x - 3\,sin\,x$, prove that $\frac{d^2y}{dx^2}$ +y = 0

3 MARK QUESTIONS

Q.NO	QUESTIONS WITH SOLUTIONS
1	If $x = a(\cos t + \log(\tan\frac{t}{2}))$, $y = a \sin t$ find the value of $\frac{d^2y}{dx^2}$ at t= $\frac{\pi}{4}$ **SOLUTION:** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $= \frac{a\cos t}{a(-\sin t+\frac{1.\sec^2\frac{t}{2}\frac{1}{2}}{\tan\frac{t}{2}})}$ $= \frac{a\cos t}{a(-\sin t+\frac{1./\cos^2\frac{t}{2}}{2\sin\frac{t}{2}/\cos\frac{t}{2}})}$ $= \frac{a\cos t}{a(-\sin t+\frac{1./\cos^2\frac{t}{2}}{2\sin\frac{t}{2}/\cos\frac{t}{2}})}$ $= \frac{a\cos t}{a(-\sin t+\frac{1}{\sin t})}$ $= \frac{\cos t}{(-\sin t+\frac{1}{\sin t})}$ $= \frac{\cos t\sin t}{1-\sin^2 t}$ $= \frac{\cos t\sin t}{\cos^2 t}$ = tan t $\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{1}{a}$ sect $= (1/a)\ \sec^3(\frac{\pi}{4})$ $= \frac{1}{a} 2\sqrt{2}$
2	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ **SOLUTION:** Put x = sin α, y = sin β cosα + cos β = a (sin α – sin β) 2 Cos $(\frac{\alpha+\beta}{2})$ Cos $(\frac{\alpha-\beta}{2})$ = a 2 Cos $(\frac{\alpha+\beta}{2}) Sin\ (\frac{\alpha-\beta}{2})$ $\cot\left(\frac{\alpha-\beta}{2}\right) = a$ $(\frac{\alpha-\beta}{2}) = \cot^{-1}a$ $\alpha-\beta = 2\cot^{-1}a$ $\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$ $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

	$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
3	If y = $e^{acos^{-1}x}$,-1≤ x ≤ 1 , show that (1 - x^2)$\frac{d^2y}{d^2x}$ – x $\frac{dy}{dx}$ - a^2y = 0 **SOLUTION:** y = $e^{acos^{-1}x}$ $\frac{dy}{dx} = \frac{-ae^{acos^{-1}x}}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\,\frac{dy}{dx} = -ae^{acos^{-1}x}$ $\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{-2x}{\sqrt{1-x^2}} = a^2e^{acos^{-1}x}.\frac{1}{\sqrt{1-x^2}}$ (1 - x^2)$\frac{d^2y}{d^2x}$ – x $\frac{dy}{dx}$ - a^2y = 0
4	If x $\sqrt{1+y}$ + y$\sqrt{1+x}$ = 0,for,-1<x<1,prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ **SOLUTION:** x $\sqrt{1+y}$ + y$\sqrt{1+x}$ = 0 x $\sqrt{1+y}$ = y$\sqrt{1+x}$ $x^2(1+y) = y^2$(1+x) $x^2(1+y) = y^2$(1+x) x^2+x^2y = y^2+y^2x x^2-y^2+x^2y-y^2x=0 (x + y)(x-y)+x y (x-y)=0 (x – y)[x + y + x y] = 0 [x + y + x y] = 0 y(1+x) =-x y = $\frac{-x}{1+x}$ $\frac{dy}{dx} = \frac{(1+x)(-1)-(-x)}{(1+x)^2}$ $= \frac{-1}{(1+x)^2}$

5	If the function f(x) = $\begin{cases} 3ax + b\,, x > 1 \\ 11,\ \ x = 1 \\ 5ax - 2b\,,\ \ x < 1 \end{cases}$ is continuous at x=1.Find the value of a & b. **SOLUTION:** At x=1 LHL= $\lim_{x \to a^-} f(x)$ = $\lim_{h \to 0} f(1-h)$ = $\lim_{h \to 0} 5a(1-h) - 2b$ = 5a-2b RHL = $\lim_{x \to a^+} f(x)$ = $\lim_{h \to 0} f(1+h)$ = $\lim_{h \to 0} 3a(1+h) + b$ = 3a + b Since f (x) is continuous LHL=RHL=f(1) 5a-2b = 3a+b = 11 On solving a= 3 b= 2
6	If x= a (cost +t sin t) and y = a (sin t -t cos t) . Find $\frac{dy}{dx}$. & . $\frac{d^2y}{dx^2}$ **SOLUTION:** $\frac{dx}{dt}$ = a (-sin t + t cos t + sin t) = at cos t $\frac{dy}{dt}$ = a (cos t –(-t sin t +cos t)) = at sin t $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ = tan t $\frac{d^2y}{dx^2} = sec^2t \cdot \frac{1}{at\cos t} = \frac{sec^3t}{at}$
7	If y = $\sqrt{sinx + \sqrt{sinx + \sqrt{sinx + \cdots \ldots to\ \infty}}}$. Prove that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$ **SOLUTION:** y = $\sqrt{sinx + \sqrt{sinx + \sqrt{sinx + \cdots \ldots to\ \infty}}}$ y= $\sqrt{\sin x + y}$

	y^2 = sin x + y 2y $\frac{dy}{dx}$ = cos x + $\frac{dy}{dx}$ $\frac{dy}{dx}$ (2y-1) =cos x $\frac{dy}{dx} = \frac{\cos x}{2y-1}$
8	If y= log (x+ $\sqrt{x^2+a^2}$) ,prove that $(x^2+a^2)\frac{d^2y}{dx^2}$ + x $\frac{dy}{dx}$ =0 **SOLUTION:** y= log (x+ $\sqrt{x^2+a^2}$) $\frac{dy}{dx} = \frac{1}{\text{x+ }\sqrt{x^2+a^2}}$ [1+ $\frac{2x}{2\sqrt{x^2+a^2}}$] $= \frac{1}{\text{x+ }\sqrt{x^2+a^2}} \cdot \frac{\text{x+ }\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$ $= \frac{1}{\sqrt{x^2+a^2}}$ $\frac{dy}{dx}\sqrt{x^2+a^2}$ =1 $\sqrt{x^2+a^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{2X}{2\sqrt{x^2+a^2}}$=0 $(x^2+a^2)\frac{d^2y}{dx^2}$ + x $\frac{dy}{dx}$ =0
9	If x y = e^{x-y}. Find $\frac{dy}{dx}$. **SOLUTION:** x y = e^{x-y} log x y = log e^{x-y} log x y = x-y $\frac{x\frac{dy}{dx}+y}{xy}$= 1- $\frac{dy}{dx}$ $x\frac{dy}{dx}+y$ = x y – x y $\frac{dy}{dx}$ $\frac{dy}{dx}$ (x + x y) = (x y -y) $\frac{dy}{dx} = \frac{(\text{x y} - \text{y})}{(\text{x} + \text{x y})}$
10	If y= $\sin^{-1} x$, prove that (1-x^2) $\frac{d^2y}{dx^2}$ -x $\frac{dy}{dx}$ =0 **SOLUTION:** y= $\sin^{-1} x$

	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\,\frac{dy}{dx}$ =1 $\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{-2x}{\sqrt{\sqrt{1-x^2}}} = 0$ (1-x^2) $\frac{d^2y}{dx^2}$ -x $\frac{dy}{dx}$ =0

EXERCISE

1	Let $f(x) = \begin{cases} \frac{1-cos4x}{x^2} & if\ x < 0 \\ a & if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}} & if\ x > 0 \end{cases}$ Determine a so that f(x) is continuous at x = 0 Ans: $a = 8$
2	If $(\cos x)^y = (\sin y)^x, find\ \frac{dy}{dx}$. Ans: $\frac{ytanx+logcosy}{xtany+logcosx}$
3	If x = a (cost + t sin t) and y = a (sin t – t cost), 0 < t <$\frac{\pi}{2}$, then find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\ and\ \frac{d^2y}{dx^2}$ Ans: -at sin t+ a cost , at cos t+ a sin t, $\frac{sec^3t}{at}$
4	If y= $(xcosx)^x + (xsin\,x)^{\frac{1}{x}}$ .Find $\frac{dy}{dx}$ Ans: $(xcosx)^x$[1-tanx+log(xcosx)+$(xsinx)^{1/x}[\frac{xcotx+1-logxsinx}{x^2}]$
5	If $x = sin\ t\ and\ y = sin\ pt$, prove that (1-$x^2$) $\frac{d^2y}{dx^2}$ -x$\frac{dy}{dx}$ +$p^2y = 0$

5 MARK QUESTIONS

Q.NO	QUESTIONS WITH SOLUTIONS
1	Differentiate the function f(x) = $x^{sinx} + sinx^{cosx}$ with respect to x. **SOLUTION:** Y = $x^{sinx} + sinx^{cosx}$ Y = u + v $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------(1) u= $\boldsymbol{x^{sinx}}$ log u = log $\boldsymbol{x^{sinx}}$ log u = sin x log x

	$\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x}$ +cos x log x $\frac{du}{dx} = \boldsymbol{x^{sinx}}$ [$\frac{\sin x}{x}$ +cos x log] ------(2) V= $\boldsymbol{sinx^{cosx}}$ log v = log $\boldsymbol{sinx^{cosx}}$ log v = cos x log sinx $\frac{1}{v}\frac{dv}{dx} = \frac{cos^2 x}{\sin x} - \sin x \log \sin x$ $\frac{dv}{dx}$ = $\text{sinx}^{\text{cosx}}$ [$\frac{cos^2 x}{\sin x} - \sin x \log \sin x$]------(3) Substituting in (1) $\frac{dy}{dx} = \boldsymbol{x^{sinx}}$ [$\frac{\sin x}{x}$ +cos x log] + $\boldsymbol{sinx^{cosx}}$ [$\frac{cos^2 x}{\sin x} - \sin x \log \sin x$]
2	Differentiate the function f(x) = sin^{-1} ($\frac{2^{x+1}}{1+4^x}$) with respect to x. **SOLUTION:** y = $sin^{-1}(\frac{\boldsymbol{2^{x+1}}}{\boldsymbol{1+4^x}})$ = $\boldsymbol{sin^{-1}}(\frac{\boldsymbol{2^x.2}}{\boldsymbol{1+(2^x)^2}})$ Put 2^x = tan θ Y = $sin^{-1}(\frac{2tan\,\theta}{1+tan^2\theta})$ Y = $\sin^{-1} sin\, 2\theta$ Y = 2θ Y = $2\tan^{-1} 2^x$ $\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = 2\,\frac{1}{1+(2^x)^2} \cdot \frac{d}{dx}(2^x)$ $\frac{dy}{dx} = \frac{2}{1+(4^x)}$. 2^x log 2 = $\frac{2^{x+1}}{1+(4^x)}$ log 2

3	Determine the values of a, b and c for which the function $$f(x) = \begin{cases} \dfrac{\sin(a+1)\,x + \sin x}{x} & for\ x < 0 \\ c\ , \quad for\ \ x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} & , x > 0 \end{cases}$$ Is continuous at x=0. **SOLUTION:** LHL = $\lim_{x\to 0^-} f(x)$ $= \lim_{h\to 0} f(0-h)$ $= \lim_{h\to 0} \frac{\sin(a+1)(0-h)+\sin(0-h)}{0-h}$ $= \lim_{h\to 0} \frac{-\sin(a+1)h}{-h} + \frac{\sin(-h)}{-h}$ = a+1 +1 =a+2 RHL = $\lim_{x\to 0^+} f(x)$ $=\lim_{h\to 0} f(0+h)$ $= \lim_{h\to 0} \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}} \cdot \frac{\sqrt{x+bx^2}+\sqrt{x}}{\sqrt{x+bx^2}+\sqrt{x}}$ $= \lim_{h\to 0} \frac{x+bx^2-x}{bx^{\frac{3}{2}}(\sqrt{x+bx^2}+\sqrt{x})}$ $= \lim_{h\to 0} \frac{\sqrt{x}}{\sqrt{x}(\sqrt{1+bx}+1)}$ $= \frac{1}{2}$ = f (0) = c Since f (x) is continuous LHL=RHL=f(o) a+2 = $\frac{1}{2}$ = c a = $\frac{-3}{2}$, c=$\frac{1}{2}$, b any real number other than 0.

EXERCISE

1	Find $\frac{dy}{dx}$ if y=$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, 0 <$\lvert x\rvert < 1$. Ans: $\frac{-x}{\sqrt{1-x^4}}$
2	If (a+ bx) $e^{\frac{y}{x}}$ = x then prove that x. $\frac{d^2y}{dx^2}$ = $(\frac{a}{a+bx})^2$
3	If $x^p y^q = (x+y)^{p+q}$, Prove that $\frac{dy}{dx} = \frac{y}{x}$

CASE BASED QUESTIONS (4 MARKS)

1	Sonia was noticing the path traced by a crawling insect and she observed that the path traced is given by x= at^2 , y = 2at Based on the above information, answer the following questions. (i) Find $\frac{dx}{dt}$ (ii) Find $\frac{dy}{dx}$ (iii) Find $\frac{d^2y}{dx^2}$ at t =4 OR Find $\frac{d^2x}{dy^2}$ at t =4 **SOLUTION:** x= at^2 , y = 2at (i) $\frac{dx}{dt}$ = 2at (ii) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$ (iii) $\frac{d^2y}{dx^2} = \frac{-1}{t^2}$ x $\frac{1}{2at} = \frac{-1}{2at^3}$ At t = 4 $= \frac{-1}{128a}$ OR $\frac{d^2x}{dy^2}$ $at\ t = 4 = \frac{1}{2a}$
2	Let f(x) be a real valued function. Then its • Left Hand Derivative (LHD): Lf'(a)= $\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$ • Right Hand Derivative (RHD): Rf'(a)= $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

Also a function f(x) is said to be differentiable at x= a if its LHD and RHD at x=a exist and both are equal.

For the function $f(x) = \begin{cases} |\boldsymbol{x-3}|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$

Answer the following questions.

(i) What is RHD of f(x) at x= 1?

(ii) What is LHD of f(x) at x=1?

(iii) (a) Check if the function f(x) is differentiable at x=1

OR

(b) Find $f'(2)$ and $f'(-1)$

SOLUTION:

(i) RHD of f(x) at x= 1 = $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{|1+h-3|-|-2|}{h}$

$= \lim_{h\to 0} \frac{2-h-2}{h}$ = -1

(ii) LHD of f(x) at x=1 = $\lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h\to 0} \frac{-1}{h}\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]$

$= \lim_{h\to 0} \frac{h^2-2h+1-6+6h+13-8}{-4h}$

$= \lim_{h\to 0} \frac{h^2+4h}{-4h}$ = -1

(iii) (a) Since LHD of f(x) at x=1 = RHD of f(x) at x=1, f(x) is differentiable at x=1

OR

(b) $f(x) = \begin{cases} x-3, & x \geq 3 \\ 3-x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$

$[f'(x)]$ at x=2 = 0-1=-1

$[f'(x)]$ at x=-1= $\frac{2(-1)}{4} - \frac{3}{2}$ = -2

3 A potter made a mud vessel, where the shape of the pot is based on

f(x)= |x-3|+|x-2|, where f(x) represents the height of the pot.

Based on the above information answer the following questions.

(i) Find the value of [f (2.3)] where [a] represent the greatest integer ≤ a.

	(ii) What is the value of $f'(x)$ at x=4 (iii) Show that f is continuous at x=2 OR Show that f is not differentiable at x=3 **SOLUTION:** (i) f(x) = |x-3|+|x-2| f(2.3) = |2.3-3|+|2.3-2| = 0.7+0.3 =1 [f(2.3)] =[1] =1 (ii) $f(x) = \begin{cases} 5-2x, & x<2 \\ 1, & 2 \le x \le 3 \\ 2x-5, & x>3 \end{cases}$ f' (4) = 2 (iii) LHL = $\lim\limits_{x \to 2^-} \lvert x-3 \rvert + \lvert x-2 \rvert$ $=\lim\limits_{h \to 0} \lvert 2-h-3 \rvert + \lvert 2-h-2 \rvert$ =1 RHL = $\lim\limits_{x \to 2^+} \lvert x-3 \rvert + \lvert x-2 \rvert$ $=\lim\limits_{h \to 0} \lvert 2+h-3 \rvert + \lvert 2+h-2 \rvert$ =1 f(2) = |2-3|+|2-2| = 1 Since LHL=RHL=f(2), f is continuous at x=2. OR $f(x) = \begin{cases} 5-2x, & x<2 \\ 1, & 2 \le x \le 3 \\ 2x-5, & x>3 \end{cases}$ Lf'(3) =0, and Rf'(3) =2 Since Lf'(3) ≠ Rf'(3), f is not differentiable at x=3.

EXERCISE

1	If a relation between x & y is such that y cannot be expressed in terms of x, then y is called an implicit function of x. When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation w.r.t x, remembering that a term in y is first differentiated w.r.t y and then multiplied by $\frac{dy}{dx}$. Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following (i) $x^3 + x^2y + xy^2 + y^3$ =81 (ii) $x^y = e^{x-y}$ (iii) $e^{siny} = xy$ OR

	$sin^2x + cos^2y = 1$ Ans: $(i)\,\frac{-(3x^2+2xy+y^2)}{(x^2+2xy+3y^2)}$,(ii) $\frac{x-y}{x(logx+1)}$, $(iii)\,\frac{1/x}{(cosy-\frac{1}{y})}$ or $\frac{sin2x}{sin2y}$
2	If y= f (u) is a differentiable function of u and u= g(x) is a differentiable function of x ,then y= f(g(x))is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} X \frac{du}{dx}$. This rule is known as CHAIN RULE. Based on the above information find the value of $\frac{dy}{dx}$ in each of the following (i) $\cos\sqrt{x}$ (ii) $7^{x+\frac{1}{x}}$ (iii) $\frac{1}{b}\tan^{-1}\frac{x}{b} + \frac{1}{a}\tan^{-1}\frac{x}{a}$ OR $\sec^{-1} x + cosec^{-1}\frac{x}{\sqrt{x^2-1}}$ Ans: (i) $\frac{-sin\sqrt{x}}{2\sqrt{x}}$ (ii) $7^{x+\frac{1}{x}}\log 7\,(1-\frac{1}{x^2})$ (iii) $\frac{1}{x^2+b^2} + \frac{1}{x^2+a^2}$ or $\frac{2}{x\sqrt{x^2-1}}$

CHAPTER: APPLICATION OF DERIVATIVES

SYLLABUS: *Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).*

Definations and Formulae:

Derivative as Rate of Change

- Let $y = f(x)$ be a function. Then $\frac{dy}{dx}$ denotes the rate of change of $y\ w.r.t\ x$.
- The value of $\frac{dy}{dx}$ at $x = x_0$ i.e $\left(\frac{dy}{dx}\right)_{x=x_0}$ i.e. represents the rate of change of $y\ w.r.t\ x\ at$ $x = x_0$
- If two variables x and y are varying with respect to another variable t, i.e., if $x = f(t)$ and y= g(t), then by Chain Rule $\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)$, provided $\frac{dx}{dt} \neq 0$
- $\frac{dy}{dx}$ is positive if y *increases as* x increases and is negative if y *decreases as* x *increases.*

Increasing and Decreasing Functions

- A function y = f (x) is said to be **increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in (a, b)$

  Alternatively, a function y = f (x) is said to be increasing if $f'(x) \geq 0$ for each x in (a, b)

  (a) **strictly increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ (a, b).

  Alternatively, a function y = f (x) is said to be strictly increasing if $f'(x) > 0$ for each x in (a, b)

  (b) **decreasing** on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in$ (a, b). Alternatively, a function y = f (x)) is said to be decreasing if $f'(x) \leq 0$ for each x in (a, b)

  (c) **strictly decreasing** on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ (a, b).

  Alternatively, a function y = f (x) is said to be strictly decreasing if $f'(x)<0$ for each x in (a, b)

  (d) **constant** function in (a, b), if f (x) = c for all $x \in$ (a, b), where c is a constant.

  Alternatively , f(x) is a constant function if $f'(x) = 0$.

A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called **a critical point** of f.

Maxima and Minima

Definition: Let f be a function defined on an interval I. Then

1) f is said to have a maximum value in I, if there exists a point c in I such that f(c) > f(x), for all x ∈ I. The number f(c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.
2) f is said to have a minimum value in I, if there exists a point c in I such that f (c) < f (x), for all x ∈ I. The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.
3) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I. The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

Local Maxima and Local Minima

Definition: Let f be a real valued function and let c be an interior point in the domain of f. Then

(a) c is called a point of local maxima if there is an h > 0 such that $f\ (c)\ \geq\ f\ (x)$, for all x in $(c - h, c\ +\ h), x\ \neq\ c$. The value f(c) is called the local maximum value of f.

(b) c is called a point of local minima if there is an h > 0 such that f (c) ≤ f (x), for all x in (c – h, c + h). The value f(c) is called the local minimum value of f.

Geometrically, the above definition states that if x = c is a point of local maxima of f, then the graph of f around c will be as shown in Fig.(a) below. Note that the function f is increasing (i.e., f ′(x) > 0) in the interval (c – h, c) and decreasing (i.e., f ′(x) < 0) in the interval (c, c + h). This suggests that f ′(c) must be zero,

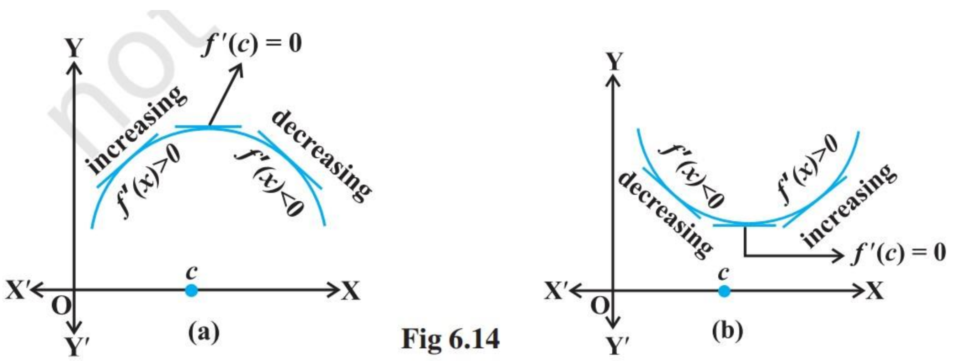


Fig 6.14

Similarly ,if x = c is a point of local minima of f, then the graph of f around c will be as shown in Fig.(b) above. Note that the function f is decreasing (i.e., f ′(x) < 0) in the interval (c – h,

c) and increasing (i.e., $f'(x) > 0$) in the interval $(c, c + h)$. This again suggests that $f'(c)$ must be zero,

Theorem: Let f be a function defined on an open interval I. Suppose $c \in I$ be any point. If f has a local maxima or a local minima at $x = c$, then either $f'(c) = 0$ or f is not differentiable at c.

Definition: A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a **critical point** of f.

Theorem: (***First Derivative Test***) Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

1) If $f'(x)$ changes sign from positive to negative as x increases through c, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of c, and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.
2) If $f'(x)$ changes sign from negative to positive as x increases through c, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of c, and $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of local minima.
3) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called **point of inflection**.

Theorem: ***(Second Derivative Test)*** Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then

1) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$ The value $f(c)$ is local maximum value of f.
2) $x = c$ is a point of local minima if $f'(c)\ 0 =$ and $f''(c) > 0$ In this case, $f(c)$ is local minimum value of f.
3) The test fails if $f'(c) = 0$ and $f''(c) = 0$. In this case, we go back to the first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

Working rule for finding absolute maximum value and/or absolute minimum value

Step 1: Find all critical points of f in the interval, i.e., find points x where either $f'(x) = 0$ or f is not differentiable.

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3.

This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

MULTIPLE CHOICE QUESTIONS

S.NO	QUESTIONS WITH SOLUTIONS
1	The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is (a) 10.8cm (b) 10.8 cm^2 (c) 10.8 cm^2/s (d) 10.8 cm/s Solution: (c) as $\frac{dx}{dt}$ *= 0.3 cm/s ,x is edge of a cube.* *Surface area* $S = 6x^2$ *Then* $\frac{ds}{dt} = 12x \cdot \frac{dx}{dt} = 12x \times 0.3 \frac{dS}{dt} = 3.6x$ And $\frac{dS}{dt}$ at $x = 3$ is $3.6 \times 3 = 10.8\ cm^2/s$
2	The total revenue in ₹ received from the sale of x units of an article is given by R(x) =$3x^2$+36x+5. The marginal revenue when x=15 is (in ₹) (a) 126 (b) 116 (c) 96 (d) 90 Solution: (a), as R′ (x)=6x+36 R′(15)=90+36=126
3	The point on the curve $y = x^2$ where the rate of change of x −coordinate is equal to the rate of change of y −coordinate is (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) (1,1) Sol: (C), as $y=x^2 \Rightarrow \frac{dy}{dt} = 2x\frac{dx}{dt}$; given $\frac{dy}{dt} = \frac{dx}{dt}$ $1 = 2x \Rightarrow x = \frac{1}{2}$ Substituting in the equation of curve, we get point as $\left(\frac{1}{2}, \frac{1}{4}\right)$
4	The interval on which the function f(x)=$2x^3+9x^2+12x-1$ is decreasing ,is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1,1]$ Sol: (b) We have, f(x)=$2x^3+9x^2+12x-1$ f′(x)=$6x^2+18x+12=6(x^2+3x+2)= 6(x+2)(x+1)$ for f(x) to be decreasing, we must have $f'(x) < 0 \Longrightarrow\ 6(x+2)(x+1) \leq 0$ $\Longrightarrow\ (x+2)(x+1) \leq 0$ $-2 \leq x \leq -1$. Hence, f(x) is decreasing on $(-2, -1)$.
5	If at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Then the value of a is (a) 124 b) -124 c) 120 d) -120 Sol: (c) As f′(x) = $4x^3$-124x+a,

	Given x=1 is point of maximum $\Rightarrow f'(1)=0 \;\Rightarrow\; 4 - 124 + a = 0 \;\Rightarrow\; a = 120$
6	The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly (a) increasing in $\left(\pi, \frac{3\pi}{2}\right)$ b) decreasing in $\left(\frac{\pi}{2}, \pi\right)$ c) decreasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ d) decreasing in $\left(0, \frac{\pi}{2}\right)$ Sol: (b) we have, $f(x) = 4\sin^3 x - 6\sin^2 + 12\sin + 100$ $\Rightarrow f'(x) = 12(\sin^2 x - \sin x + 1)\cos x = \left\{\left(\sin x - \frac{1}{2}\right)^2 + \frac{3}{4}\right\}\cos x$ $\Rightarrow$ Sign of $f'(x)$ is same as that of cos x $\Rightarrow f'(x) < 0$ on $\left(0, \frac{\pi}{2}\right) \Rightarrow f(x)$ is decreasing on $\left(\frac{\pi}{2}, \pi\right)$
7	Which of the following functions is decreasing in $(0, \pi/2)$. (a) $\sin 2x$ b) $\tan x$ c) $\cos x$ d) $\cos 3x$ Sol :(c) We find that $\frac{d}{dx}(\cos x) = -\sin x < 0$ for all $x \in (0, \pi/2)$ So, $\cos x$ *is decreasing on* $(0, \pi/2)$
8	The function $f(x)=2x^3-3x^2-12x+4$ has (a) two points of local maximum (b) two points of local minimum (c) one maximum and one minimum (d)no maximum, no minimum Sol: (c) We have, $f(x)=2x^3-3x^2-12x+4$ $\Rightarrow$ $f'(x)=6x^2+18x+12$ and $f''(x)=12x-6$ At points of local maximum or minimum, we have $\Rightarrow$ $f'(x)=0 \Rightarrow 6(x^2-x-2)=0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$ At x = -1, we obtain : $f''(-1) = -18<0$. So, x = -1 is a point of local maximum. At x = 2, we obtain : $f''(2) = 24 - 6 = 18>0$. So, x = 2 is a point of local minimum.
9	The interval on which the function $f(x)=x^3+6x^2+6$ is strictly increasing is (a) $(-\infty,-4) \cup (0, \infty)$ (b) $(-\infty,-4)$ (c) $(-4,0)$ (d) $(-\infty,0) \cup (4, \infty)$ Sol: (a) We have, $f(x) = x^3+6x^2+6$ $\Rightarrow$ $f'(x) = 3x^2+12x = 3x(x+4)$ For f(x) to be increasing, we must have $f'(x)>0 \Rightarrow 3x(x+4)>0 \Rightarrow x(x+4) > 0 \Rightarrow x<-4$ or, $x>0$ Hence, f(x) is increasing on $(-\infty,-4) \cup (0, \infty)$.
10	The rate of change of the area of a circle with respect to its radius r *at* $r = 6cm$ is: (a) 10π cm^2/cm (b) 12π cm^2/cm (c) 8π cm^2/cm (d)11π cm^2/cm Sol: (b) Area of circle $(A) = \pi r^2$ $\Rightarrow \frac{dA}{dr} = 2\pi r$ $\Rightarrow \frac{dA}{dr}]r = 6$ $= 2\pi \times 6 = 12\pi$

11	The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2+36x+5$. The marginal revenue, when $x = 15$ is: (a) 116 b) 96 c) 90 d) 126 Sol: (d) Total revenue $R(x) = 3x^2+36x+5$ Marginal revenue = $\frac{d}{dx}R(x) = 6x + 36 = 6 \times 15 + 36 = 126$
12	The maximum value of the function $f(x) = 5 + sin2x$ is (a) 1 b) 6 c) 4 d) -1 Sol: (b) $-1 \le sin\, 2x \le 1$ $\Rightarrow 5 - 1 \le 5 + sin\, 2x \le 5 + 1$ $\Rightarrow 4 \le f(x) \le 6$ Maximum value of $f(x) = 6$
13	The function $f(x) = x - sin\, x$ decreases for (a) $all\ x$ (b) $x < \pi/2$ (c) $0 < x < \pi/4$ (d) $no\ value\ of\ x$ Sol: (d) We have, $f(x) = x - sin\, x$ $\Rightarrow f'(x) = 1 - \cos x \ge 0$ for all x , since $-1 \le cosx \le 1$ $\Rightarrow f(x)$ is increasing for all $x \in R \Rightarrow f(x)$ decreases for no value of x.
14	The absolute maximum value of $f(x) = x^3 - 3x + 2$ in $0 \le x \le 2$ is (a) 4 b) 6 c) 2 d) 0 Sol: (a) As $f'(x) = 3x^2-3$, $f'(x) = 0 \Rightarrow x = \pm1$. $f(0) = 2$, $f(1) = 1 - 3 + 2 = 0$, $f(-1) = -1 + 3 + 2 = 4$, $f(2) = 8 - 6 + 2 = 4$

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
APPLICATION OF DERIVATIVES	https://youtu.be/0Zk2RUMBcWU	

EXERCISE

1	For the function y = x^3+21, the value of x, when y increases 75 times as fast as x, is (a) ± 3 (b) $\pm 5\sqrt{3}$ c) ± 5 d) none of these **Answer: c**
2	The maximum value of $\left(\frac{1}{x}\right)^x$ is (a) e (b) e^e c) $e^{\frac{1}{e}}$ d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$ **Answer: c**
3	The function f(x) = cos x -2px is monotonically decreasing for (a) p < $\frac{1}{2}$ b) p > $\frac{1}{2}$ c) p < 2 d) p > 2 **Answer: b**
4	The maximum value of xy, subject to x+y = 8 is (a) 8 b) 16 c) 20 d) 24 **Answer: b**
5	A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. The x-coordinates of the points on the curve at which y-coordinate is changing twice as fast as x-coordinate is (a) 1 b) ± 1 c) $\frac{5}{3}$ d) $\frac{1}{3}$ **Answer: b**

ASSERTION AND REASONING QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of reason(R). Choose the correct answer out of the following choices.(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true and (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

1	Let f(x) be a polynomial function in a degree 6 such that $\frac{d}{dx}$(f(x)) = $(x-1)^3(x-3)^2$, then **Assertion (A)**: $f(x)$ *has a minimum at* $x = 1$. **Reason (R)**: When $\frac{d}{dx}$(f(x))<0, $\forall\ x \in (a-h, a)$ and $\frac{d}{dx}$ (f(x))>0, $\forall\ x \in (a, a+h)$; where 'h' is an infinitesimally small positive quantity, then f(x) has a minimum at x=a, provided f(x) is continuous at x=a. **Sol : (a)** $\frac{d}{dx}$(f(x)) = $(x-1)^3(x-3)^2$ Assertion : f(x) has a minimum at x=1 is true as $\frac{d}{dx}$(f(x))<0, $\forall\ x \in (1-h, 1)$ and $\frac{d}{dx}$ (f(x))>0, $\forall\ x \in (1, 1+h)$; where, 'h' is an infinitesimally small positive quantity, which is in accordance with the Reason statement.
2	Let C be the circumference and A be the area of a circle. **Assertion(A):** The rate of change of the area with respect to radius is equal to C. **Reason(R):** The rate of change of the area with respect to diameter is $\frac{C}{2}$. **Sol: (b)**

	Let r be the radius of the circle. Then, A = πr^2 and C = 2 πr ⇒ $\frac{dA}{dr}$ = 2 πr = C. So, **(A)** is true. Let x be the diameter of the circle. Then, A = $(\frac{x}{2})^2 = \frac{x}{4}x^2$ and C = $\pi x \Rightarrow \frac{dA}{dx} = \frac{\pi x}{2} \Rightarrow \frac{dA}{dx} = \frac{C}{2}$ So, (R) is also true but (R) is not a correct explanation for (A).
3	Let the radius, surface area and volume of sphere be r, S and V respectively. **Assertion(A):** The rate of change of volume of sphere with respect to its radius is equal to S. **Reason(R):** The rate of change of volume of sphere with respect to S is $\frac{r}{2}$. **Sol: (b)** We have, V = $\frac{4}{3}\pi r^3$ and S = 4 $\pi r^2 \Rightarrow \frac{dV}{dr}$ = 4 $\pi r^2$and $\frac{dS}{dr}$ = 8 πr ⇒ $\frac{dV}{dr}$ = $S\ , (A) is\ true\ and\ \frac{dV}{dS} = \frac{dV/dr}{dS/dr} = \frac{4\,\pi r^2}{8\,\pi r} = \frac{r}{2}$,(R)is true Thus, both (A) & (R) are true but (R) is not a correct explanation for (A).
4	**Assertion(A):** If the area of a circle increases at a uniform rate, then its perimeter varies inversely as its radius. **Reason(R):** The rate of change of area of a circle with respect to its perimeter is equal to the radius. **Sol: (a)** Let r be the radius, P be the perimeter and A be the area of a circle. Then, $(A) = \pi r^2$ and P =2 πr ⇒ $\frac{dA}{dr}$ = 2 πr and $\frac{dP}{dr} = 2\,\pi$ ⇒ $\frac{dA}{dP} = \frac{dA/dr}{dP/dr} = \frac{2\pi r}{2\pi}$ = r So, (R) is true. Now, $\frac{dA}{dP}$ = r ⇒ $\frac{dA/dt}{dP/dt} = r \Rightarrow \frac{dP}{dt} = \frac{1}{r}\frac{dA}{dt}$ If $\frac{dA}{dt}$ = constant (=k,say). Then, $\frac{dP}{dt} = \frac{k}{r} \Rightarrow \frac{dP}{dt} \propto \frac{1}{r}$ ⇒Perimeter varies inversely as the radius. So, (A) is also true and (R) is a correct explanation for (A).
5	Let f(x) =$1-x^3- x^5$ **Assertion(A):** f(x) is an increasing function **Reason(R):** $3x^2+5x^4>0$, for all x ≠ 0. **Sol: (d)** f(x) = $1-x^3- x^5$ ⇒ f′(x)= $-3x^2-5x^4$ = - $(3x^2+5x^4)<0$ ∵$3x^2+5x^4>0$ ∴ f(x) is decreasing [(A) is false] x^2+x^4 always >0 for all x>0 ∴ $3x^2+5x^4>0$, for all x≠0 [R is true]
6	$Let\ f(x)\ =\ 2sin3x + 3cos3x$ **Assertion(A):** f(x) does not have a maximum or minimum at x = $\frac{5\pi}{6}$ **Reason(R):** f′($\frac{5\pi}{6}$) =0

	Sol: (c) f(x) = 2sin3x+3cos3x f′(x)= 6cos3x-9sin3x for maximum or minimum f′(x)=0 f′($\frac{5\pi}{6}$)= 6cos $\frac{5\pi}{2}$ - 9sin$\frac{5\pi}{2}$ = 0-9 = -9 ≠ 0 ∴ f(x) does not have a maximum at x = $\frac{5\pi}{6}$ ⇒ (A) is true f′$\left(\frac{5\pi}{6}\right) = -9 \neq 0$ ∴ (R) is false ∴ (A) is true but (R) is false.
7	Let f(x) = $2x^3$-$3x^2$-12x+4 **Assertion(A):** x = -1 is a point of local maximum **Reason(R):** f′′ (-1) >0 **Sol: (c)** f(x) = $2x^3$-$3x^2$-12x+4 f′(x)= $6x^2$-6x-12 = 6(x^2-x-2) =6(x-2) (x+1) f′(x) = 0 ⇒ x=2 or x=-1 = f′′ (x) = 6(2x-1) f′′(1) = 6(-2-1) = -18<0 ∴ x = -1 is a point of local maximum, [(A) is true.] f′′(-1) = -18<0 ∴ (R) is false.
8	$Let\ f(x) = x + cosx$ **Assertion(A):** f(x) is an increasing function on R **Reason(R):** $-1 \leq sinx \leq 1$ **Sol: (a)** f$(x) = x + cosx$ f′(x)= 1-sinx ≥0, for all x (∵-1≤sinx≤1) ⇒ f is an increasing function. (A)is true $-1 \leq sinx \leq 1$ ⇒(R)is true Thus, both (A) & (R) are true and(R) is the correct explanation for (A).
9	**Assertion(A) :** The function f(x) = x^3+5x+1, $x \in \mathbb{R}$ is always increasing **Reason(R):** f′(x)>0 for $x \in \mathbb{R}$, for increasing function **Sol: (a)** f(x) = x^3+5x+1 f′(x)= $3x^2$+5>0 for all $x \in \mathbb{R}$ ∴ f(x) is always increasing ((A)is true) (R)is also true and R is the correct explanation of A.
10	$Let\ f(x) = sinx$ **Assertion(A) : f**(x) is increasing in $\left(0, \frac{\pi}{2}\right)$ **Reason(R):** cosθ is positive for all $\theta \in \left(0, \frac{\pi}{2}\right)$ Sol: (a)

	f(x) = sinx ⇒ f′(x)= cosx>0, for all $x \in \left(0, \frac{\pi}{2}\right)$ ⇒ f is increasing in $\left(0, \frac{\pi}{2}\right)$ [(A)is true] (R)is also true and R is the correct explanation of A.

EXERCISE

In the following questions, a statement of assertion (A) is followed by a statement of reason(R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true and (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

1	**Assertion (A) :**The minimum value of x^2 – 8x + 17 is 4. **Reason (R) :** A function f(x) is minimum at x=c if $f'(c) = 0$ and $f''(c)$ is positive. **Answer: d**
2	**Assertion (A) :**The absolute minimum value of $x^3 - 18x^2 - 96x$ in [0,9] is 0. **Reason (R) :** A function always attains absolute minimum in the interval $[a, b]$ $at\ x = a$ **Answer: c**
3	**Assertion (A) :** Let f(x)=$e^x$is an increasing function $\forall x \in R$. **Reason (R) :** If $f'(x) \leq 0$ then f(x) is an increasing function. **Answer: c**
4	**Assertion (A) :** f(x) =log$x$ is defined for all $x \in (0, \infty)$. **Reason (R) :** If $f'(x) > 0$ then f(x) is strictly increasing function. **Answer: b**
5	**Assertion (A) :** f(x) = sin 2x+3 is defined for all real values of x. **Reason (R) :** Minimum value of f(x) is 2 and Maximum value is 4. **Answer: b**

2 MARK QUESTIONS

1	Radius of variable circle is changing at the rate of 5cm/s. What is the radius of the circle at a time when its area is changing at the rate of $100\text{cm}^2/s$? Solution: The area A of a circle with radius r is given by A = πr^2 $\frac{dA}{dt} = 2\,\pi r \frac{dr}{dt} \Rightarrow 100 = 2\,\pi r \times 5 \Rightarrow r = \frac{10}{\pi}$cm. $[\frac{dr}{dt} = 5cm/s, \frac{dA}{dt} = 100\text{cm}^2/\text{s}]$ Hence, radius of the circle is r = $\frac{10}{\pi}$cm.
2	The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its perimeter. Sol: Let the side of the triangle be ‘a’ then $\frac{da}{dt} = 0.5\ cm/s$ Perimeter of the triangle, P =3a $\Rightarrow \frac{dP}{dt} = 3\frac{da}{dt}$ =3 x 0.5 cm/s = 1.5 cm/s

3	If the rate of change of volume of a sphere is equal to the rate of change of its radius, then find the radius. Sol: Given, $\frac{dV}{dt} = \frac{dr}{dt}$ $\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = \frac{dr}{dt}$ $\Rightarrow 4\pi r^2 . \frac{dr}{dt} = \frac{dr}{dt}$ $\Rightarrow 4\pi r^2 = 1$ $\Rightarrow r^2 = \frac{1}{4\pi} \Rightarrow r = \frac{1}{2\sqrt{\pi}}$ units
4	A balloon which always remain spherical has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x. Sol: Diameter of the balloon = $\frac{3}{2}(2x+1)$ $\therefore$ r = radius of the balloon = $\frac{3}{4}(2x+1)$ Volume of the balloon, $V = \frac{4}{3}\pi[\frac{3}{4}(2x+1)]^3$ $= \frac{9}{16}\pi(2x+1)^3$ $\Rightarrow \frac{dV}{dx} = \frac{9}{16}\pi.3(2x+1)^2.2 = \frac{27}{8}\pi(2x+1)^2$
5	x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of Second Square with respect to the area of the first square. Sol: The area A_1 of square of side x is given by $A_1 = x^2$ and area A_2 of square of side y is given by $A_2 = y^2 = (x - x^2)^2$ $\frac{dA_1}{dx} = 2x$, $\frac{dA_2}{dx} = 2(x - x^2)(1 - 2x)$ $\frac{dA_2}{dA_1} = \frac{dA_2}{dx} \div \frac{dA_1}{dx} = \frac{2(x-x^2)(1-2x)}{2x}$ $= (1 - x)(1 - 2x) = 1 - 3x - 2x^2$
6	Find the intervals in which $f(x) = x^2 - 2x + 15$ is strictly increasing or strictly decreasing. Sol: We have, $f(x) = -x^2 - 2x + 15$ $\Rightarrow f'(x) = -2x - 2 = -2(x+1)$ For $f(x)$ to be increasing, we must have $f'(x)>0$ $\Rightarrow -2(x+1)>0$ $\Rightarrow x+1<0$ $\Rightarrow x < -1 \Rightarrow x \in (-\infty, -1)$ Thus, $f(x)$ is increasing on the interval $(-\infty, -1)$. For $f(x)$ to be decreasing, we must have $f'(x) < 0$ $\Rightarrow -2(x+1)<0$ $\Rightarrow x+1>0$ $\Rightarrow x > -1 \Rightarrow x \in (-1, \infty)$ So, $f(x)$ is decreasing on $(-1, \infty)$.
7	Show that the function $f(x) = (x^3 - 6x^2 + 12x + 18)$ is an increasing function on R. Sol: $f(x) = (x^3 - 6x^2 + 12x + 18)$ $\Rightarrow f'(x) = 3x^2 + 12x + 12$ $= 3(x^2 + 4x + 4) = 3(x-2)^2 \geq 0$ for all $x \in R$. Thus, $f'(x) \geq 0$ for all $x \in R$. Hence, $f(x)$ is an increasing function on R.

8	Find the intervals on which the function $f(x) = (5 + 36x + 3x^2 - 2x^3)$ is increasing. Sol: $f(x) = (5 + 36x + 3x^2 - 2x^3)$ $\Rightarrow f'(x) = 36 + 6x - 6x^2$ $= -6(x^2 - x - 6) = -6(x+2)(x-3)$ $f(x)$ is increasing $\Rightarrow f'(x) \geq 0$ $\Rightarrow -6(x+2)(x-3) \geq 0$ $\Rightarrow (x+2)(x-3) \leq 0$ $\Rightarrow -2 \leq x \leq 3$ $\Rightarrow x \in [-2,3]$. $\therefore f(x)$ is increasing on$[-2,3]$.
9	Find the maximum and the minimum values of the function $f(x) = x + 2, x \in (0,1)$. Sol: $f(x) = x + 2$ $f'(x) = 1$ so for no value of x, $f'(x) = 0$. So $f(x)$ has no critical points. Hence, $f(x)$ has neither local maximum nor local minimum.
10	Amongst all pairs of positive numbers with sum 24, find those whose product is maximum. Sol: Let the numbers be x and $(24 - x)$. Let $P = x(24 - x) = (24x - x^2)$ Then, $\frac{dP}{dx} = (24 - 2x)$and $\frac{d^2P}{dx^2} = -2$. Now, $\frac{dP}{dx} = 0 \Rightarrow (24 - 2x) = 0 \Rightarrow x = 12$. Thus,$\left\{\frac{d^2P}{dx^2}\right\}_{x=12} = -2<0$. $\therefore$ x=12 is a point of maximum. Hence, the required numbers are 12 and 12.
11	Find the local maxima and local minima, if any of the function f, given by f(x) = sin x + cos x, $0 < x < \frac{\pi}{2}$. Sol: f(x) = sin x + cos x $f'(x) = \cos x - \sin x$, for points of local maxima or minima $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$ $f''(x) = -\sin x - \cos x$, $f''\left(\frac{\pi}{4}\right)<0$ $\therefore x = \frac{\pi}{4}$ is a point of local maximum Local maximum value = $f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$ $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$
12	Find the interval/s in which the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $(x) = xe^x$, is increasing. Sol: $f(x) = xe^x \Rightarrow f'(x) = e^x(x + 1)$ For $f(x)$ to be increasing, $f'(x) \geq 0$ $\Rightarrow e^x(x + 1) \geq 0 \Rightarrow x \geq -1$ as $e^x > 0, \forall x \in \mathbb{R}$

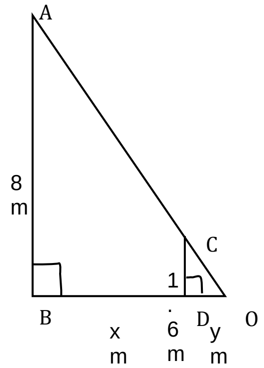
	Hence, $f(x)$ increases in $[-1, \infty)$.
13	If $f(x) = \frac{1}{4x^2+2x+1}$; $x \in \mathbb{R}$, then find the maximum value of $f(x)$. Sol: We have $f(x) = \frac{1}{4x^2+2x+1}$ Let $g(x) = 4x^2 + 2x + 1 = 4\left(x^2 + 2x\frac{1}{4} + \frac{1}{16}\right) + \frac{3}{4}$ $= 4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$ $\Rightarrow$minimum value of $g(x) = \frac{3}{4}$. $\therefore$ maximum value of $f(x) = \frac{4}{3}$.
14	Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees. Sol: $P(x) = 72 + 42x - x^2$ $P'(x) = 42 - 2x$, $P''(x) = -2$ For maxima or minima, $P'(x) = 0 \Rightarrow 42 - 2x = 0 \Rightarrow x = 21$ $P''(x) = -2 < 0$ So, $P(x)$ is maximum at x = 21. The maximum value of $P(x)$ $= P(21) = 72 + (42 \times 21) - (21)^2 = 513$ i.e., the maximum profit is Rs.513.
15	Check whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + x$, has any critical point/s or not. If yes, then find the point/s. Sol: $f(x) = x^3 + x$, for all $x \in \mathbb{R}$. $f'(x) = 3x^2 + 1 > 0$ for all $x \in \mathbb{R}$, $(x^2 \geq 0)$ $\Rightarrow f'(x) \neq 0$ Hence, no critical point exists.

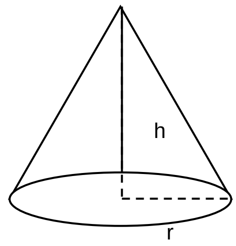
EXERCISE

1	The total cost C(x) associated with the production of x units of an item is given by C(x) = $= 0.005x^3 - 0.02x^2 - 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output. **Answer: 29.985**
2	Find the intervals in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing & decreasing. **Answer: Increasing. in $(-\infty, -2) \cup (3, \infty)$. Decreasing. in $(-2, 3)$.**
3	A ladder, 5-meter-long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10cm/sec, find the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 meters from the wall. **Answer: $\frac{1}{20}$ radians/sec**
4	Let x and y be the radii of two circle such that $y = x^2 + 1$. Find the rate of change of circumference of second circle w.r.t the circumference of the first circle. **Answer: $2x$**

5	Find the least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval [0,9]. **Answer: 0**

3 MARK QUESTIONS

<table>
<tr><td>1</td><td>
A man 1.6 m tall walks at the rate of 0.5 m/s away from a lamp post, 8 meters high. Find the rate at which his shadow is increasing and the rate with which the tip of shadow is moving away from the pole.
Solution: Let AB be the lamp post and CD the height of the man.
Let distance of the man from the lamp post be x m
and from tip of shadow be y m.


$\frac{dx}{dt}$ = 0.5 m/s
In similar triangles ABO ans CDO
$\frac{8}{1.6} = \frac{x+y}{y}$
⇒ 5y = x+y ⇒ y = $\frac{1}{4}x$
∴ $\frac{dy}{dt} = \frac{1}{4}\frac{dx}{dt} = \frac{0.5}{4}$ = 0.125 m/s
∴ Rate at which shadow is increasing 0.125 m/s
Rate of change of tip of shadow = $\frac{d}{dt}(x + y)$
= $\frac{dx}{dt} + \frac{dy}{dt}$
= 0.5+0.125 = 0.625 m/s
</td></tr>
<tr><td>2</td><td>
The area of an expanding rectangle is increasing at the rate of $48cm^2/s$. The length of the rectangle is always equal to square of breadth. At what rate, the length is increasing at the instant when breadth is 4.5 cm?
Solution: Let the length of the rectangle be l and its breadth b.
Then $l = b^2 \Rightarrow A = l.\sqrt{l} = l^{\frac{3}{2}}$
$\frac{dA}{dt} = \frac{3}{2}\sqrt{l}.\frac{dl}{dt}$
⇒ 48 = $\frac{3}{2}$ x 4.5 x $\frac{dl}{dt}$ …… (b= $\sqrt{l}$)
⇒ $\left.\frac{dl}{dt}\right]_{b=4.5\ cm} = \frac{320}{45} = \frac{64}{9}$ = 7.11 cm/s
</td></tr>
<tr><td>3</td><td>
Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?
Solution: Let h be the height, V the volume and r the radius of the base of cone at the time t.
Given $h = \frac{1}{6}r$
$\Rightarrow r = 6h$
$\frac{dV}{dt}$ = 12 cm^3/s
Volume of the cone, $V = \frac{1}{3}\pi r^2$h
= $\frac{1}{3}\pi(6h)^2$h = 12 πh^3
∴ $\frac{dV}{dt}$ = 12 π . $3h^2.\frac{dh}{dt}$
⇒ 12 = 36 πh² $\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{3\pi h^2}$


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</table>

	$\Rightarrow \frac{dh}{dt}\Big]_{h=4} = \frac{1}{3\pi X 16} = \frac{1}{48\pi} cm/s.$
4	Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly increasing. **Solution:**We have, $f(x) = 2x^3 - 9x^2 + 12x + 15$ $\Rightarrow f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$ (i) For $f(x)$ to be increasing, we must have $f'(x) > 0$ $\Rightarrow 6(x^2 - 3x + 2) > 0$ $\Rightarrow x^2 - 3x + 2 > 0$ [$\because 6 > 0 \therefore 6(x^2 - 3x + 2) > 0 \Rightarrow x^2 - 3x + 2 > 0$] $\Rightarrow (x-1)(x-2) > 0$ (See fig) $\Rightarrow x < 1$ or $x > 2$ $\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$. So, $f(x)$ is increasing on $(-\infty, 1) \cup (2, \infty)$.  Fig. Signs of $f'(x)$ for different values of x
5	Show that $f(x) = tan^{-1}(cosx + sinx)$ is a strictly increasing function on the interval $(0, \frac{\pi}{4})$. **Solution:** $f(x) = tan^{-1}(cosx + sinx)$ $\Rightarrow f'(x) = \frac{1}{1+(cosx+sinx)^2} \cdot \frac{d}{dx}(cosx + sinx)$ $= \frac{(-sinx+\cos x)}{1+cos^2x+sin^2x+2\sin x\cos x}$ $= \frac{cos\, x - sin\, x}{(2+\sin 2x)}$ Now, when $0 < x < \frac{\pi}{4}$, we have $cosx > sinx$ & $sin2x > 0$ $\therefore (cosx - sinx) > 0$ and $(2 + sin2x) > 0$. $\therefore f'(x) > 0$ for all x when $0 < x < \frac{\pi}{4}$ Hence**,** $f(x)$ is strictly increasing in $(0, \frac{\pi}{4})$.
6	Separate $[0, \frac{\pi}{2}]$ into subintervals in which $f(x)$ = sin 3x is (a) increasing (b) decreasing. **Solution:** $f(x)$ = sin 3x $\Rightarrow f'(x)$ = 3cos 3x Also, $0 \le x \le \frac{\pi}{2} \Rightarrow 0 \le 3x \le \frac{3\pi}{2}$ (a) $f(x)$ is increasing $\Rightarrow f'(x) \ge 0$ $\Rightarrow 3\cos 3x \ge 0 \Rightarrow \cos 3x \ge 0$ $\Rightarrow 0 \le 3x \le \frac{\pi}{2}$ $\Rightarrow 0 \le x \le \frac{\pi}{6}$ $\Rightarrow x \in [0, \frac{\pi}{6}]$. $\therefore f(x)$ is increasing on $[0, \frac{\pi}{6}]$. (b) $f(x)$ is decreasing $\Rightarrow f'(x) \le 0$ $\Rightarrow 3\cos 3x \le 0 \Rightarrow \cos 3x \le 0$

$\Rightarrow \frac{\pi}{2} \leq 3x \leq \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{6} \leq x \leq \frac{\pi}{2}$

$\Rightarrow x \in [\frac{\pi}{6}, \frac{\pi}{2}]$.

7 Show that $f(x) = 2x + cot^{-1}x + log(\sqrt{1+x^2} - x)$ is increasing in R.

Solution: We have,

$f(x) = 2x + cot^{-1}x + log(\sqrt{1+x^2} - x)$

$\Rightarrow f'(x) = 2 + (\frac{-1}{1+x^2}) + \frac{1}{\sqrt{1+x^2}-x} \ (\frac{1}{2\sqrt{1+x^2}} .2x - 1)$

$= 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}-x} . \frac{x-\sqrt{1+x^2}}{\sqrt{1+x^2}}$

$= 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}$

$= \frac{2+2x^2-1-\sqrt{1+x^2}}{1+x^2} = \frac{1+2x^2-\sqrt{1+x^2}}{1+x^2}$

For increasing function, $f'(x) \geq 0$

$\Rightarrow \frac{1+2x^2-\sqrt{1+x^2}}{1+x^2} \geq 0$

$\Rightarrow 1 + 2x^2 \geq \sqrt{1+x^2}$

$\Rightarrow (1 + 2x^2)^2 \geq 1 + x^2$

$\Rightarrow 1 + 4x^4 + 4x^2 \geq 1 + x^2$

$\Rightarrow 4x^4 + 3x^2 \geq 0$

$\Rightarrow x^2(4x^2 + 3) \geq 0$

which is true for any real value of x.

Hence, $f(x)$ is increasing in R.

8 Prove that $f(x) = \sin x + \sqrt{3}\cos x$ has maximum, value at $x = \frac{\pi}{6}$.

Solution: We have, $f(x) = \sin x + \sqrt{3}\cos x$

$\therefore f'(x) = \cos x + \sqrt{3}(-sinx)$

$= \cos x - \sqrt{3}sinx$

For $f'(x) = 0, \Rightarrow \cos x - \sqrt{3}sinx = 0$

$\Rightarrow \cos x = \sqrt{3}sinx \ \Rightarrow \ \tan x = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{\pi}{6}$

Again, differentiating $f'(x)$, we get

$f''(x) = -\sin x - \sqrt{3}\cos x$

$f''\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} - \sqrt{3}\cos\frac{\pi}{6}$

$= -\frac{1}{2} - \sqrt{3} . \frac{\sqrt{3}}{2}$

$= -\frac{1}{2} - \frac{3}{2} = -2 < 0$

Hence $x = \frac{\pi}{6}$ is the point of local maxima.

9 Find the local maximum and the local minimum values of the function f(x) = $\frac{-3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

Solution: $f'(x) = -3x^3 - 24x^2 - 45x$

$= -3x(x^2 + 8x + 15)$

$= -3x(x + 3)(x + 5)$

$f''(x) = -9x^2 - 48x - 45$

$f'(x) = 0 \Rightarrow -3x(x + 3)(x + 5) = 0$

$\Rightarrow x = 0, x = -3, x = -5$

	$f''(0)$ = -45<0 So x=0 is a point of local maxima . $f''(-3)$=$+18 > 0$ So x=-3 is a point of local minima $f''(-5)$=$-30 < 0$ So x=-5 is a point of local maxima
10	**A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs.300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs.1 per one subscriber will discontinue the service. Find what increase will bring maximum profit?** **Solution:** Consider that company increases the annual subscription by Rs. x So**,** x subscribers will discontinue the service ∴ Total revenue of company after the increment is given by R(x) = (500-x)(300+x) =15 x $10^4 + 500x - 300x - x^2$ = $-x^2 + 200x + 150000$ On differentiating both sides w.r.t x, we get R' (x) = $-2x + 200$ Now, R' (x) = 0 ⇒ 2x = 200 ⇒ x=100 ∴ R'' (x) = -2<0 So, R(x) is maximum when x=100 Hence, the company should increase the subscription fee by Rs.100, so that it has maximum profit.

EXERCISE

1	Find two positive numbers whose sum is 16 and sum of whose cubes is minimum. **Answer: 8, 8**
2	Show that y = log(1+x) - $\frac{2x}{2+x}$, x> -1 is an increasing function of x throughou its domain .
3	Show that the function $f(x) = x^3 - 3x^2 + 3x, x \in R$ is increasing on R.
4	Find the intervals in which the function $f(x)$=$2x^3 - 9x^2 + 12x - 15$ is (i) increasing. (ii) decreasing **Answer:** : $(-\infty, \mathbf{1}) \cup (\mathbf{2}, \infty)$
5	The total revenue received from the sale of x units of a product is given by R(x) $= 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when x=5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant. At what value of x is R(x) minimum **Answer: 66**

5 MARK QUESTIONS

1	Find the intervals in which the function f(x) =3x^4 -4x^3-12x^2+5 is increasing or decreasing. **Solution:**

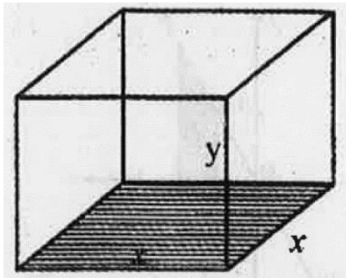
$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
$f'(x) = 12x^3 - 12x^2 - 24x$
$= 12x(x^2 - x - 2)$
$=12x(x-2)(x+1)$
$f^1(x) = 0 \Rightarrow x = 0,2,-1$

Intervals	Sign of f'(x)	Nature of f(x)
(-∞,-1)	-ve	decreasing
(-1,0)	+ve	increasing
(0,2)	-ve	decreasing
(2,∞)	+ve	increasing

Hence f is increasing in $(-1,0) \cup (2,\infty)$ and decreasing in $(-\infty,-1) \cup (0,2)$.

2 Show that the surface area of a closed cuboid with a square base and given volume is minimum, when it is a cube.

Solution: Let x be side of the square base and y be the height of the cuboid.
Volume (V)= x.x.y=x^2y …….. (i)



y=$\frac{V}{x^2}$
Surface area (S)=2(x.x+x.y+x.y)
$=2x^2 + 4xy = 2x^2 + 4x\frac{v}{x^2}$
S=$2x^2 + \frac{4^v}{x} \Rightarrow \frac{ds}{dx} = 4x - \frac{4v}{x^2}$
For minimum surface area,
$\frac{ds}{dx} = 0 \Rightarrow 4x - \frac{4^v}{x^2} = 0 \Rightarrow x^3 = v$
x=$\sqrt[3]{v}$
$\frac{d^2s}{dx^2} = 4 + \frac{8^v}{x^3}$
$\frac{d^2S}{dx^2} = 4 + \frac{8v}{v} > 0$
For x=$\sqrt[3]{v}$, surface area is minimum
x^3 = V
$x^3 = x^2y$ [from (i)]
x=y cuboid is a cube.

3 Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Solution: Let a cone of base radius x and height y be inscribed in a sphere of radius R.
$R^2=(y-R)^2 + x^2$
$x^2 = 2Ry-y^2$ [in right triangle OAB]…(i)
Volume of the cone, V=$\frac{1}{3}\pi x^2y$

$\frac{1}{3}\pi y(2Ry - y^2)$

$= \frac{1}{3}\pi(2Ry^2 - y^3)$ [from (i)]……(ii)

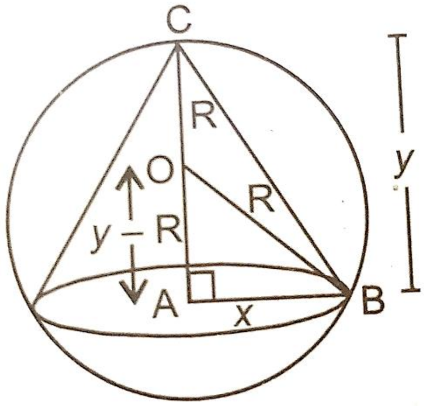
$\frac{dV}{dy} = \frac{1}{3}\pi(4Ry - 3y^2)$

For maximum volume,

$\frac{dV}{dy} = 0$

$\Rightarrow 4Ry = 3y^2$

$\Rightarrow y = \frac{4R}{3}$



$$\frac{d^2v}{dy^2} = \frac{\pi}{3}(4R - 6y)$$

$$\frac{d^2v}{dy^2} < 0, \quad \text{for } y = \frac{4R}{3}$$

$$V_{max.} = \frac{1}{3}\pi\left[2R\left(\frac{4R}{3}\right)^2 - \left(\frac{4R}{3}\right)^3\right]$$

$$= \frac{1}{3}\pi\left[\frac{32R^3}{9} - \frac{64R^3}{27}\right]$$

$$= \frac{32\pi R^3}{81} cm^3$$

$$= \frac{8}{27}\left(\frac{4}{3}\Pi R^3\right)$$

$$\frac{8}{27}\ (Volume\ of\ the\ sphere)$$

4 Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

Solution:

Let x be radius of base and y height of a cylinder which is inscribed in a sphere of radius R.

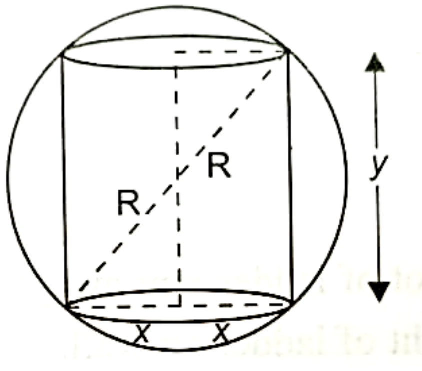
$4x^2 + y^2 = 4R^2$ …….(i)

Volume of cylinder

$V = \pi x^2 y = \pi y\left(\frac{4R^2 - y^2}{4}\right)$

$= \frac{\pi}{4}(4R^2 y - y^3)$

[from(i)] ……(ii)



$$\frac{dv}{dy} = \frac{\pi}{4}(4R^2 - 3y^2)$$

For maximum volume, $\frac{dV}{dy} = 0$

$Y = \frac{2R}{\sqrt{3}}$

$$\frac{d^2v}{dy^2} = \frac{-3\pi y}{2}$$

$\frac{d^2v}{dy^2} < 0$ for $y = \frac{2R}{\sqrt{3}}$

Now substituting the value of y in equation (ii), we get maximum volume $= \frac{4\pi R^3}{3\sqrt{3}}$ cubic units.

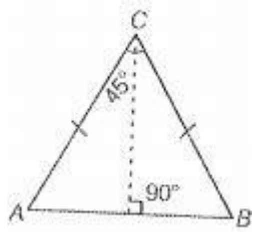
5 Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45^0 to each other. If they travel by different roads, then find the rate at which they are being separated.

Solution: Let two men start from the point C with velocity v each at the same time.

Also, $\angle BCA = 45^0$

Since, A and B are moving with same velocity v, so they will cover same distance in same time.

Therefore, ΔABC is an isosceles triangle with AC=BC.



Now, draw CD⊥AB.

Let at any instant t, the distance between them is AB.

Let AC=BC=x and AB=y

In ΔACD and ΔDCB,

∠CAD=∠ CBD

∠CDA=∠ CDB=90^0

∠ACD=∠ DCB

∠ACD=$\frac{1}{2} X$∠ ACB

∠ ACD=$\frac{1}{2} X 45^0$

∠ ACD=$\frac{\pi}{8}$

Sin $\frac{\pi}{8} = \frac{AD}{AC}$

Sin $\frac{\pi}{8} = \frac{Y/2}{x}$

$\frac{y}{2}$ = x sin $\frac{\pi}{8}$

Y=2x.sin$\frac{\pi}{8}$

Now, differentiating both sides w.r.t. t ,we get

$\frac{dy}{dt}$ = 2.sin $\frac{\pi}{8} . \frac{dx}{dt}$

=2.sin $\frac{\pi}{8}$ .v .... [v=$\frac{dx}{dt}$]

=2v. $\frac{\sqrt{2-\sqrt{2}}}{2}$ .... [sin $\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$]

=$\sqrt{2-\sqrt{2}}$ v unit/s

Which is the rate at which A and B are being separated.

EXERCISE

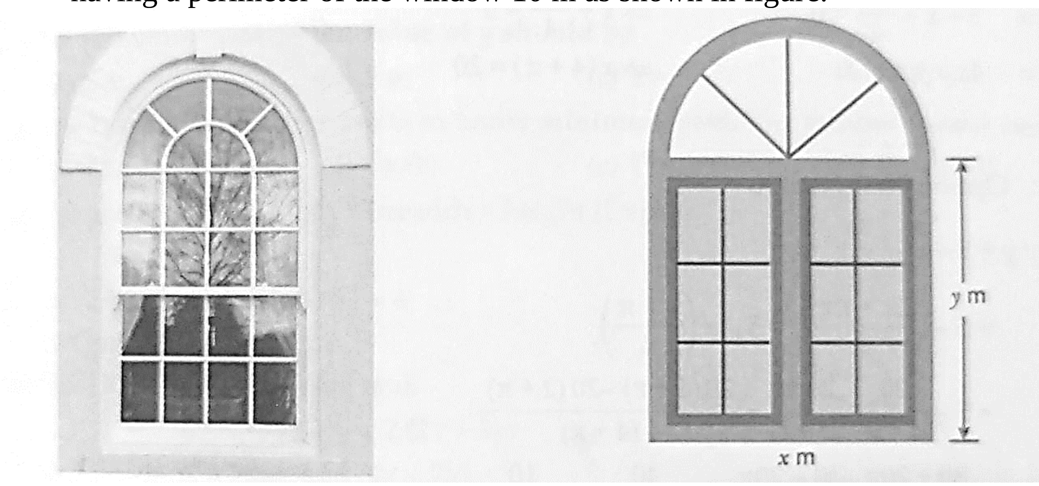
1	Find the intervals in which the function f(x)=$\frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (i) strictly increasing (ii) strictly decreasing. **Answer: (i)** $(-3, 2) \cup (4, \infty)$**. (ii)** $(-\infty, -3) \cup (2, 4)$**.**
2	The length of the sides of an isosceles triangle are 9+x^2 , 9+x^2 and 18-2x^2 units. Calculate the area of the triangle in terms of x and find the value of x which makes the area maximum. **Answer:** $x = \sqrt{3}$
3	A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle, so that its area is maximum. Also find maximum area. **Answer:** $\sqrt{2}$r , $\frac{r}{\sqrt{2}}$
4	If the sum of a side and the hypotenuse of a right- angled triangle be given, show that the area of the triangle will be maximum if the angle between the given side and the hypotenuse be 60^0.

5	Show that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is $sin^{-1}\frac{1}{3}$.
6	Show that the surface area of a closed cuboid with a square base and given volume is minimum, when it is a cube.
7	If the length of three sides of a trapezium other than the base are equal to 10 cm , then find the maximum area of the trapezium .
8	Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ with its vertex at one end of the major axis. **Answer:** $9\sqrt{3}$ sq.units

CASE STUDY QUESTIONS

Read the following and answer the questions given below.

1 Dr. Ritham residing in Delhi went to see an apartment of 3BHK in Noida. The window
of the house was in the form of a rectangle surmounted by a semicircular opening
having a perimeter of the window 10 m as shown in figure.



(i). If x and y represent the length and breadth of the rectangular region, then the relation between the variable is

a) $x + y + \frac{x}{2} = 10$ b) $x + 2y + \frac{x}{2} = 10$

c) $x + 2y + \pi\frac{x}{2} = 10$ d) $2x + 2y = 10$

(ii) The area of the window (A) expressed as a function of x is

a) $A = x - \frac{\pi x^3}{8} - \frac{x^2}{2}$ b) $A = 5x - \frac{-x^2}{2} - \frac{\pi x^2}{8}$

c) $A = 5x - \frac{x^2}{2} - \frac{3x^2}{8}$ d) $A = 5x + \frac{x^2}{2} + \frac{\pi x^2}{8}$

(iii) Dr. Ritam is interested in maximizing the area of the whole window. For this to
Happen the value of x should be

(a) $\frac{20}{\pi}$ b) $\frac{20}{4-\pi}$ c) $\frac{20}{2+\pi}$ d) $\frac{20}{4+\pi}$

Solution:

(i) Since perimeter of window = $x + y + y$ + perimeter of semicircle

$= x + 2y + \frac{1}{2} \times 2\pi \times \frac{x}{2}$ $\left[Here\ radius\ of\ semicircle\ is\ \frac{x}{2}\right]$

$= x + 2y + \frac{\pi x}{2}$

Option (c) is correct.

(ii) A = x x y + $\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$$= xy + \frac{\pi x^2}{8} = x\left(5 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$$

$$= 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} \quad = \; 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

Option (b) is correct.

(iii) For maximum value of A

$\frac{dA}{dx} = 0$

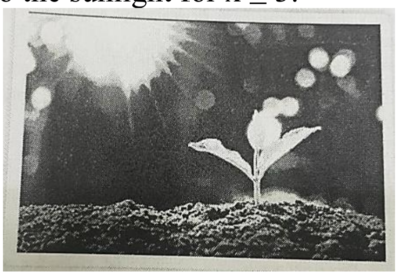
$\Rightarrow 5 - x - \frac{\pi x}{4} = 0 \qquad \Rightarrow x + \frac{\pi x}{4} = 5$

$\Rightarrow 4x + \pi x = 20 \qquad \Rightarrow x(4 + \pi) = 20$

$\Rightarrow x = \frac{20}{4+\pi}$

Option (d) is correct

2 Read the following passage and answer the questions given below:
The relation between the height of the plant (‘y” in cm) with respect to its exposure
to the sunlight is governed by the following equation y = 4x - $\frac{1}{2}$ x ^{2} , where ‘ x ‘ is the
number of days exposed to the sunlight for x ≤ 3.



(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

Solution:

(i)The rate of growth of the plant with respect to the number of days exposed to sunlight is given by $\frac{dy}{dx}$ = 4 – x.

(ii) Let rate of growth be represented by the function g (x) = $\frac{dy}{dx}$.

Now, g$'$(x) = $\frac{d}{dx}$ (.$\frac{dy}{dx}$) = -1 < 0

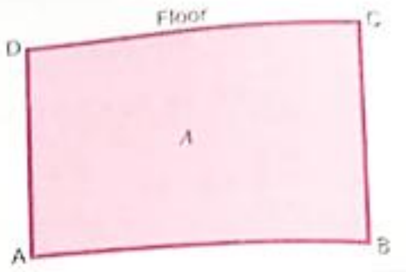
g(x) decreases.

So the rate of growth of the plant decreases for the first three days.

Height of the plant after 2 days is y = 4 x 2 - $\frac{1}{2}$ (2) ^{2} = 6 cm.

3

A rectangular hall is to be developed for a meeting of farmers in an agriculture college to aware them for new techniques in cultivation. It is given that the floor has a fixed
perimeter *P* as shown below. And if x & y represents the length & breadth of the rectangular region.



Answer the following question.

(i) The area of the rectangular region 'A' expressed as a function of *x* is

(a) $\frac{1}{2}(P + x^2)$ (b) $\frac{1}{2}(P\,x - 2x^2)$ (c) $\frac{1}{2}(P\,x + 2x^2)$ (d) $P\,x - 2x^2$

(ii) Principal of agriculture college is interested in maximizing the area of floor 'A' . For this to happen the value of *x* should be

(a) P *(b)* $\frac{P}{2}$ *(c)* $\frac{2P}{3}$ *(d)* $\frac{P}{4}$

Solution:

(i) $A = x.y$

$$= x.\frac{P-2x}{2}$$

$$= \frac{Px-2x^2}{2} \quad \begin{bmatrix} since, given\ Perimeter = P \\ 2(x + y) = \\ Y = \frac{P}{2} - x = \frac{P-2x}{2} \end{bmatrix}$$

Option (b) is correct.

(ii) $A = \frac{Px-2x^2}{2} \Rightarrow \frac{dA}{dx} = \frac{P-4x}{2}$

For maximum or minimum value of x

$\frac{dA}{dx} = 0 \Rightarrow \frac{P-4x}{2} = 0$

$\Rightarrow P - 4x = 0 \Rightarrow x = \frac{P}{4}$

$Also, \frac{d^2A}{dx^2} = -2 < 0\ at\ x = \frac{P}{4}$

$\therefore$ *A is maximum*

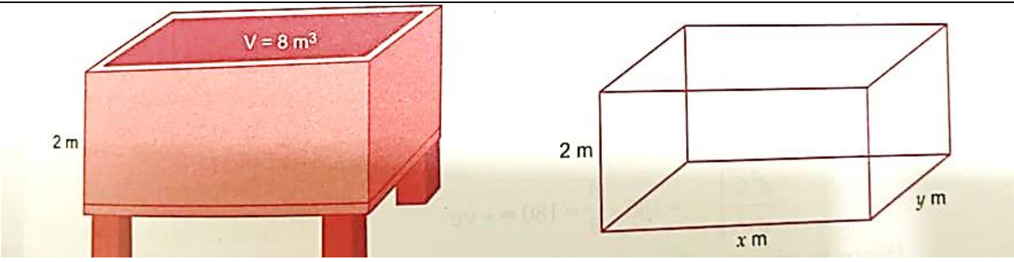
Option (d) is correct.

EXERCISE

1

Q1.Read the following and answer the questions given :

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m ^{3} as shown below:



(i) If x and y represents the length and breadth of its rectangular base, then the relation between the variables is

(a) $x + y = 8$ (b) $x \,.\, y = 4$ (c) $x + y = 4$ (d) $\frac{x}{y} = 4$

(ii) If construction of tank cost ₹70 per sq. meter for the base and ₹45 per square meter or sides, then making cost ' C' expressed as a function of x is

(a) C = 80 + 80 ($x + \frac{4}{x}$) (b) C = 280x +280 ($x + \frac{4}{x}$)

(c) C = 280 + 180($x + \frac{4}{x}$) (d) C = 70x + 70 ($x + \frac{x}{4}$)

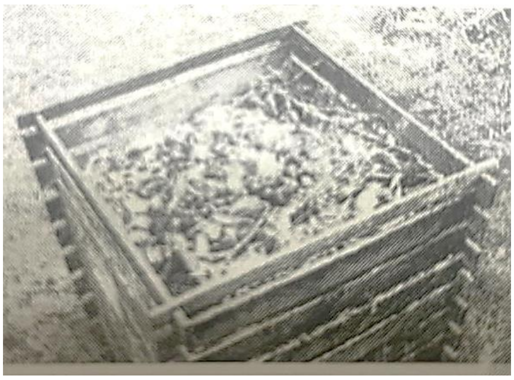
(iii) The owner of a construction agency is interested in minimizing the cost 'C' of whole tank, for this to happen the value of x should be

(a) 4 m (b) 3 m (c) 1 m (d) 2 m

Answer: $(i) b$ (ii) c (iii) d

2 Read the following text and answer the following questions on the basis of the same:

In a residential society comprising of 100 houses, there were 60 children between the ages of 10 – 15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycle, For this purpose, instead of each child doing it for only his/her house, children convinced the Residents welfare association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of ₹ 50 per square meter for space so that there is no misuse of the space and Resident welfare association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ₹400 x $(\text{depth})^2$. Association will like to have minimum cost.



i).Let side of square plot is x m and its depth is h meters, then cost C for the pit is

(a) $\frac{50}{h}$ + $400h^2$ (b) $\frac{12500}{h}$ + $400h^2$ (c) $\frac{250}{h}$ + h^2 (d) $\frac{250}{h}$ +$400h^2$

ii).Value of h (in m) for which $\frac{dc}{dh}$ = 0 is

(a) 1.5 (b) 2 (c) 2.5 (d) 3

iii). Value of x (in m) for minimum cost is

(a)5 (b)$10\sqrt{\frac{5}{3}}$ (c)$5\sqrt{5}$ (d) 10

Answer: (i) b (ii) c (iii) d

3	Read the following text and answer the following questions, on the basis of the same: $P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.  (i) What will be the production when the profit is maximum? (a) 37,500 (b) 12.5 (c) -12.5 (d) -37,500 (ii) What will be the maximum profit? (a) ₹38,28,125 (b) ₹38,28,.25 (c) ₹39,000 (d) None of these (iii) Check in which interval the profit is strictly increasing. (a) (12.5, ∞) (b) for all real numbers (c) for all positive real numbers (d) (0,12.5) **Answer: (i) b (ii) b (iii) d**

CHAPTER: INTEGRALS

SYLLABUS:

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{a^2 - x^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$, $\int \sqrt{x^2 \pm a^2}\, dx$, $\int \sqrt{a^2 - x^2}\, dx$,
$\int \sqrt{ax^2 + bx + c}\, dx$ $\int \frac{px+q}{ax^2+bx+c} dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

Formulae and Definitions:

Indefinite Integrals

1. $\int 1dx = x + c$
2. $\int x\, dx = x^2 + c$
3. $\int sinx\, dx = -cosx + c$
4. $\int cosxdx = sinx + c$
5. $\int tanx\, dx = \log secx + c$
6. $\int cosecx\, dx = \log|(cosecx - cotx)| + c$
7. $\int secx\, dx = \log|secx + tanx| + c$
8. $\int cotxdx = log|sinx| + c$
9. $\int sec^2xdx = tanx + c$
10. $\int cosec^2xdx = -cotx + c$
11. $\int secx.tanxdx = secx + c$
12. $\int cosecx.cotxdx = -cosecx + c$
13. $\int e^x dx = e^x + c$
14. $\int \frac{dx}{x} = \log x + c$
15. $\int a^x dx = \frac{a^x}{\log a} + c$
16. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + c$
17. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$
18. $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

19. $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c$

20. $\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c$

21. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}+c$

22. $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

23. $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$

24. $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

25. $\int e^x\left(f(x)+f^l(x)\right)dx = e^x f(x)+c$

26. $\int u.vdx = u\int v\,dx - \int u^l\,[\int v\,dx]dx$

Partial fractions

- The rational function $\frac{P(x)}{Q(x)}$ is said to be proper if the degree of P(x) is less than the degree of Q(x)
- Partial fractions can be used only if the integrand is proper rational function

Definite Integrals

S.No	Form of rational function	Form of Partial fraction
1	$\frac{1}{(x-a)(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}$
2	$\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}$
3	$\frac{px^2+qx+c}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$
4	$\frac{1}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$
5	$\frac{1}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
6	$\frac{px+q}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
8	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ where x^2+bx+c cannot be factorized further	$\frac{A}{(x-a)}+\frac{Bx+C}{x^2+bx+c}$

Properties of Definite Integrals

1. $\int_a^a f(x)dx$=0
2. $\int_a^b f(x)dx$=$\int_a^b f(t)dt$
3. $\int_a^b f(x)dx$=$-\int_b^a f(x)dx$

4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where a<c<b

5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

6. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

7. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

8. $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(2a-x) = f(x) \\ 0 & if\ f(2a-x) = -f(x) \end{cases}$

9. $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(-x) = f(x) \\ 0 & if\ f(-x) = -f(x) \end{cases}$

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND SOLUTION S
1.	Evaluate $\int \frac{1}{x-\sqrt{x}}$ dx ? a) 2 log $\sqrt{x}$ + C b) log ($\sqrt{x}$ - 1) + C c) 2 log ($\sqrt{x}$ - 1) + C d) None of the above Ans. I = $\int \frac{1}{\sqrt{x}(\sqrt{x}-1)}$ dx Put $\sqrt{x} - 1$ = t then I = 2 logt = 2 log ($\sqrt{x} - 1$) + C (c)
2.	Evaluate $\int \frac{sec^2x}{\sqrt{tan^2x+4}}$ dx ? a) log$\left\|\sqrt{tan^2x+4}\right\| + C$ b) log $\left\|tanx + \sqrt{tan^2x+4}\right\| + C$ c) $\frac{1}{2}$ log $\|tan\,x\| + C$ d) tanx + C Ans. Sub tan x = t then I = $\int \frac{dt}{\sqrt{t^2+2^2}}$ =log $\left\|tanx + \sqrt{tan^2x+4}\right\| + C$ (b)
3.	Evaluate $\int cos^3x\,.e^{\log sinx}$ dx ? $a)\frac{1}{3}\sin^3 x + C$ b) $-\frac{1}{2}\cos^4 x + C$ c) $-\frac{1}{4}\cos^4 x + C$ d) $-\frac{1}{3}\sin^3 x + C$ Ans. Here $e^{\log sinx}$ = sinx then I = $\int cos^3x\,.sinx\,dx$ Let cosx = t Ans : $-\frac{1}{4}\cos^4 x + C$ (c)
4	$\int_0^{\frac{\pi}{6}} sec^2\left(x-\frac{\pi}{6}\right) dx$ is equal to : (a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$ Ans: $\int_0^{\frac{\pi}{6}} sec^2\left(x-\frac{\pi}{6}\right) dx = tan\left(x-\frac{\pi}{6}\right)\Big]_0^{\frac{\pi}{6}} = \tan 0 - tan\left(-\frac{\pi}{6}\right) = tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ Option: a

5.	Evaluate $\int \frac{dx}{e^x - 1}$ a) log $\|e^x - 1\|$ +C b) $\log\|1 - e^{-x}\|$+C c) $\log\|1 - e^{x}\|$+C d) $\log\| e^{-x} - 1\|$+C Ans. Dividing the numerator and denominator with e^x I = $\int \frac{e^{-x}}{1 - e^{-x}}$ dx = $\log\|1 - e^{-x}\|$+C (b)
6.	Evaluate $\int_0^1 sin^{-1} \left(\frac{2x}{1+x^2} \right)$ dx $a)\frac{\pi}{2}$ - log2 b) log 2 c) 0 d) 1 Ans. Let x = tant then t from 0 to $\frac{\pi}{4}$ .and apply sin2x formula then $I =$ $\int_0^{\frac{\pi}{4}} 2t.sec^2t\, dt = \frac{\pi}{2}$ - log2 (a)
7.	If $\frac{d}{dx}$ (f(x)) = logx , then f(x) equals : $a)\frac{1}{x}$ +C b) x (logx + x)+C c) x (log x -1) +C d) - $\frac{1}{x}$ +C Ans : x (log x -1)+C (c)
8.	Evaluate $\int_{-1}^{1} \frac{\|x-2\|}{x-2}$ dx ? a)1 b) -1 c) 2 d) -2 Ans. Apply $\|x - 2\|$ property to get the -2 (d)
9.	Evaluate $\int e^x (cosx - sinx)$ dx ? a) – e^x sinx +C b) e^x sinx +C c) e^x cosx+C d) - e^x cosx+C Ans. Apply $\int e^x$ [f(x) + f^1(x)] dx = e^x f(x) formula I= e^x cosx (c)
10.	Find $\int_{-5}^{5} f(x)\, dx$ where f(x) = $\|x - 2\|$ a) 25 b) 29 c) 15 d) 20 Q. Apply modulus function definition to get $\int_{-5}^{5} \|x - 2\|$ dx = $\int_{-5}^{2} \|x - 2\|$ dx + $\int_{2}^{5} \|x - 2\|$ dx=29 (b)

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
INTEGRALS	https://youtu.be/VBMyRVKOvck	

EXERCISE

1.	Q. Evaluate $\int \frac{cosx}{\sqrt{1+sinx}}$ dx ? $a)\ 1\ + sinx$ b) $1 - \ sinx$ c) $2\ \sqrt{1\ + sinx}$ d) $2\ \sqrt{1 - sinx}$ Ans(c)
2.	Q. Evaluate $\int \frac{x^2}{1+x^3}$ dx ? a) log(1 + x^3) b) $\frac{1}{3}$ log(1 – x^3) c) log (1 – x^3) d) $\frac{1}{3}$ log(1 + x^3) Ans(d)
3.	Q.Evaluate $\int\ [\frac{1}{\log x} - \frac{1}{(logx)^2}]$ dx ? $a)\ \frac{logx}{x}$ b) $\frac{x}{\log x}$ c) $\frac{x-1}{logx}$ d) $\frac{\log x}{x-1}$ Ans(b)
4.	Q. Evaluate $\int_0^{\frac{\pi}{4}} 2\ tan^3 x$ dx ? a)1 b) log 2 c) 1 – log2 d) 1 + log2 Ans(c)
5.	Q. Evaluate $\int \frac{1}{1-sinx}$ dx ? a) secx – tanx b) tanx + secx c) tanx d) Secx Ans(b)

ASSERTIONS AND REASONING QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

A) Both Assertion (A) and Reason (R) true and Reason (R) is the correct explanation of Assertion (A).

B) Both Assertion (A) and Reason (R) true and Reason (R) is not the correct explanation of Assertion (A)

C) Assertion (A) is true but Reason (R) is false

D) Assertion (A) is false but Reason (R) is true

1	Assertion: $\int \sin x\, dx\ = -\cos x\ +\ C.$ Reason: sin x is an odd function and the integral of an odd function is -f(x) + C. Solution: Assertion is true but reason is false. The integral of sin x is -cos x + C, but sin x is not an odd function. Sin x is an even function. Correct option: (C) Assertion is true but reason is false
2	Assertion: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x\ dx = 0$ Reason: If f(x) is odd function $\int_{-a}^{a} f(x)dx = 0$ **Answer:** A) both assertion and reasoning are correct and reason is the correct explanation
3	Assertion: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cos\ x\ dx = 0$

	Reason: If f(x) is odd function $\int_{-a}^{a} f(x)dx = 0$ **Answer:** D) assertion is False and reasoning is correct
4	Assertion: $\int \boldsymbol{x^2 - 3x + 2\ dx = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + c}$ Reason: The integral of a polynomial function ax^n is $\frac{a}{n+1}x^{n+1} + c$, where 'C' is the constant of integration. **Answer:** A) both assertion and reasoning are correct and reason is the correct explanation
5	Assertion: $\int \boldsymbol{\frac{2x}{x^2+1}\ dx = log\left\|x^2+1\right\| + c}$ Reason: $\int \boldsymbol{\frac{f'(x)}{f(x)}\ dx = log\|f(x)\| + c}$ **Answer:** A) both assertion and reasoning are correct and reason is the correct explanation
6	Assertion: $\int \boldsymbol{\sin^2 x\ dx = \frac{x}{2} - \frac{sin\, 2x}{4} + c}$ Reason: $\boldsymbol{1 - \cos 2x = 2\sin^2 x}$ **Answer:** A) both assertion and reasoning are correct and reason is the correct explanation
7	Assertion: $\int \boldsymbol{e^x(cos\, x + sin\, x)\ dx = e^x cos\, x + c}$ Reason: $\int \boldsymbol{e^x(f(x) + f'(x))\ dx = e^x f(x) + c}$ **Answer:** D) assertion is False and reasoning is correct
8	Assertion: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2 x\ dx = -2$ Reason: If f(x) ≥ 0 on [a, b] then $\int_a^b f(x)\ dx \geq \ \boldsymbol{0}$ **Solution:** $sec\, x$ is not defined at x = $\frac{\pi}{2}$ in $[\frac{\pi}{4}, \frac{3\pi}{4}]$ **Answer:** D) assertion is False and reasoning is correct
9	Assertion: $\int \boldsymbol{log(log\, x) + \frac{1}{logx} dx = xlog(logx) + c}$ Reason: $\int \boldsymbol{e^x(f(x) + f'(x))\ dx = e^x f(x) + c}$ **Answer:** B) Both Assertion (A) and Reason (R) true and Reason (R) is not the correct explanation of Assertion (A)
10	Assertion: $\int \boldsymbol{(sin^{-1}x + cos^{-1}x)dx = \frac{\pi}{2}x + c}$ Reason: $\boldsymbol{sin^{-1}x + cos^{-1}x = \frac{\pi}{2}}$ **Answer:** A) both assertion and reasoning are correct and reason is the correct explanation

2 MARK QUESTIONS

1.	Evaluate $\int x\,.\,tan^{-1}x\,.\,dx$? **Solution:** $\int tan^{-1}x.x\,.\,dx$ Apply $\int u.v\; dx$ formula $\tan^{-1}x.\,\frac{x^2}{2} - \int \frac{1}{1+x^2}\,.\,\frac{x^2}{2}\,dx$ $= \frac{x^2}{2}\,.\,\tan^{-1}x - \frac{x}{2} + \frac{tan^{-1}x}{2} + C$
2.	Evaluate $\int e^x\,(\,tanx + \log secx\,)dx$? **Solution:** $\int e^x\,(\,\log secx + tanx\,)dx$ Apply $\int e^x\left(f(x) + f^1(x)\right)dx$ formula to get $e^x\; log\; Secx\; + \;C$
3.	Evaluate $\int \sqrt{4-9x^2}$ dx? **Solution:** $\int \sqrt{2^2-(3x)^2}$ dx Apply $\int \sqrt{a-(x)^2}$ dx formula to get $\frac{x}{2}\,.\,\sqrt{4-9x^2} - \frac{2}{3}\sin^{-1}(\,\frac{3x}{2}\,)+C$
4.	Evaluate $\int_0^4 \lvert x-1\rvert$ dx? **Solution:** $\int_0^1 (1-x)\;dx + \int_1^4 (x-1)\;dx = 5$
5.	Evaluate $\int \frac{2+Sin2x}{1+Cos2x}\;e^x\,.\,dx$? **Solution:** $\int \frac{2+2Sinx.cosx}{2\,Cos^2x}\;e^x\,.\,dx$ $\int(\,Sec^2x + Tanx\,)\,e^x\,.\,dx$ Apply $\int e^x\left(f(x) + f^1(x)\right)dx$ formula to get $e^x.Tanx$ + C
6.	Evaluate $\int \frac{Sec^2(logx)}{x}$ dx? **Solution:** Substitute $log\;x\; = \;t$ then $\int Sec^2t.dt = \;tan\;(log\;x\,)\; + \;C$
7.	Evaluate $\int \frac{dx}{x^2-4x+8}$? **Solution:** $\int \frac{1}{(\,x-2\,)^2+2^2}$ dx $= \frac{1}{2}\;tan^{-1}(\,\frac{x-2}{2}\,)$ + C
8.	Evaluate $\int_0^{\frac{\pi}{2}} sin2x\,.\,logtanx\;dx$? **Solution:** $I = \int_0^{\frac{\pi}{2}} sin\;2\left(\frac{\pi}{2} - \;x\right)\log tan\left(\frac{\pi}{2} - \;x\right)dx$ $= -\int_0^{\frac{\pi}{2}} sin2x\,.\,logtanx\;dx$ =>2I=0 =>I=0
9.	Evaluate $\int \frac{1\;+cotx}{x+\log sinx}\;dx$? **Solution:** Substitute $x\; + \;log\;sinx\; = \;t$ => log (x + log sinx) + C
10.	If f(x) = $\int_0^x t\,.\,sint\,.\,dt$ then find $f'(x)$? **Solution:** f(x) = $\int_0^x t\,.\,sint\,.\,dt$ Apply $\int u.v\;dx$ formula then f(x) = $[\,-\mathrm{t}\,.\,\mathrm{cost}\; + \;\sin \mathrm{t}\,]_0^x$

	$f(x) = -x\,cosx + Sinx$ $f'(x)$ = x. sinx

EXERCISE

1.	Evaluate $\int_1^2 e^x \left[\frac{1}{x} - \frac{1}{x^2}\right] dx$?
2.	Evaluate $\int sinx \,.\sqrt{1 + cos2x}$ dx?
3.	Evaluate $\int e^{ax}\, sinbx.dx$?
4.	Evaluate $\int \frac{dx}{\sqrt{15-8x^2}}$?
5.	Evaluate $\int \frac{1}{(1+x)(2+x)}$ dx

3 MARK QUESTIONS

1.	Evaluate $\int \frac{x}{(x+1)(x+2)}$ dx? **Solution:** $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ $A = -1, B = 2$ Ans : log $\left(\frac{(x+2)^2}{(x+1)}\right)$ + C
2.	Evaluate $\int \frac{cos\,x}{(1-sinx)(2-sinx)}$ dx? **Solution:** Let cos x = t then $\int \frac{Cos\,x}{(1-sinx)(2-sinx)}$ dx = $\int \frac{dt}{(1-t)(2-t)}$ Apply partial fraction method, A = 1, B = -1 I= log $\left(\frac{2-sinx}{1-sinx}\right)$ + C
3.	Evaluate $\int \frac{x^3+x+1}{x^2-1}$ dx? **Solution:** $\frac{x^2+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$ (Convert it into proper rational function) Now, $\int \left(x + \frac{2x+1}{x^2-1}\right)$ dx = $\int x\,dx + \int \frac{2x+1}{x^2-1}\,dx$ $= \frac{x^2}{2} + \log(x^2-1) + \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + C$
4.	Evaluate $\int \frac{1}{x(x^4-1)}$ dx? **Solution:** Multiplying num. and denom. by x^{4-1} I = $\int \frac{x^3}{x^4(x^4-1)}$ dx Let x^4 = t , then I = $\frac{1}{4}\int \frac{dt}{t(t-1)}$ dt

	Apply partial fractions then I= $\frac{-1}{4}$ log ($\frac{x^4}{x^4-1}$) + C
5.	Evaluate $\int \frac{2x}{(x^2+1)(x^2+3)}$ dx **Solution:** Let x^2 = t, then $\int \frac{2x}{(x^2+1)(x^2+3)}$ dx = $\int \frac{dt}{(t+1)(t+3)}$ Apply partial fractions then I= $\frac{1}{2}$log($\frac{x^2+1}{x^2+3}$) + C
6.	Q. Evaluate $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx$? **Solution:** $\int \frac{6x+7}{\sqrt{x^2-9x+20}}\, dx = \int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx$ 6x + 7 = A $\frac{d}{dx}(x^2 - 9x + 20)$ + B A = 3 , B = 34 substitute the values then I=6 $\sqrt{x^2 - 9x + 20}$ + 34 log [x - $\frac{9}{2}$ +$\sqrt{x^2 - 9x + 20}$] + C
7.	Evaluate $\int \frac{1}{1+cotx}$ dx ? **Solution:** $\int \frac{1}{1+Cotx}$ dx = $\int \frac{sinx}{Sinx+Cosx}$ dx = $\frac{1}{2}\int \frac{2sinx}{Sinx+Cosx}$ dx = $\frac{1}{2}\int \frac{(sinx+Cosx)-(Cosx-sinx)}{Sinx+Cosx}$ dx = $\frac{x}{2}$ - $\frac{1}{2}\int \frac{Cosx-sinx}{Sinx+Cosx}$ dx Put $sinx + cosx = t$ then I= $\frac{x}{2}$ - $\frac{1}{2}$ $log\,(sinx + cosx) + C$
8.	Evaluate $\int \frac{1}{Cos\,(x-a)Cos\,(x-b)}$ dx **Solution:** $\int \frac{1}{Cos\,(x-a)Cos\,(x-b)}$ dx = $\frac{1}{Sin\,(a-b)} \int \frac{Sin(a-b)}{Cos\,(x-a)Cos\,(x-b)}$ dx = $\frac{1}{Sin\,(a-b)} \int \frac{Sin[(x-b)-(x-a)]}{Cos\,(x-a)Cos\,(x-b)}$ dx = $\frac{1}{Sin\,(a-b)}$ [$\int tan\,(x-b)\, dx - \int tan(x-a)\, dx$ = $\frac{1}{Sin\,(a-b)}$ log ($\frac{Cos\,(x-a)}{Cos\,(x-b)}$) + C
9.	Find $\int_0^{\pi} \frac{x}{1+sinx}$ dx **Solution:** Apply the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and prove that I = π
10.	Find $\int (sin^{-1}x)^2$ dx ? **Solution:** Let x = sint then I = $\int t^2\, .cost\, dt$, Apply $\int u.v\, dx$ formula I= x . ($\sin^{-1} x$)^2 + 2 $\sqrt{1 - x^2}$ $\sin^{-1}x$ -2x + C

EXERCISE

1.	Evaluate $\int \frac{Cos\, x}{(2+ sinx)(3+4\, sinx)}$ dx
2.	Evaluate $\int \frac{e^x}{\sqrt{5-4e^x- e^{2x}}}\ dx$?
3.	Evaluate $\int \frac{x.e^x}{(x+1)^2}$ dx ?
4.	Evaluate $\int_{-1}^{2} \|x^3 - x\|$ dx ?
5.	Evaluate $\int \frac{1 - x^2}{x\,(1 - 2x)}$ dx?

5 MARK QUESTIONS

1.	Evaluate $\int_0^{\pi} log\,(1 + cosx\,)dx$? **Solution:** Apply the properties of definite integral and prove that I = - $\pi.log2$
2.	Evaluate $\int_0^{\pi} \frac{x\,.tanx}{Secx + tanx}\,dx$? **Solution:** Apply $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ I = $\frac{\pi}{2}\,(\pi - 2\,)$
3.	Evaluate $\int \frac{3x + 5}{x^3- x^2- x+1}$ dx ? **Solution:** $\int \frac{3x + 5}{x^3- x^2- x+1}$ dx = $\int \frac{3x + 5}{(x-1)^2\,(x+1)}$ dx $\frac{3x + 5}{(x-1)^2\,(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$ A = $\frac{-1}{2}$, B = 4 and C = $\frac{1}{2}$ I= $\frac{1}{2}\,log\frac{x+1}{x-1}$ - $\frac{4}{x-1}$ + C

EXERCISE

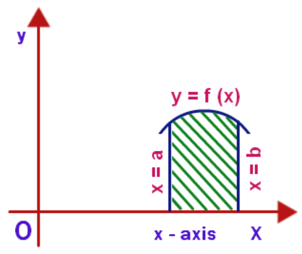
1.	Evaluate $\int \frac{x\,dx}{a^2Cos^2x+ b^2sin^2x}$?
2.	Evaluate $\int_0^{\frac{\pi}{4}} \frac{sinx + cosx}{9+16\,sin2x}$ dx ?
3.	Evaluate $\int \frac{dx}{x^3+ x^2+ x+1}$?

CHAPTER: APPLICATION OF INTEGRALS

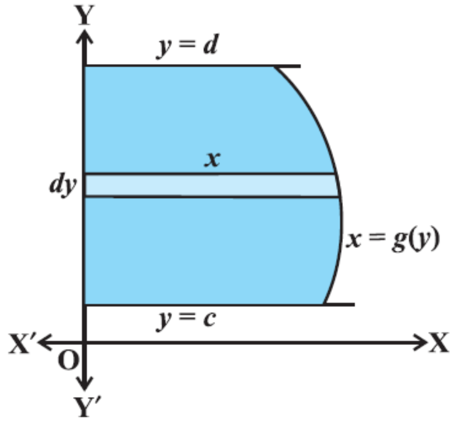
SYLLABUS: *Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)*

Definitions and Formulae:

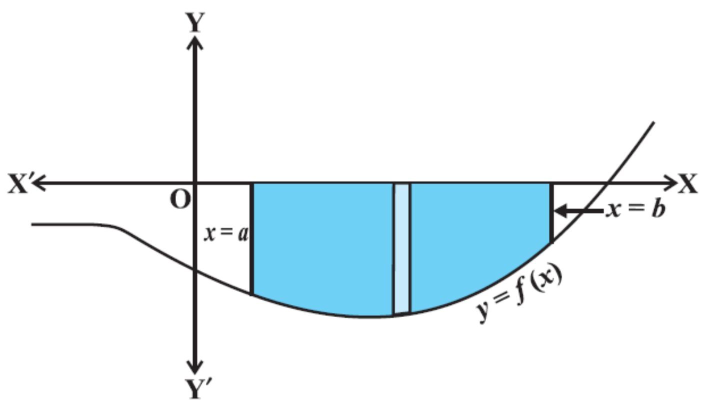
Let f(x) be a function defined in [a, b] , then the area bounded by the curve y = f(x) , x – axis and the ordinates x = a and x = b is given by $\int_a^b f(x)dx$ or $\int_a^b y\,dx$



Let $g(y)$ be a function defined in [c, d] , then the area bounded by the curve $x = g(y)$, y – axis and the ordinates $y = c$ and $y = d$ is given by $\int_c^d g(y)dy$



If the curve y = f(x) lies below x- axis ,then the area bounded by the curve y= f(x) , x-axis and the ordinates x = a and x= b is –ve. So the area is $\left|\int_a^b f(x)\,dx\right|$



MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND ANSWERS
1	Find the area bounded by $y = x^2$,x *axis and lines* $x = -1$ *and* $x = 1$ (a) $\frac{-1}{3}$ sq. unit (b) $\frac{4}{3}$ sq. unit (c) $\frac{2}{3}$ sq. unit (d) None of these **Solution:** Area of the required region $= \int_{-1}^{1} x^2 dx = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$
2	Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines x =0 and x = a (a) $\frac{\pi a^2}{4}$ sq. unit (b) $\frac{\pi}{4}$ sq. unit (c) $\frac{a^2}{4}$ sq. unit (d) $\frac{\pi a^2}{12}$ sq.unit **Solution:** Required area $= \int_0^a \sqrt{a^2 - x^2}\, dx = \frac{a^2}{2} \times \frac{\pi}{2} = \pi \frac{a^2}{4}$
3	The area of the region bounded by the curve $y = x^2$ and the line y = 16 is (a) $\frac{32}{3}$ sq. unit (b) $\frac{256}{3}$ sq. unit (c) $\frac{64}{3}$ sq. unit (d) $\frac{128}{3}$ sq.unit **Solution:** Required area= $2\left[\int_0^4 16dx - \int_0^4 x^2 dx\right] = 2\left[64 - \frac{64}{3}\right] = \frac{256}{3}$
4	Find the area bounded by y = sinx between x = 0 and x = 2π. (a) 4 sq.unit (b) 4πsq. unit (c) 2 sq.unit (d) 1 sq.unit **Solution:** Area = $2\int_0^{\pi} sinx\, dx = 4$
5	Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x =0 and x = 2 is (a) π sq. unit (b) $\frac{\pi}{2}$ sq. unit (c) $\frac{\pi}{3}$ sq. unit (d) $\frac{\pi}{4}$ sq. unit **Solution:** Area = $\int_0^2 \sqrt{4 - x^2}\, dx = \pi$
6	The area of the region bounded by the straight line x = 2y + 3 , y axis and the lines y = 1 and y = -1 is (a) 4 sq. unit (b) $\frac{3}{2}$ sq. unit (c) 6 sq. unit (d) 8 sq. unit **Solution:** Area = $\int_{-1}^{1} 2y + 3dy =$ 1+3 –(1- 3) = 6

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
APPLICATION OF INTEGRALS	https://youtu.be/p4R2unWOT9Q	

ASSERTION AND REASON QUESTIONS

The following questions consist of two statements – Assertion (A) and Reason(R), Answer the questions selecting the appropriate option given below.

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true and R is not the correct explanation for A.

(c) A is true and R is false

(d) A is false and R is true

1	**Assertion:** The area of the ellipse $2x^2 + 3y^2 = 6$ is more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$ **Reason:** The length of the semimajor axis of ellipse $2x^2 + 3y^2 = 6$ is more than the radius of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$ **Answer:** Area of ellipse $= \sqrt{6}\,\pi$ Area of circie= π . A is true Length of major axis is $2\sqrt{3}$ .Radius of circle= 1. R is true. But R is not the correct explanation of A Option b is correct
2	**Assertion:** Area enclosed by the circle $x^2 + y^2 = 36$ is equal to 36π sq.unit **Reason:** Area enclosed by circle $x^2 + y^2 = r^2$ is πr^2 **Answer:** Area of the circle is 36π sq.unit Option (a) is correct
3	**Assertion:** The area of the region bounded by y= cosx and the ordinates $x = 0\ and\ x = \pi\ is\ 2 sq.unit$ **Reason:** cos x is an increasing function in the first quadrant **Answer:** (c)

4	**Assertion:** The area of the region bounded by y = x+1 , x-axis and he lines x =2 and x=3 is $\frac{5}{2}$ sq.units **Reason:** The intercept made by the line on the x-axis and y axis is 1unit left of zero and 1unit respectively. **Answer:** (d)
5	**Assertion:** The area bounded by the curve x =y^2 , y-axis and the lines y =3 and y= 4 is$\frac{37}{2}$ **Reason:** Area = $\int_a^b f(y)$ dy **Answer:** (a)
6	**Assertion:** Area bounded by $y = \|x+2\|$ from x= -2 to x= 0 is 4 sq.unit **Reason:** $y = \|x+2\|$ is differentiable in R **Answer:** (c)
7	**Assertion:** The area of the region bounded by y = x^2 + x , x = 2 and x = 5 cannot be evaluated **Reason:** Area of the unbounded region cannot be evaluated **Answer:** (d)
8	**Assertion:** The area of the region y =sin^{2}x from 0 to π will be more than that of the curve y=sinx from 0 to π **Reason:** x^2 > x if x > 1 **Answer:** (d)

EXERCISE

The following questions consist of two statements – Assertion (A) and Reason(R) ,Anawer the questions selecting the appropriate option given below

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true and R is not the correct explanation for A.

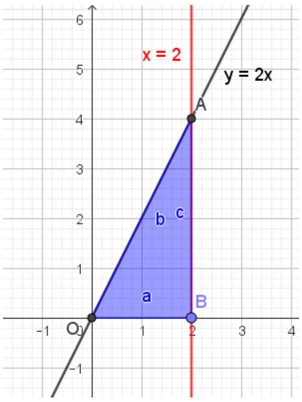
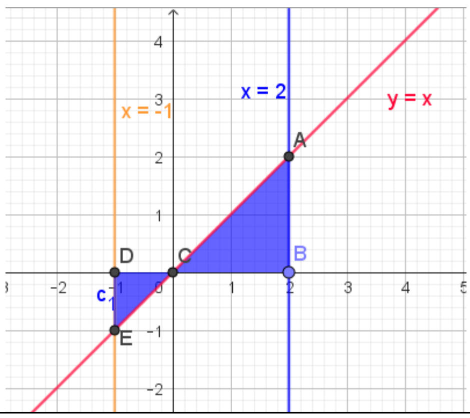
(c) A is true and R is false

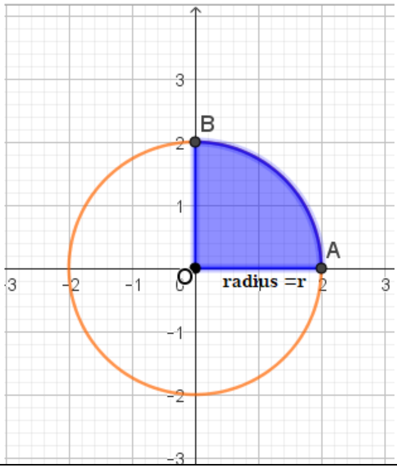
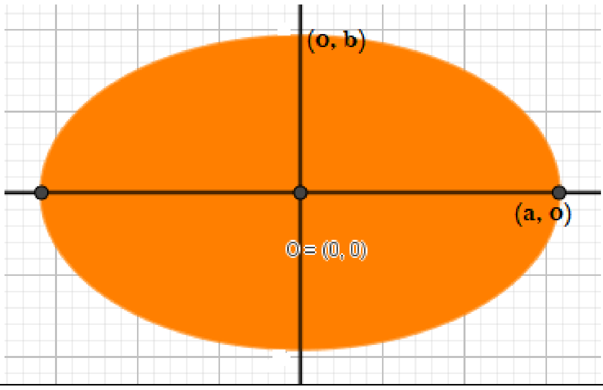
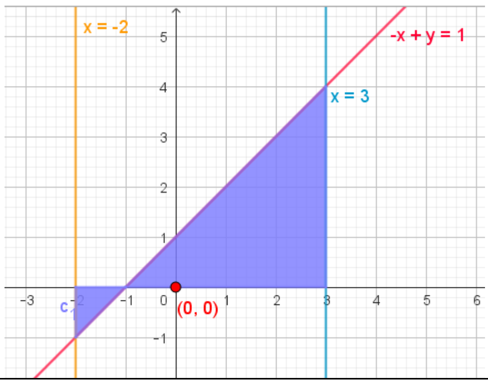
(d) A is false and R is true

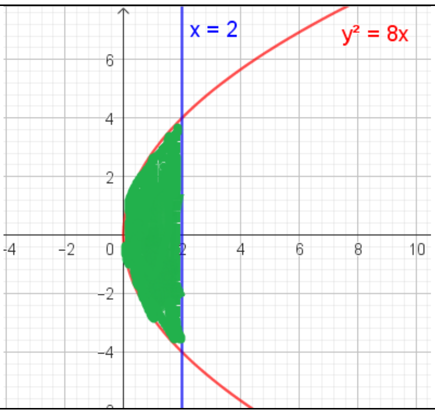
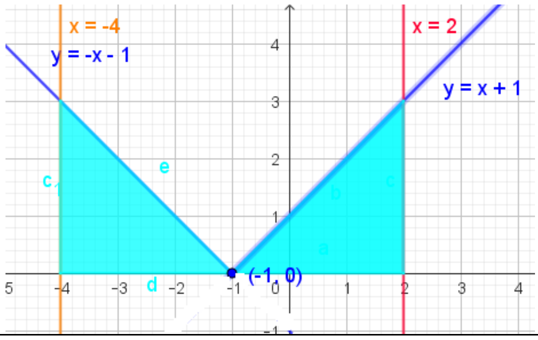
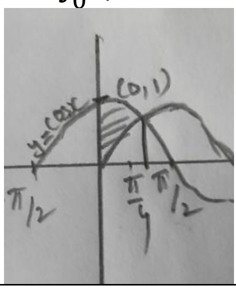
1	**Assertion:**The region bounded by the curve y=$\sqrt{4-x^2}$ is a semicircle above the x-axis **Reason:** area of the semicircle is half of the area bounded by the equation $x^2 + y^2 = 4$
2	**Assertion:** Area enclosed by $y = x\|x\|$, x-axis and the ordinates $x = -1$ *and* $x = 1$ is given by $\frac{2}{3}$ **Reason:** f(x) = $\|x\|$ = x , x≥0 and –x , x< 0

3	**Assertion:** The region bounded by the curve y=$\sqrt{4-x^2}$ is a semicircle above the x-axis **Reason:** area of the semicircle is half of the area bounded by the equation $x^2 + y^2 = 4$
4	**Assertion:** The area bounded by the curve $y = \log_e x$ and x- axis and the straight line x= e **Reason:** The most approximate value of e =2.7
5	**Assertion:** The area between x-axis and y = cosx when $0 \leq x \leq 2\pi$ is 4sq.unit **Reason:** Area= $\int_0^{2\pi} cosx\, dx = 4\int_0^{\frac{\pi}{2}} cosx\, dx$
Solutions 1. (b) 2. (b) 3 .(c) 4. (d) 5. (a)	

2 MARK QUESTIONS

1	Find the area of the region bounded by the line $y = 2x$,x-axis and x=2 **Solution:** Area = $\int_0^2 2x$ dx = 4 sq.unit. 
2	Find the area bounded by $y = x$, $the\ x-axis$ and the ordinate $x = -1$, $x = 2$. **Solution:** $A = \int_0^2 x\, dx + \left\lvert\int_{-1}^0 x\, dx\right\rvert = \frac{5}{2}$. 
3	Find the area bounded by the circle $x^2 + y^2 = r^2$ **Solution:** A= 4× $\int_0^r \sqrt{r^2 - x^2}$ dx = πr^2

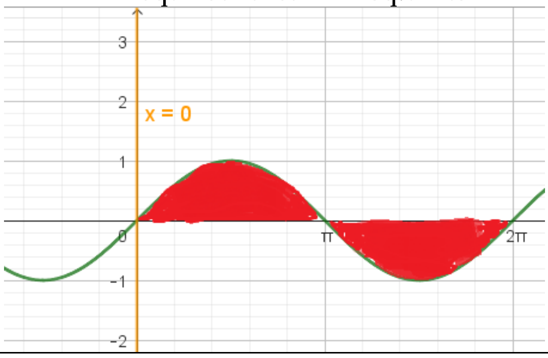
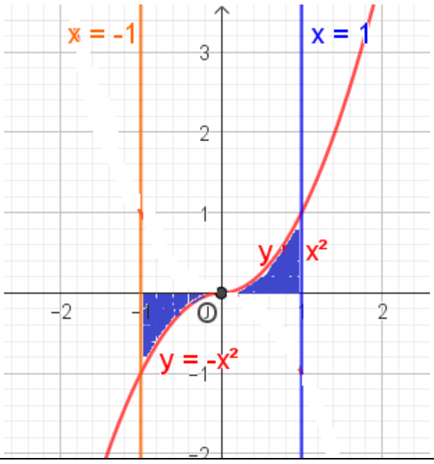
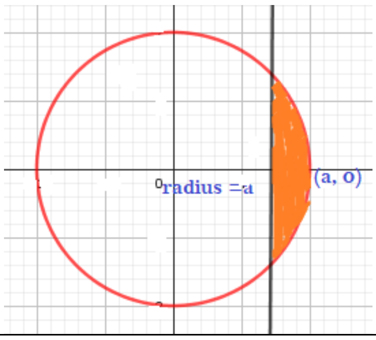
	
4	Find area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ **Solution:** $A = 4 \times \frac{b}{a}\int_0^a \sqrt{a^2-x^2}$ dx = πab 
5	Using integration find the area of the region bounded by the line $y - 1 = x$, x-axis and the ordinates $x = -2$ and $x = 3$ **Solution:** A = $\left\|\int_{-2}^{-1}(x+1)\,dx\right\| + \int_{-1}^{3}(x+1)dx = \frac{17}{2}$ 
6	Using integration find the area of the region bounded between the line $x = 2$, and the parabola $y^2 = 8$x. **Solution:** A = $2\int_0^2 \sqrt{8x}$ dx= $\frac{32}{3}$ sq.units

	
7	Draw the graph of $y = \lvert x+1 \rvert$ and find the area between $x-$ axis $x = -4$ and $x = 2$. **Solution:** $A = \left\lvert \int_{-4}^{-1} (x+1)\, dx \right\rvert + \int_{-1}^{2} (x+1)\, dx = 9$ sq.units 
8	Draw the graph of $y = sinx$, $y = cosx$ between $x = 0$ and $x = \frac{\pi}{4}$ and also find the area between the curves between $x = 0$ and $x = \frac{\pi}{4}$ **Solution:** $A = \int_0^{\frac{\pi}{4}} (cosx - sinx)\, dx = \sqrt{2} - 1$ sq.units 

EXERCISE

1	Calculate the area of the region bounded by $y^2 = 4ax$ and the line $y = mx$ is $\frac{a^2}{12}$ sq. units. Find the value of m
2	Sketch the graph of $y = \lvert x+3 \rvert$ and evaluate the area under the curve $y = \lvert x+3 \rvert$ above x- axis and between $x = -6$ and $x = 0$
3	Find the area bounded by $y^2 = 4ax$, latus rectum and x-axis
4	Find the area of the region bounded by $y^2 = 4ax$ and $x = a$, $x = 2a$, $a > 0$
Answers:	
1. m = 2 2. 9 sq.units 3. $\frac{4}{3}a^2$ sq.units 4. $\frac{56a^2}{3}$ sq.units	

3 MARK QUESTIONS

<table>
<tr><td>1</td><td>Find the area bounded by the curve y = sinx between $x = 0 \ and \ x = 2\pi$
Solution:
$\int_0^{\pi} sinx\, dx = 2\ sq.units.$
And $\left|\int_{\pi}^{2\pi} sinx\, dx\right|$ =2 sq.units.
Required area = 4 sq.units

</td></tr>
<tr><td>2</td><td>Find the area bounded by the curve $y = x|x|$ and the ordinates x = -1 and x = 1.
Solution:
A = $2\int_0^1 x^2\, dx = \frac{2}{3}$

</td></tr>
<tr><td>3</td><td>Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$
Solution:
A =2 $\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}$ dx = $\frac{a^2}{2}$ ($\frac{\pi}{2}$ - 1).

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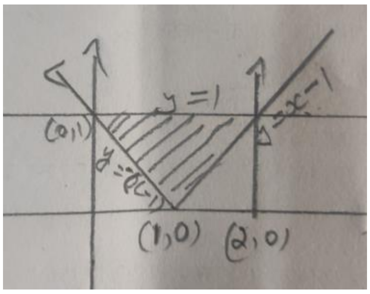
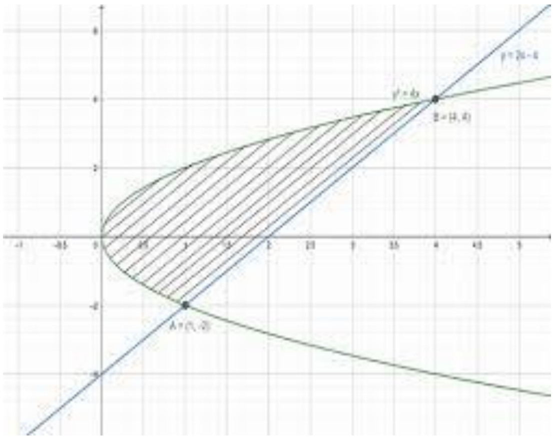
4	Area of the region bounded by the curve $y = \|x - 1\|, y = 1.$ **Solution:** $A = \int_0^2 1\, dx - 2\int_0^1 (x-1)\, dx = 1$ 

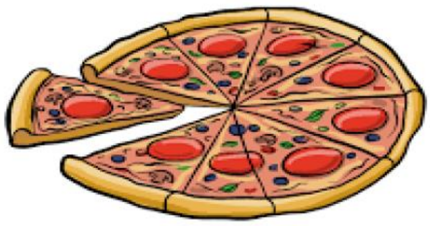
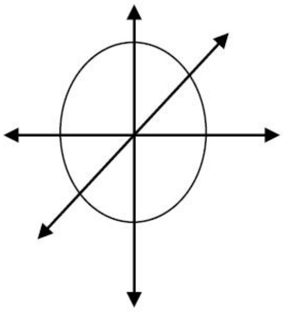
EXERCISE

1	Find the area of the circle $4x^2 + 4y^2 = 1$
2	Find the area of the region bounded by the ellipse $6x^2 + 8y^2 = 1$
3	Sketch the area lying in first quadrant and bounded by y = $9x^2$, x = 0, y = 1 and y = 4 . Find the area of this region using integration.
4	Using integration, find the area of the triangle formed by +ve x-axis, tangent and normal to the circle$x^2 + y^2 = 4$ at $(1, \sqrt{3})$
Answers: 1) $\frac{5}{8}$ sq. units 2).$\frac{\pi}{4\sqrt{3}}$. sq. units 3).$\frac{19}{4}$ sq.units 4).$2\sqrt{3}$	

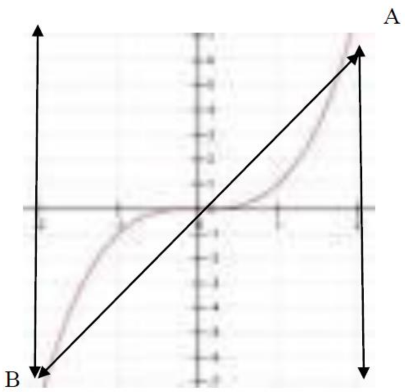
CASE STUDY QUESTIONS

I	A student designs an open air honeybee nest on the branch of a tree, whose Plane figure is parabolic, whose equation is $y^2 = 2$x and the branch of tree is given by a straight line x- y = 4  Based on the above passage answer the following questions 1. Draw the rough diagram of parabola and straight line 2. Find point of intersection of the parabola and straight line 3. Find the area enclosed by the parabola and straight line **Solution:**

	1.  2) . Solve the equations y^2 = 2x and x- y =4 the point of intersection is (2, -2) and (8,4) 3). $\int_{-2}^{4} y+4 -\frac{y^2}{2}$ dy = 18
II	A mirror is in the shape of an ellipse represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$was hanging on the wall. Arun and his sister were playing with ball inside the house , even their mother refused to do so . All of a sudden, ball hit the mirror and got a scratch in the shape of line represented by $\frac{x}{a} + \frac{y}{b} = 1$ Based on the above information answer the following questions.  1..Find the point of intersection of mirror and scratch 2. The value of $\frac{b}{a} \int_0^a \sqrt{a^2 - x^2}$ dx 3. The area of the smaller region bounded by the mirror and the scratch **Solution:** 1) Solve the equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x}{a} + \frac{y}{b} = 1$. The points are (a,0) (0,b) 2). $\frac{b}{a} \int_0^a \sqrt{a^2 - x^2} = \frac{\pi ab}{4}$ 3). $\frac{b}{a} \int_0^a \sqrt{a^2 - x}$ dx - $\frac{b}{a} \int_0^a a - x$ dx = $\frac{1}{2}$ ($\frac{\pi}{2}$ - 1).
III	A child cut a pizza with a knife. Pizza is circular in shape which is represented by

	$x^2 + y^2 = 4$ and knife represents x $=\sqrt{3}y$, On the basis of the above information answer the following.  PIZZA  1) Find the point of intersection of circle and straight line. 2) Find the area enclosed by the circle, line and x-axis. **Solution:** 1. Point of intersection is $(1,\sqrt{3})$ and $(-1, -\sqrt{3})$ 2. Required area = $\sqrt{3}\int_0^1 x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx = \frac{2\pi}{3}$.

EXERCISE

I	The location of three branches of a bank is represented by the three points A(-2, 0) , B(1,4), C (2,3) . Based on this information solve the following questions. 1. Find the equations of line AB and BC 2. Find the area of triangle ABC.
II	An insect moves on a curve represented by y = x^3 . It started from a point (-2,-8) on the curve and as soon as it reached at a point (2,8) got tired and slept. The path of its movement is given below. Based on this information answer the following questions.  1. Find the area enclosed by the curve y= x^3 , the lines x=2 and x= -2 2. If it would have moved along the line represented by y = x what is the area bounded by the curve y = x^3 and y = 6 x

Answers:

I) (1) y = $\frac{4}{3}$ (x +2), y = - x+3 (2). $\frac{7}{2}$ sq.units

II) (1). 8.sq. unit. (2)12 sq.unit

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CHAPTER: DIFFERENTIAL EQUATIONS

SYLLABUS: *Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:*

$\frac{dy}{dx}$ *+py=q, where p and q are functions of x or constants.*

$\frac{dx}{dy}$ *+ px=q, where p and q are functions of y or constants.*

Definitions and Formulae:

<u>Methods of solving First Order and First Degree Differential Equations</u>

➤ Differential Equations with Variables separables

➤ Homogeneous differential equations

➤ Linear differential equations

<u>Differential Equations with Variables separables</u>

To solve the differential equation in variable separable form, write the differential equation as

(x terms) X dx = (y terms) X dy then integrate both sides.

- Let the differential equation $\frac{dy}{dx}=\frac{f(x)}{g(y)}$

then g(y)dy = f(x)dx
then integrate on both sides $\int g(y)dy=\int f(x)dx$

- Let the differential equation $\frac{dy}{dx}=\frac{g(y)}{f(x)}$

then $\frac{dy}{g(y)}=\frac{dx}{f(x)}$
then integrate on both sides $\int\frac{dy}{g(y)}=\int\frac{dx}{f(x)}$

- Let the differential equation $\frac{dy}{dx}=f(x).g(y)$

then $\frac{dy}{g(y)}$ = f(x)dx
then integrate on both sides $\int\frac{dy}{g(y)}=\int f(x)\,dx$

<u>Homogeneous differential equations</u>

❖ A function $F(x,y)$ is said to be homogeneous function of degree n if $F(\lambda x,\lambda y)=\lambda^n F(x,y)$

- A differential equation of the form $\frac{dy}{dx} = F(x,y)$ is said to be homogeneous if F(x,y) is a homogeneous function of degree zero

  i.e. if $F(\lambda x, \lambda y) = \lambda^0 F(x,y)$

Steps to solve the homogeneous differential equation of the type: $\boldsymbol{\frac{dy}{dx} = f(\frac{y}{x})}$

- Let $y = vx$
- $\frac{dy}{dx} = v + x\frac{dv}{dx}$
- Substitute $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in $\frac{dy}{dx} = f(\frac{y}{x})$
- Then use variables and separables in terms of y and v only

Steps to solve the homogeneous differential equation of the type: $\boldsymbol{\frac{dx}{dy} = f(\frac{x}{y})}$

- Let $x = vy$
- $\frac{dx}{dy} = v + y\frac{dv}{dy}$
- Substitute $x = vy$ and $\frac{dx}{dy} = v + y\frac{dv}{dy}$ in $\frac{dx}{dy} = f(\frac{x}{y})$
- Then use variables and separables in terms of x and v only

<u>Linear differential equation</u>

Steps to solve the Linear differential equation of the type: $\boldsymbol{\frac{dy}{dx} + P(x)y = Q(x)}$

- $\frac{dy}{dx} + P(x)y = Q(x)$
- $Integratin\ Factor\ (IF) = e^{\int p(x)dx}$
- $Solution\ is\quad y.(IF) = \int (IF).Q(x)dx$

Steps to solve the Linear differential equation of the type: $\boldsymbol{\frac{dx}{dy} + P(y)x = Q(y)}$

- $\frac{dx}{dy} + P(y)x = Q(y)$
- $Integratin\ Factor\ (IF) = e^{\int p(y)dy}$
- $Solution\ is\quad x.(IF) = \int (IF).Q(y)dy$

MULTIPLE CHOICE QUESTIONS

S.NO	QUESTIONS AND SOLUTIONS
1.	Integrating factor for the differential equation $(xlogx)\frac{dy}{dx} + y = 2logx$ is (a) $log(logx)$ (b) $logx$ (c) e^x (d) x Ans (b) Equation is $\frac{dy}{dx} + \frac{1}{\text{xlogx}}.y = \frac{2}{x}$ Here p(x) = $\frac{1}{xlogx}$ Integrating factor $= e^{\int \frac{1}{xlogx}dx} = e^{\log(logx)} = logx$
2.	If $m\ and\ n$, respectively, are the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then $m + n =$ (a) 1 (b) 2 (c) 3 (d) 4 Ans: (c) The given differential equation is $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$ Differentiate w.r.t x , weget $4\left(\frac{dy}{dx}\right)^3 \frac{d^2y}{dx^2} = 0$ Here, m = 2 and n =1 Hence, m + n = 3
3.	If $p\ and\ q$ are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then the value of $2p - 3q$ is (a) 7 (b) -7 (c)3 (d) -3 Ans: (b) degree p =1 and order q = 3 ∴ $2p - 3q = 2 - 9 = -7$
4.	Find the value of m and n, where m and n are order and degree of differential equation $\frac{4\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$ (a) $m = 3, n = 2$ (b) $m = 2 , n = 3$ (c) $m = 2, n = 2$ (d) $m = 3, n = 3$ Ans (a) Solution(a) Given $\frac{4\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$ $\Rightarrow 4\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2 = (x^2 - 1)\frac{d^3y}{dx^3}$ $m = 3, n = 2$

5.	Differential equation $e^x \frac{dy}{dx} = 3y^2$ can be solved using the method of (a) Separating the variables (b) Homogenous equation (c) Linear differential equation of first order (d) None of these Ans (a)
6.	General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ (a) $e^{-y} = \frac{1}{2}e^{2x} + C$ (b) $\frac{1}{e^y} + \frac{1}{2}e^{2x} + C$ (c) $-e^{-y} = \frac{1}{2}e^{2x} + C$ (d) $e^y = \frac{1}{2}e^{2x} + C$ Ans (c) $\frac{dy}{dx} = e^{2x+y} = e^{2x}.e^y$ $\Rightarrow \int e^{-y}dy = \int e^{2x}dx$ $\Rightarrow -e^{-y} = \frac{1}{2}e^{2x} + C$
7.	The particular solution of the differential equation $\frac{dy}{dx} = ytanx$, given that y = 1 when x = 0 is (a) $y = cosx$ (b) $y = secx$ (c) $y = tanx$ (d) $y = secxtanx$ Ans (b) Solution: $\int \frac{dy}{dx} = \int tanx\, dx$ ⇒ log|y| = log$\|secx\| + logC$ ⇒y =cosecx Given y = 1, x = 0 ⇒1 = sec0 ⇒C = 1 Solution is $y = secx$
8.	Degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = x$ is (a) 1 (b)2 (c) 3 (d) Does not exist Ans (b) Solution: $\left(\frac{d^3y}{dx^3}\right)^2 = x^3$
9.	Integrating factor of the differential equation $\frac{dy}{dx} = \frac{cosy}{1-xsiny}$ is (a) $cosy$ (b) $-secy$ (c) $secy$ (d) $tany$ Ans (c) Solution: $\frac{dy}{dx} = \frac{cosy}{1-xsiny}$ $\Rightarrow \frac{dx}{dy} = \frac{1-xsiny}{cosy}$ $\Rightarrow \frac{dx}{dy} + tany.x = secy$ Now, P(y) = tany; Q(y) = secy I.F. = $e^{\int P\,dy} = e^{\int tany\,dy} = secy$

10.	Differential equation $x\frac{dy}{dx} = y(logy - logx + 1)$ can be solved using method of (a) Separating the variables (b) Homogenous equation (c)Linear differential equation of first order (d)None of these Solution : (b)

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
DIFFERENTIAL EQUATIONS	https://youtu.be/4T5yvAwh4dM	

EXERCISE

1	The degree of differential equation $\left(1 + \frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ (i) 1 (ii) 2 (iii)3 (iv)4 **Solution: (iii)**
2	General solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ is (i) logy = Cx (ii) y = Cx (iii) xy = C (iv)y = Clogx **Solution: (ii)**
3	Integrating factor for the differential equation $(xlogx)\frac{dy}{dx} + y = 2logx$ is (i) log(logx) (ii) logx (iii)e^x (iv)x **Solution: (ii)**
4	Integrating factor for the differential equation $sin^2x\frac{dy}{dx} + y = \cot x$ is (i) e^{-cotx}

	(ii) $cotx$ (iii)$-cotx$ (iv)e^{cotx} **Solution:(i)**
5	Integrating factor for the solution of differential equation $(x-y^3)dy+ydx=0$ is (i) $\frac{1}{y}$ (ii) logy (iii)y (iv)y^2 **Solution: (iii)**

ASSERTION REASONING BASED QUESTIONS

	In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true
1	**Assertion (A):** The solution of differential equation $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$ is $\cos\left(\frac{y}{x}\right)=xc$ **Reason (R):** $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$ we can clearly see that it is an homogenous equation substituting Y=vx $\Rightarrow\frac{dy}{dx}=v+\frac{dv}{dx}$ $\Rightarrow$ v + x$\frac{dv}{dx}=v+tanv$ Separating the variables and integrating we get $\int\frac{1}{\tan v}dv=\int\frac{1}{x}dx$ Log(sinv) = logx + logC Sin(v) =xC $\Rightarrow$sin$\left(\frac{y}{x}\right)=xC$ Is the solution where, C is constant. Solution: (d)
2	**Assertion (A):** The degree of the differential equation given by $\frac{dy}{dx}=\frac{x^4-y^4}{(x^2+y^2)xy}$ is 1 **Reason (R):** The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction. The given differential equation has first order derivative which is free from radical and fraction with power = 1, thus it has a degree of 1. Solution: (a)
3	**Assertion (A):** Solution of the differential equation $\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$ is $\frac{e^{2y}}{3}=\frac{e^{3x}}{3}+\frac{x^2}{2}+C$

	Reason (R): $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$ $\frac{dy}{dx} = e^{-2y}(e^{3x} + x^2)$ Separating the variables e^{2y}dy =$(e^{3x} + x^2)dx$ $\int e^{2y}dy = \int(e^{3x} + x^2)dx$ $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^2}{2} + C.$ Solution: (d)
4	**Assertion (A):** The order and degree of differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are 2 and 1 respectively **Reason (R):** The differential equation $\left(\frac{dy}{dx}\right)^3 + 2y^{\frac{1}{2}} = x$ is of order 1 and degree 3 Solution: (b)
5	**Assertion (A):** Order of differential equation is $\frac{dy}{dx} + 4y = \sin x$ is 1 **Reason (R):** Since order of the differential equation is defined as order of the highest derivative occurring in the differential equation, i.e., for nth derivative $\frac{d^ny}{d^xn}$ if n = 1 then it's order = 1. Given differential equation contains only $\frac{dy}{dx}$ derivative with variable and constants. Solution: (a)
6	**Assertion (A): order of differential equation $\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2}$= sinx is 1** **Reason (R):** Order of the differential equation is the order of the highest order differential present in the equation Solution: (d)
7	Assertion (A**):** $\frac{dy}{dx} + x^2y = 5$ **is a first order linear differential equation** **Reason (R): If Pand Q are functions of x only or constant then differential equation of the form $\frac{dy}{dx} + Py = Q$ is a first order linear differential equation** Solution: (a)
8	**Assertion (A): $\frac{dy}{dx} = \frac{x^3-xy^2+y^3}{x^2y-x^3}$ is a homogeneous differential equation.** **Reason (R): the function F(x,y)= $\frac{x^3-xy^2+y^3}{x^2y-x^3}$ is homogenous** Solution: (a)
9	**Assertion (A):** The degree of the differential equation $\frac{d^2y}{dx}$ +3$\left(\frac{dy}{dx}\right)^2$ = x^2log$\left(\frac{d^2y}{dx^2}\right)$ is not defined. **Reason (R):** If the differential equation is a polynomial in terms of its derivatives, then its degree is defined. Solution: (a)

EXERCISE

	Assertion reasoning-based question In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true
1	**Assertion (A):** Degree of differential equation $\frac{dy}{dx} + log\left(\frac{dy}{dx}\right) = 0$ is not defined. **Reason (R):** Differential equation cannot be written as polynomial of derivatives. Solution: (b)
2	**Assertion (A):** The differential equation $\frac{dy}{dx} = 1 + \frac{y}{x}$ is homogenous differential equation. **Reason (R):** For a homogenous equation $\frac{dy}{dx} = f\left(\frac{dy}{dx}\right)$. Solution: (c)
3	**Assertion (A):** To solve the differential equation $\frac{dy}{dx} = \sin(x + y)$ we first substitute x +y =t **Reason (R):** If x +y = t, then $\frac{dy}{dx} = \frac{dt}{dx} - 1$ Solution: (a)
4	**Assertion (A):** Integrating factor for the differential equation x$\frac{dy}{dx} - y = x^2 sinx$ is -x. **Reason (R):** $e^{alogx} = e^{logx^a} = x^a$ Solution: (d)
5	**Assertion (A):** The differential equation $\frac{dy}{dx} = \frac{x+\sqrt{y^2-x^2}}{x}$ is homogenous equation. **Reason (R):** $f(\lambda x, \lambda y) = f(x, y)$ for homogenous equation. Solution: (a)
6	**Assertion (A):** The differential equation $x^2 = y^2 + xy\frac{dy}{dx}$is an ordinary differential equation. **Reason (R):** An ordinary differential equation involves derivatives of the dependent variable with respect to only one dependent variable. Solution: (a)

7	For the differential equation $\frac{d^2y}{dx^2}$ + y = 0, let its solution be y = $\emptyset_1$(x) =sin $\left(x+\frac{\pi}{4}\right)$. **Assertion (A):** The function y = $\emptyset_1$(x) is called the particular solution. **Reason (R):** The solution which is free from arbitrary constant, is called a particular solution. Solution: (a)
8	**Assertion (A):** The differential equation $\frac{dx}{dy}$+x $= \cos y\, and\ \frac{dx}{dy}+\frac{-2x}{y}=y^2e^{-y}$ are first order linear differential equations. **Reason (R):** The differential equation of the form $\frac{dx}{dy}+P_1x=Q_1$ Where, P_1 and Q_1 are constants or functions of y only, is called first order linear differential equations. Solution: (a)
9	**Assertion (A):** The differential equation y^3dy + $(x+y^2)$dx = 0 becomes homogeneous if we put $y^2=t$ **Reason (R):** All differential equation of first order first degree becomes homogeneous if we put y = tx. Solution: (c)
10	Let a solution y = y(x) of the differential equation x$\sqrt{x^2-1}dy-y\sqrt{y^2-1}dx=0$ satisfy $y(2)=\frac{2}{\sqrt{3}}$ **Assertion (A):** y(x) = sec $\left(\sec^{-1}x-\frac{\pi}{6}\right)$ **Reason (R):** y(x) is given by $\frac{1}{y}=\frac{2\sqrt{3}}{x}-\sqrt{1-\frac{1}{x^2}}$ Solution: (c)
11	**Assertion (A):** The degree of the differential equation $\frac{d^3y}{dx^3}+2\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ + 2y = 0 is zero **Reason (R):** The degree of a differential equation is not defined if it is not a polynomial eqn in its derivatives. Solution: (d)
12	**Assertion (A):** $\frac{dy}{dx}=\frac{x^3-xy^2+y^3}{x^2y-x^3}$ is a homogeneous differential equation. **Reason (R):** The function F(x,y) = $\frac{dy}{dx}=\frac{x^3-xy^2+y^3}{x^2y-x^3}$ is homogenous. Solution: (a)
13	**Assertion (A):** = $\frac{dy}{dx}+x^2y^5$ is a first order linear differential equation.

	Reason (R): If P and Q are functions of x only or constant then differential equation of the form $\frac{dy}{dx}$ + Py = Q is a first order linear differential equation. Solution: (a)

2 MARKS QUESTIONS

Q.NO	QUESTIONS WITH SOLUTIONS
1	Question: Write the integrating factor of the differential equation: $\sqrt{x}\left(\frac{dy}{dx}\right)$ + y $=e^{-2\sqrt{x}}$ Solution: The given differential equation is: $\frac{dy}{dx}+\left(\frac{1}{\sqrt{x}}\right)y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ I.F. = $e^{\int\frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$
2	Question: Find the general solution of the differential equation ydx – (x + $2y^2)dy=0$ Solution: Given differential equation can be written as y $\frac{dy}{dx}-x=2y^2$ or $\frac{dx}{dy}-\frac{1}{y}.x=2y$ Integrating factor is $e^{-logy}=\frac{1}{y}$ ∴Solution is x. $\frac{1}{y}=\int 2dy$ = 2y + C or x = $2y^2$ + Cy
3	Write the order and degree of the differential equation y = px +$\sqrt{1+p^2}$, where $p=\frac{dy}{dx}$ solution: Order: 1, Degree: 2
4	Solve the differential equation $\frac{dy}{dx}= e^{x-y}+x^3e^{-y}$ Solution: $\int e^y dy=\int(e^x+x^3)dx$ ⇒$e^y=e^x+\frac{x^4}{4}+C$
5	Find the general solution of differential equation $$\frac{dy}{dx}=\frac{x+1}{2-y},(y\neq 2).$$ Solution: $\int(2-y)dy=\int(x+1)dx$ ⇒$2y-\frac{y^2}{2}=\frac{x^2}{2}+x+C$ is the required solution.
6	Solve the differential equation xydy =(y + 5)dx, given that y(5) = 0 Solution: $\int\frac{y}{y+5}dy=\int\frac{dx}{x}$ ⇒ $\int\frac{(y+5)-5}{y+5}dy=\int\frac{dx}{x}$ ⇒ $\int\{1-\frac{5}{y+5}\}dy=\int\frac{dx}{x}$ ⇒y – 5log|y + 5| = log|x| + C Given when x = 5, y = 0 ⇒0 – 5log5 = log5 +C ⇒C = -6log5

	Substituting in (i), we get y – 5log|y + 5| = log|x| - 6log5 is required solution.
7	For the differential equation, $\sqrt{a+x}\frac{dy}{dx}+x=0$ Find the general solution Solution: dy =$\frac{-x}{\sqrt{a+x}}dx \Rightarrow \int dy = -\int \frac{x}{\sqrt{a+x}}dx$ $\Rightarrow \int dy = -\int \left(\sqrt{a+x}-\frac{a}{\sqrt{a+x}}\right)dx$ $\Rightarrow$y = -$\frac{2}{3}(a+x)^{\frac{3}{2}}+2a\sqrt{a+x}$+C is the required solution.
8	For the differential equation, $e^x\sqrt{1-y^2}dx+\frac{y}{x}dy=0$ Solution: $\int xe^x dx + \int \frac{y}{\sqrt{1-y^2}}dy = 0$ $\Rightarrow xe^x - \int 1.e^x\, dx - \frac{1}{2}X\, 2\sqrt{1-y^2} = C$ $\Rightarrow$x$e^x - e^x - \sqrt{1-y^2}$ =C is the required solution.
9	Write the solution of differential equation $\frac{dy}{dx}=2^{-y}$ Solution: Given $\frac{dy}{dx}=2^{-y} \Rightarrow \int 2^y dy = \int dx$ $\Rightarrow \frac{2^y}{log_e 2} = x + C$ is the required solution
10	Find the general solution of the differential equation $$(x-y^3)dy+ydx=0$$ Solution: $\frac{dx}{dy}=\frac{y^3-x}{y}$ $\Rightarrow \frac{dx}{dy}+\frac{1}{y}.x=y^2; I.F.=e^{\int\frac{1}{y}dy}=y$ Solution is $y.x$ = $\int y.y^2 dy = \int y^3 dy$ $\Rightarrow$yx = $\frac{y^4}{4}+C$ $\Rightarrow x = \frac{y^3}{4}+\frac{C}{y}$
11.	Find the general solution of the differential equation $$(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}$$ Solution: $\frac{dy}{dx}+\frac{2x}{x^2-1}.y=\frac{2}{(x^2-1)^2}$ I.F. = $e^{\int\frac{2x}{x^2-1}dx}=e^{log(x^2-1)}=(x^2-1)$ Solution is $(x^2-1)y=\int\frac{2}{x^2-1}dx$ $\Rightarrow(x^2-1)y=log\left\|\frac{x-1}{x+1}\right\|+C$
12.	Find the particular solution of differential equation x$\frac{dy}{dx}+y=x^3$, given that y = 1, when x = 2 solution: $\frac{dy}{dx}+\frac{1}{x}y=x^2, I.F.= e^{\int\frac{1}{x}dx} = e^{logx}=x$

	Solution x.y = $\int x^3 dx \Rightarrow$xy =$\frac{x^4}{4} + C$ given y = 1, x = 2 ∴2 = 4 + C ⇒C = -2 y = $\frac{x^3}{4} - \frac{2}{x}$ is the particular solution
13.	Find the general solution of the differential equation $$\frac{dx}{dy} + x = 1 + e^{-y}.$$ Solution: $\frac{dx}{dy} + x = 1 + e^{-y}, I.F. = e^{\int 1.dy} = e^y,$ Solution is $e^y.x = \int e^y(1 + e^{-y})dy = \int(e^y + 1)dy$ $\Rightarrow e^y.x = e^y + y + C$ is the required solution.
14.	Question: Find the general solution of the differential equation: $\frac{dy}{dx} = tan^2 2x$ Solution: The given differential equation is $\frac{dy}{dx} = tan^2 2x$ On separating variable, we get: dy = $tan^2 2x$ dx $\Rightarrow \int dy = \int (\sec^2 2x - 1)dx$ ⇒y = $\frac{1}{2} tan2x - x$ + C This is the required solution.

EXERCISE

1	For the differential equation, find a particular solution satisfying the given condition $(1 + y^2)(1 + logx)dx + xdy = 0$ given that y = 1 when x = 1. Answer: Log|x| +$\frac{(logx)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$
2	Find the equation of curve passing through the point (-2,3) given that slope of the tangent to the curve at point (x,y) is $\frac{2x}{y^2}$. Answer: $\frac{y^3}{3} = x^2 + 5$
3	Solve the differential equation $x\frac{dy}{dx} = y(logy - logx + 1).$ Answer: $\log\left(\frac{y}{x}\right) = Cx$
4	Find the particular solution of the differential equation $\frac{dy}{dx} + 2ytanx = six,$ given that y = 1, when x= $\frac{\pi}{3}$ Answer: y = cosx + C$cos^2 x$
5	Find the general solution of differential equation ydx - $(x + 2y^2)dy = 0$ Answer: x = $2y^2$ + Cy

3 MARK QUESTIONS

Q.NO	QUESTIONS WITH SOLUTIONS
1	Find the particular solution of differential equation $(1+e^{2x})dy + (1+y^2)e^x dx = 0$, given that when x=0,y =1 Solution: from differential equation $\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}}dx$ For $\int \frac{e^x}{1+e^{2x}}dx = \int \frac{1}{1+t^2}dt$ $\quad \left\vert \begin{array}{l} let\ e^x=t \\ \Rightarrow e^x dx=dt \end{array} \right.$ $= tan^{-1}t = -tan^{-1}e^x$ From (i), we get $tan^{-1}y = -tan^{-1}e^x + C$ When x = 0, y= 1 $\Rightarrow tan^{-1}1 = -tan^{-1}1 + C$ $\Rightarrow$C = $\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ $\quad (e^0 = 1)$ Substituting in (ii), we get $tan^{-1}y = -tan^{-1}e^x + \frac{\pi}{2}$ $\Rightarrow tan^{-1}y + tan^{-1}e^x = \frac{\pi}{2}$ is the required solution
2	Find the particular solution of the differential of the equation $\frac{dy}{dx}$=1+x+y+xy, given that y=0, when x=1. Solution: Consider equation $\frac{dy}{dx}$=1+x+y+xy $\Rightarrow \frac{dy}{dx}$=1(1+ x) +y(1+x)= (1+x)(1+y) $\Rightarrow \frac{dy}{1+y}$=(1+x)dx on integrating both sides $\Rightarrow \int \frac{dy}{1+y} = \int (1+x)\,dx$ $\Rightarrow$log |1+y|= x+ $\frac{x^2}{2}$ + C Given y=0, when x=1 $\Rightarrow$log |1+0| =1+ $\frac{1}{2}$+ C $\quad \Rightarrow$C =$-\frac{3}{2}$ substituting in (i),we get log |1+y|= x+ $\frac{x^2}{2} - \frac{3}{2}$ is the required solution.
3	Find the particular solution of the differential equation e^xtanydx+$(2-e^x)sec^2$ydy=0, given that y = $\frac{\pi}{4}$ when x=0. Solution: consider equation e^xtanydx+$(2-e^x)sec^2$ydy=0 $\Rightarrow(2-e^x)sec^2$ydy=$-e^x$tanydx $\Rightarrow \frac{sec^2 y}{tany}dy = \frac{e^x}{e^x-2}dx$ Integrating both sides we get $\int \frac{sec^2 y}{tany}dy = \int \frac{e^x}{e^x-2}dx$ $\Rightarrow$log|tany| =log$(e^x-2) + logC$ = log$\lvert C(e^x-2)\rvert$ $\Rightarrow$tany =$C(e^x-2)$ Given y = $\frac{\pi}{4}$ when x=0. $\Rightarrow$tan$\frac{\pi}{4}$=$C(e^0-2)$

	$\Rightarrow$1= -C $\Rightarrow$ C =-1 Subsituting in (i), we get tany = -$(e^x - 2)$ or tany =$2 - e^x$ is particular solution
4	For the differential equation, find the particular solution satisfying the given condition $(1 + sin^2x)dy + (1 + y^2)cosxdx$= 0, given that when x = $\frac{\pi}{2}$, y = 0 Solution: $$\int \frac{dy}{(1+y^2)} = \int \frac{-cosx}{(1+sin^2x)}dx$$ On integrating , we get $$tan^{-1}y = -tan^{-1}(sinx) + C$$ [by substituting sinx=t] When x = $\frac{\pi}{2}$, y=0 0= - 4 +C C = Substituting in (i), we get $tan^{-1}y = -tan^{-1}(sinx) + \frac{\pi}{4}$ is requires solution
5	Solve the differential equation x$\frac{dy}{dx} = y - xtan\left(\frac{y}{x}\right)$. Solution: $\frac{dy}{dx} = \frac{y}{x} - tan\left(\frac{y}{x}\right)$, homogenous equation Let y =vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = v - tanv$ $\Rightarrow \int \frac{1}{tanv}dv = -\int \frac{1}{x}dx$ $\Rightarrow \int cotv\, dv = -\int \frac{1}{x}dx$ $\Rightarrow log\lvert sinv\rvert = -log\lvert x\rvert + logC$ $\Rightarrow Log\lvert sinv\rvert = log\left\lvert\frac{C}{x}\right\rvert$ $\Rightarrow xsin\frac{y}{x} = C$ is the required solution.
6	Sole the differential equation 2y$e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ Solution: 2y$e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ $\Rightarrow \frac{dx}{dy} = \frac{-\left(y-2xe^{\frac{x}{y}}\right)}{2ye^{\frac{x}{y}}}$ ….(i) Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ Substituting in (i) we get $v + y\frac{dv}{dy} = \frac{-(y-2vye^v)}{2ye^v} = \frac{-1+2ve^v}{2e^v}$ $\Rightarrow$ y $\frac{dv}{dy} = \frac{-1+2ve^v}{2e^v} - v$ $= \frac{-1+2ve^v-2ve^v}{2e^v} = -\frac{1}{2e^v}$

	$\Rightarrow e^v dv = -\frac{1}{2y}dy \Rightarrow \int e^v dv = -\frac{1}{2}\int \frac{1}{y}dy$ $\Rightarrow e^v = -\frac{1}{2}\log\lvert y\rvert + C$ $\Rightarrow e^{\frac{x}{y}} = -\frac{1}{2}\log\lvert y\rvert + C$ is the required equation.
7	Solve the differential equation y+x.sin$\left(\frac{y}{x}\right) = x\frac{dy}{dx}$ Solution: $\frac{dy}{dx} = \frac{y}{x} + sin\left(\frac{y}{x}\right)$ Let y =vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow$v+x$\frac{dv}{dx} = v + sinv \Rightarrow \int \frac{1}{sinv}dv = \int \frac{dx}{x}$ $\Rightarrow \int cosecv\, dv = \int \frac{dx}{x}$ $\Rightarrow$log|cosecv – cotv| = log|x| + logC $\Rightarrow$cosecv – cotv = xC $\Rightarrow \left(cosec\frac{y}{x} - cot\frac{y}{x}\right) = xC$ is the required solution.
8	Solve the differential equation $$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$ Solution: let x=vy $\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$, We get $(1 + e^v)\left(v + y\frac{dv}{dy}\right) + e^v(1 - v) = 0$ $\Rightarrow$v+v$e^v + y(1 + e^v)\frac{dv}{dy} + e^v - ve^v = 0$ $\Rightarrow \frac{dv}{dy} = \frac{-(e^v+v)}{y(1+e^v)} \quad \Rightarrow \int \frac{(1+e^v)}{e^v+v}dv = -\int \frac{1}{y}dy$ $\Rightarrow log\lvert e^v + v\rvert = -\log\lvert y\rvert + logC$ $\Rightarrow e^{\frac{x}{y}} + \frac{x}{y} = \frac{C}{y}$ $\Rightarrow ye^{\frac{x}{y}} + x = C$ is the required solution.
9	Question: Find the general solution of the differential equation: $\frac{dy}{dx} = \frac{5x+3}{2y+7}$,y$\neq -\frac{7}{2}$ Solution: $\frac{dy}{dx} = \frac{5x+3}{2y+7}$.This is a variable separable differential equation. On separation of variables, we get (2y+7)dy = (5x+3)dx $\Rightarrow \int(2y + 7)$ dy = $\int(5x + 3)$dx $\Rightarrow \frac{1}{4}(2y + 7)^2 + C_1 = \frac{(5x+3)^2}{10} + C_2$ On multiplying throughout by 20, we get: $5(2y+7)^2 + 20C_1, = 2(5x+3)^2 + 20C_2$ $\Rightarrow 5(2y+7)^2 = 2(5x+3)^2 + 20C_2 - 20C_1$ $\Rightarrow 5(2y+7)^2 = 2(5x+3)^2 + C$ This is the required solution.
10	Question: Solve the differential equation: $(x + y + 1)\frac{dy}{dx}$ =1. Solution:The given equation $(x + y + 1)\frac{dy}{dx} = 1$

	$\Rightarrow \frac{dx}{dy} = x + y + 1$ $\Rightarrow \frac{dx}{dy} - x =$y +1(L.D.E) $\therefore$ I.F. = $e^{\int -dy} = e^{-y}$ Now solution is x e^{-y}= $\int (y+1)e^{-y}$dy $\Rightarrow$ x $e^{-y} = \frac{(y+1)e^{-y}}{-1} - \int \frac{e^{-y}}{-1} dy$ $\Rightarrow$ x $e^{-y} = -(y+1)e^{-y} - e^{-y} + \text{C}$ $\Rightarrow$x = C $e^{y} - (y+2)$
11	Question: Find the particular solution of the differential equation $\frac{dy}{dx}$ + 2ytanx = sinx, given that y=0 when x= $\frac{\pi}{3}$ Solution: $\frac{dy}{dx}$ +(2tanx)y = sinx IF = $e^{\int 2tanx} dx = e^{2logsecx} = sec^2x$ y $sec^2x = \int sec^2x.sinxdx\ + C$ =$\int \frac{sinx}{cos^2x} dx + c$ = $\int tanxsecxdx + C$ ysec^2x= secx +C
12	Question: Find the general solution of the differential equation $\frac{dy}{dx} - y = sinx$ Solution: Given differential equation is $\Rightarrow$ IF = $e^{\int(-1)dx} = e^{-x}$ $\therefore$Solution is y. $e^{-x} = \int sinx\,.e^{-x}dx = \text{I}_1$ $\Rightarrow \text{I}_1 = -sinx.e^{-x} + \int cosxe^{-x}dx$ $= -sinx.e^{-x} + [-cosxe^{-x} - \int sinxe^{-x}dx]$ $\Rightarrow \text{I}_1 = \frac{1}{2}[-sinx - cosx]e^{-x}$ $\therefore$Solution is y. $e^{-x} = \frac{1}{2}$(-sinx-cos x) e^{-x}+c Or y =$-\frac{1}{2}$(sin x+ cos x) + c. e^{-x}

EXERCISE

1	Solve the differential equation $\left(y - x\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right)$. Answer: y = C(x + a)(1 – ay)
2	For the differential equation, finf a particular solution satisfying the given condition x$(x^2-1)\frac{dy}{dx} = 1; y = 0$ when x = 2 Answer: y =$\frac{1}{2}log\left\|1 - \frac{1}{x^2}\right\| - \frac{1}{2}log\frac{3}{4}$

3	Solve the differential equation xdy -ydx = $\sqrt{x^2+y^2}dx$. Answer: y + $\sqrt{x^2+y^2}$ = Cx^2
4	Show that the differential equation $\left(xe^{\frac{y}{x}} + y\right) dx = xdy$ is homogenous. Find the particular solution of this differential equation, given that x = 1 when y = 1. Answer: log|x| + $e^{\frac{y}{x}} = \frac{1}{e}$
5	Find the particular solution of the differential equation dy = cosx(2 – ycosecx)dx, given that y = 2, when x =$\frac{\pi}{2}$. Answer: sinx.y = -$\frac{1}{2}cos2x + \frac{3}{2}$

5 MARK QUESTIONS

Q.NO	QUESTIONS WITH SOLUTIONS
1	Question: Find the particular solution of the differential equation: $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ given $y = 1, when\ x = 1$ Solution: The given differential equation is $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ $\Rightarrow \frac{dy}{dx} = (1 + x^2) + y^2(1 + x^2)$ $\Rightarrow \frac{dy}{dx} = (1 + x^2)(1 + y^2)$ It is the variable separable differential equation $\Rightarrow \int \frac{dy}{(1+y^2)} = \int (1 + x^2)dx$ $\Rightarrow tan^{-1}y = \frac{x^3}{3}$ + x + C $\quad\quad \Rightarrow$ C = $\frac{\pi}{4} - \frac{4}{3}$ The particular solution of the given variable separable differential equation is $tan^{-1}y = \frac{x^3}{3} + x + \frac{\pi}{4} - \frac{4}{3}$
2	Question: Solve: (xlog x)$\left(\frac{dy}{dx}\right)$ + y = $\frac{2logx}{x}$ Solution: Give differential equation: (x log x)($\frac{dy}{dx}$) + y = $\frac{2logx}{x}$ $\Rightarrow \frac{dy}{dx} + \left(\frac{1}{xlogx}\right) y = \frac{2}{x^2}$ This is a linear equation of the form: $\frac{dy}{dx}$ + *f(x)*y = g(x) I.F. = $e^{\int \frac{1}{xlogx}dx} = e^{\log(\log x)}$ = log x Now the solution of the differential equation will be: y(I.F.) = ∫ g(x)(I.F.)dx ⇒ y(I.F) = $\int \frac{2logxdx}{x^2} = \int$(logx)($2x^{-2}$) dx

	Now on integrating RHS by parts, we get ylogx= log x$\left(\frac{2x^{-1}}{-1}\right)$ - $\int \frac{1}{x}\left(\frac{2x^{-1}}{-1}\right) dx$ ⇒ylog.x = $\frac{-2logx}{x} + \int 2x^{-2}$dx ⇒ylogx = $\frac{-2logx}{x} + \ 2\frac{x^{-1}}{-1} + C$ ⇒ylogx = $\frac{-2logx}{x} - \frac{2}{x} + C$
3	Question: Find the particular solution of the differential equation $e^{x}tany\,dx + (2 - \ e^{x})sec^{2}ydy = 0$ given that y = $\frac{\pi}{4}$ when x = 0 Solution: $e^{x}tanydx = (e^{x} - 2)sec^{-2}y\ dy$ = 0 ⇒ $\int \frac{sec^{2}y}{tany}dy$ = $\int \frac{e^{x}}{(e^{x}-2)}dx$ Put $tany\ =\ t$ ⇒ $sec^{2}xy\ dy = dt$ ⇒$\int \frac{dt}{t} = \int \frac{e^{x}}{(e^{x}-2)}dx$ Again put $(e^{x} - 2) = u$ ⇒ $e^{x}dx = du$ ⇒ $logt\ =\ log\,u\ +\ logC$ ⇒$log(tany)\ =\ log[(e^{x} - 2)C]$ ⇒tany =C$(e^{x} - 2)$ Put y = $\frac{\pi}{4}$ ⇒tan$\frac{\pi}{4}$ =C(1-2) ⇒ C = -1 ⇒tany = $-(e^{x} - 2)$
4	Question: Solve the differential equation $(tanx^{-1}x - y\ )dx$ = (1+ $x^{2})dy$ Solution: Given differential equation can be written as (1+x²) $\frac{dy}{dx} + y =\ tan^{-1}$x ⇒ $\frac{dy}{dx}$ + $\frac{1}{1+x^{2}}$ y = $\frac{tan^{-1}x}{1+x^{2}}$ ⇒I.F. = $e^{\int\left(\frac{1}{1+x^{2}}\right)dx}$ = $e^{tan^{-1}x}$ ∴Solution is y. $e^{tan^{-1}x} = \int tan^{-1}x.\,e^{tan^{-1}x}.\frac{1}{1+x^{2}}dx$ ⇒ y. $e^{tan^{-1}x} =\ e^{tan^{-1}x}.(tan^{-1}$x -1) + C Or y = $(tan^{-1}$x -1) + C . $e^{-tan^{-1}x}$
5	Question: Find the particular solution of the differential equation 2y $e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy$ = 0 given that $x = 0\ when\ y\ =\ 1$. Solution: the given differential equation 2y $e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy$ = 0 On dividing Nr and Dr of RHS by y,we get $\frac{dx}{dy} = \frac{-\left[1-2\left(\frac{x}{y}\right)e^{\left(\frac{x}{y}\right)}\right]}{2e^{\left(\frac{x}{y}\right)}}$

	It is homogenous differential equation of second type ∴Put x =vy and $\frac{dx}{dy} = v + y\left(\frac{dv}{dy}\right)$ v + y$\left(\frac{dv}{dy}\right) = \frac{2ve^{v}-1}{2e^{v}}$ ⇒ y$\left(\frac{dv}{dy}\right) = \frac{2ve^{v}-1}{2e^{v}} - v$ ⇒ y$\left(\frac{dv}{dy}\right) = -\frac{1}{2e^{v}}$ ⇒2 $\int e^{v}$dv = $\int \frac{dy}{y}$ ⇒ 2 $e^{v} = -log\|y\| + C$ ⇒ 2 $e^{\frac{x}{y}} = -log\|y\| + C$ Put x = 0 and y =1 $2e^{0} = -\log 1 + C$ ⇒ 2 = 0+ C ⇒C =2 ∴Particular solution of given homogenous differential equation is ⇒ 2 $e^{\frac{x}{y}} = -log\|y\| + 2$
6	Question: Solve: (1 + x^2) $\frac{dy}{dx}$ + 2xy = $4x^2$ subject to the initial condition y(0) = 0 Solution: The given differential equation: (1 + x^2) $\frac{dy}{dx}$ + 2xy = $4x^2$ ⇒ $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{4x^2}{x^2+1}$ It a linear differential equation of the form $\frac{dy}{dx} + f(x)y = g(x)$ Where f(x) = $\frac{2x}{x^2+1}$ and g(x) = $\frac{4x^2}{x^2+1}$ I.F. = $e^{\int f(x)dx} = e^{\int\left(\frac{2x}{x^2+1}\right)dx} = e^{\log(x^2+1)}$ ⇒ I.F. = $x^2 + 1$ ∴Solution of the given linear differential equation y(I.F.) = $\int g(x)(\text{I.F.})$dx y$(x^2 + 1) = \int 4x^2 dx = \frac{4x^2}{3} + C$ ⇒ y$(x^2 + 1) = \frac{4x^2}{3}$ + C To find the value of C, put x=0 and y =0 in the above equation 0(0+1) = $\frac{4 X 0}{3} + C$ ⇒ $\text{C} = 0$ ⇒ y$(x^2 + 1) = \frac{4x^2}{3}$
7	Question: Find the particular solution of the differential equation $e^{x}\sqrt{1-y^2}dx + \left(\frac{y}{x}\right)dy = 0\ , x = 0, y = 0.$ Solution: The given differential equation is: $e^{x}\sqrt{1-y^2}dx + \left(\frac{y}{x}\right)dy = 0$ ⇒ $\left(\frac{y}{x}\right)dy = -\,e^{x}\sqrt{1-y^2}dx$ ⇒$\left(\frac{y}{\sqrt{1-y^2}}\right)dy = -xe^{x}$dx

	$\Rightarrow\int\left(\frac{y}{\sqrt{1-y^2}}\right)dy = -\int xe^x\,dx$ $\Rightarrow -\frac{1}{2}\int\frac{-2ydy}{\sqrt{1-y^2}} = -\int xe^x$dx $\Rightarrow\sqrt{1-y^2} = xe^x - e^x + C$ To find the particular solution put $x = 0$ and $y = 0$ in the above equation $\sqrt{1-0} = 0e^0 - e^0 + C \qquad \Rightarrow \text{C} = 2$ ⇒the particular solution is $\sqrt{1-y^2} = xe^x - e^x + 2$
8	Question: Find the particular solution of the differential equation $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$,given y =1, when x = 1 Solution: The given differential equation is $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$ $\Rightarrow x^2(y-1)dy = y^2(x^2+1)dx$ $\Rightarrow\int\frac{(y-1)}{y^2}dy = \frac{(x^2+1)}{x^2}dx$ $\Rightarrow\int\left[\frac{1}{y}-\frac{1}{y^2}\right]dy = \int\left[1+\frac{1}{x^2}\right]dx$ ⇒logy + $\frac{1}{y} = x - \frac{1}{x} + C$ To find the particular solution, Put x = 1 and y = 1 in equation log(1) + $\frac{1}{1} = 1 - \frac{1}{1} + C$ ⇒ 0 +1 = C ⇒C =1 The particular solution of the given differential equation is logy + $\frac{1}{y} = x - \frac{1}{x} + 1$
9	Question: Find the particular solution of the differential equation: log $\left(\frac{dy}{dx}\right) =$ $3x + 4y,$ given y =0, when x = 0 Solution: The given differential equation is log$\left(\frac{dy}{dx}\right) = 3x + 4y$ $\Rightarrow \frac{dy}{dx} = e^{3x+4y} = e^{3x}.e^{4y}$ $\Rightarrow\int e^{-4y}dy = \int e^{3x}\,dy$ $\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$ $\Rightarrow \frac{-1}{4e^{4y}} = \frac{e^{3x}}{3} + C$ $\Rightarrow\frac{1}{4e^{4y}} + \frac{e^{3x}}{3} = C$ To find the value of C ,put x = 0 and y = 0 in equation

	$\frac{1}{4e^0} + \frac{e^0}{3} = C$ $\Rightarrow \frac{1}{4} + \frac{1}{3} = C$ $\Rightarrow$ C $= \frac{7}{12}$ Hence $\frac{1}{4e^{4y}} + \frac{e^{3x}}{3} = \frac{7}{12}$ is the particular solution of given variable separable equation.
10	Question: For the differential equation xy $\left(\frac{dy}{dx}\right)$ = (x+2)(y+2), find the solution curve passing through the point (1,-1) Solution: The given differential equation xy $\left(\frac{dy}{dx}\right)$ = (x+2)(y+2) It is a variable separable differential equation. $\therefore \int\left(\frac{y}{y+2}\right)dy = \int\left(\frac{x+2}{x}\right)dx$ $\Rightarrow \int\left(\frac{(y+2-2)}{y+2}\right)dy = \int\left(\frac{x+2}{x}\right)dx$ $\Rightarrow \int\left(1-\frac{2}{y+2}\right)dy = x + 2logx + C$ $\Rightarrow$ y -2log(y+2) = x+2log(x) + C This curve passing through a point (1, -1) Put x =1 and y = -1 in it. -1 -2log (1) = 1 + 2log(1) + C $\Rightarrow$ -1 – 0 =1 + 2 $\times$ 0 + C $\Rightarrow$ -1 = 1+ C $\Rightarrow C = -2$ On putting $C = -2$ in the curve equation: $y - 2log\|y+2\| = x + 2log\|x\| - 2$

EXERCISE

1	Find the general solution of differential equation $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x$ Answer: y = $\frac{1}{2}$log$\|x+1\| + \frac{3}{4}log\|x^2+1\| - \frac{1}{2}tan^{-1} x + C$
2	Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy-x^2}$ is homogenous and also solve it.

	Answer: $\frac{y}{x} - \log\lvert y\rvert = C$
3	Find the particular solution of the differential equation $\tan x \,.\frac{dy}{dx} = 2x\tan x + x^2 - y;\ (\tan x \neq 0)$ given that y = 0 when x =$\frac{\pi}{2}$. Answer: $\sin x.y = x^2\sin x - \frac{\pi^2}{4}$

CASE BASED QUESTIONS

Q.NO	QUESTIONS AND SOLUTIONS
1	Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$where x denotes the number of weeks and y the number of children who have been given the drops. (a) State the order of the above given differential equation. (b) Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50 - y)$? (c) the solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ is given by, (i) $\log\lvert 50 - y\rvert = kx + C$ (ii) $-\log\lvert 50 - y\rvert = kx + C$ (iii) $\log\lvert 50 - y\rvert = \log(\lvert kx\rvert) + C$ (iv) $50 - y = kx + C$ (d) The value of c in the particular solution given that $y(0) = 0$ *and* $k = 0.049$ is (i) log 50 (ii) $\log\left(\frac{1}{50}\right)$ (iii) 50 (iv) - 50 **Solution:** (a) Order is 1 (b) (i), Variable separable method (c) (ii), $-\log\lvert 50 - y\rvert = kx + C$ (d) (ii), $\log\frac{1}{50}$
2	An equation involving variables as well as derivative of the dependent variable with respect to only one independent variable is called an ordinary differential equation. e.g. $\frac{dy}{dx} + \frac{d^2y}{dx^2}$-2=0

From any given relation between the dependent and independent variables, a differential equation can be formed by differentiating it with respect to the independent variable and eliminating arbitrary constants involved.

(a) The degree of the differential equation
$\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$ is

(i) 1 (ii) 3 (iii) 2 (iv) 4

(b) The order of differential equation

(i) 1 (ii) 2 (iii)4 (iv)3

(c) The number of arbitrary constants in general solution of differential equation of third order is

(i) 0 (ii)2 (iii)3 (iv)4

(d) The degree of differential equation
$x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$ is

(i) 1 (ii) 0 (iii) 4 (iv) Is not defined

Solution: (a)-(i), The highest order derivative is $\frac{d^2y}{dx^2}$ whose degree is one. So, degree of differential equation is 1.

(b) - (iv), The highest order derivative present in the differential equation is y'''. Therefore, its order is 3.

(c) - (iii), We know that the number of arbitrary constant in the general solution of a differential equation of order n is equal to its order.
Therefore, the number of constants in the general equation of the third-order differential equation is three.

(d) - (iv), as differential equation can't be written as polynomial of derivative.

3 A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F.The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using *Newton law of cooling which is governed by the differential equation*: $\frac{dT}{dt} \propto$(T-70), where 70°F is the room temperature and T is the temperature of the object at time t.
Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ where k is a constant of proportion, time of death is calculated

a) State the degree of the above given differential equation.

(b) Write method of solving a differential equation helped in calculation of the time of death?

(c) If the temperature was measured 2 hours after 11.30 pm, will the time of death change? (Yes/No)

(d) Find the solution of the differential equation
$\frac{dT}{dt} = k(T - 70)$

(e) If t = 0 when T is 72, then find the value of c

Solution: (a) Degree is 1, (b) Variable separable method, (c) No,
(d) $log|T - 70| = kt + C, (e)\ log2$

4 A function of the form y = ∅(x) + C which satisfies given differential equation, is called the solution of the differential equation. The solution which contains as many arbitrary constants as the order of the differential equation, is called the general solution of the differential equation.

A solution obtained by giving particular values to arbitrary constants in the general solution of a differential equation, is called the particular solution.

(a) y^2= 4ax is solution of differential equation

(i) $yy' = xyy'' + xy'^2$ (ii) $xy' = xy' + xy''$
(iii) $2xy' - y = 0$ (iv) $xy'' + y' = xy$

(b)The solution of the differential equation $sec^2x.tanydx + sec^2ytanxdy = 0$ is

(i) $tanx = C$ (ii) $tanx.tany = C$
(iii) $tanx + tany = C$ (iv) $\frac{tanx}{tany}$ = C

(c) The genral solution of differential equation
(i) $\frac{\tan^{-1} y}{\tan^{-1} x} = C$
(ii) $\tan^{-1} x . \tan^{-1} y = C$
(iii) $\frac{\tan^{-1} x}{\tan^{-1} y} = C$
(iv) $\tan^{-1} y = \tan^{-1} x + C$

(d) The solution of differential equation
$e^x \tan y\, dx + (1 + e^x)sec^2ydy = 0$

(i) $\tan x = C(1 - e^y)$
(ii) $\tan y = C(1 - e^y)$
(iii) $\tan y = C(1 - e^x)$
(iv) $\tan y = C(1 + e^x)^{-1}$

Solution: (a) - (iii) ,(b) – (ii), (c) – (iii), (d) – (iv)

5	Friends are revising differential equations $\frac{dy}{dx} = \frac{y^2}{xy-x^2}$, answered. few questions are required to be (i) Which method is used to solve the given differential equation? (ii) What is the degree of the given differential equation? (iii) Find the general solution of the given differential equation. Solution: (i) Method of homogenous differential equation, (ii) degree =1 ,(iii) $\frac{y}{x} - \log\lvert y\rvert = C$
6	Two friend together are preparing for board examination and they were revising differential equations. They were asking each other one by one concepts related to differential equations, then one of the friends asked how to solve the differential equation, (2x-5y+3)dx + (4x-10y-9)dy = 0. : During conversation they came across the following questions (i)Differential equations can be solved using (a) seperating the variables (b) method for solving homogeneous equations (c) method for solving linear differential equations of first order (d) using substitution method (ii) We can start with (a) separating variables (b) substituting y = vx (c) finding integrating factor (d) substituting 2x - 5y = t (iii) $\int \frac{2x-9}{3x-1} dx$ is equal to (a) $\frac{2}{3} - \frac{25}{9} log\lvert 3x-1\rvert + C$ (b) $\frac{2}{3}x - \frac{25}{9} log\lvert 3x-1\rvert + C$ (c) $\frac{2}{3} - \frac{25}{3} log\lvert 3x-1\rvert + C$ (d) $\frac{2}{3}x - \frac{25}{3} log\lvert 3x-1\rvert + C$ (iv) $\int 3\,.dx$ is equal to (a) 3 +C (b) Log3 +C (c) 3x +C (d) $\frac{3^2}{2} + C$ (v) solution of differential equation is (a) $\frac{2}{9}(2x-5y) - \frac{25}{21} log\lvert 18x-45y-3\rvert = x + C$ (b) $(2x-5y) - log\lvert 18x-45y-3\rvert = x + C$

	(c) $\frac{2}{9}(2x-5y)-log\|18x-45y-3\|=x+C$ (d) $\frac{2}{9}(2x-5y)-\frac{25}{27}log\|18x-45y-3\|=C$ Solution: (i)-(d), (ii) – (d), (iii) – (b), (iv) – (c), (v) – (a)
7	As during COVID-19 board examinations have been postpond, so friends are revising the syllabus again and again and the topic in question is differential equations and given differential equation is $(1+y^2)$dx = $(\tan^{-1}y-x)$dy with the above information answer the following: (i) What is the degree of the differential equation? (a) not defined (b) 0 (c) 1 (d) 2 (ii) The differential equation can be solved using the method of solution by (a) seperating the variables (b) using method for linear differential equation of type $\frac{dy}{dx}$ + P(x).y = Q(x) (c) using method for linear differential equation of the type $\frac{dx}{dy}$ + P(y).x = Q(y) (d) using method for homogeneous equation (iii) To solve the differential equation, it can be written as (a) $\frac{dy}{dx}=\frac{1+y^2}{\tan^{-1}y-x}$ (b) $\frac{dx}{dy}=\frac{\tan^{-1}y-x}{1+y^2}$ (c) $\frac{dx}{dy}+\frac{1}{1+y^2}x\ =\tan^{-1}y$ (d) $\frac{dy}{1+y^2}=\frac{dx}{\tan^{-1}y-x}$ (iv) For the solution the differential equation integrating factor is (a) $\tan^{-1}y$ (b) $e^{\tan^{-1}y}$ (c) $e^{\frac{1}{1+y^2}}$ (d) $e^{-\tan^{-1}y}$ (v) Solution is (a) x = $\tan^{-1}y-1+C.e^{-\tan^{-1}y}$ (b) y = $\tan^{-1}x-1+C.e^{-\tan^{-1}x}$ (c) $\tan^{-1}y=log(\tan^{-1}y-x)+C$ (d) $y=\tan^{-1}x+\ C$ Solution: (i) – (c), (ii) –(c), (iii) – (b), (iv) – (b), (v) – (a)

CHAPTER: VECTORS

SYLLABUS: *Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.*

Definitions and Formulae:

Scalar: those physical quantities which have only magnitude are called scalar

for example: time period, distance, work done, area, volume, mass, density, speed, temperature, money voltage, resistance etc

Vector: Those physical quantities which are defined by both magnitude and direction are called vector

For example: force, velocity, acceleration, displacement, weight, momentum, electric field intensity etc

Zero vector: a vector whose initial and terminal points are same is called a zero vector or null vector and is denoted by $\vec{0}$

Co initial vectors: two or more vectors having the same initial point is called coinitial vectors

Collinear vectors: two or more vectors are said to be collinear if they are parallel to the same line irrespective of their magnitudes

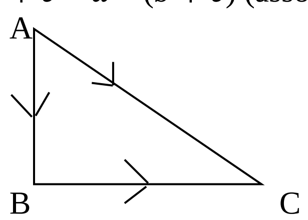
Equal vectors: two vectors $\vec{a}$and$\vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points and written as $\vec{a} = \vec{b}$

Negative of a vector: a vector whose magnitude is the same as that of a given vector but direction is opposite to that of it is called negative of the given vector.

$\overrightarrow{BA}$ is the negative of $\overrightarrow{AB}$

Important Properties

1. For any two vectors $\vec{a}$ and $\vec{b}$, $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative property)
2. For any three vectors $\vec{a}$, $\vec{b}$ and$\vec{c}$, $(\vec{a} + \overrightarrow{b)} + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (associative property)
3. Triangle law of vector addition:



In the given triangle $\overrightarrow{AC} = \overrightarrow{AB}+\overrightarrow{BC}$

4. If $\vec{r}$ = x$\hat{\imath}$ + y$\hat{\jmath}$ +z$\hat{k}$ then the magnitude of $\vec{r}$ is $\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$

5. Let $\vec{a}$ be a given vector and λ a scalar. Then the product of the vector $\vec{a}$ by the scalar λ is denoted as $\lambda\vec{a}$ and $\left|\lambda\vec{a}\right| = \left|\lambda\right| \left|\vec{a}\right|$

6. The unit vector of $\vec{a}$ is $\hat{a} = \frac{\vec{a}}{\left|\vec{a}\right|}$

7. Direction ratios if $\vec{r}$ = x$\hat{\imath}$ + y$\hat{\jmath}$ +z$\hat{k}$ then x , y , z are the direction ratios of vector $\vec{r}$

8. Direction cosines: the direction cosines of $\vec{r}$ are denoted by l, m, n (cosα, cosβ, cosγ)

$l = \frac{x}{\left|\vec{r}\right|}$, m= $\frac{y}{\left|\vec{r}\right|}$, $n = \frac{z}{\left|\vec{r}\right|}$

9. The product of two vectors is commutative $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

10. If l ,m ,n are the direction cosines then $l^2+m^2+n^2 = 1$

11. $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

12. $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$

13. $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{k} \times \hat{\jmath} = -\hat{\imath} = \hat{\imath} \times \hat{k} = -\hat{\jmath}$

14. $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

15. $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\ \boldsymbol{cos}\Theta$

16. $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\ \boldsymbol{sin}\Theta\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ *and* $\vec{b}$,

such that $\vec{a}, \vec{b}, \hat{n}$ form right handed system of coordinate axes

MULTIPLE CHOICE QUESTIONS

S.NO	QUESTIONS AND ANSWERS
1	Which of the following is correct? a) $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$ b) $\vec{a} \times \vec{a} = \vert\vec{a}\vert^2$ c) $\vec{a} \times \vec{a} = \vert\vec{a}\vert$ d) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$ **Solution:** d) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$
2	Two vectors $\vec{a}$ and $\vec{b}$ are parallel if a) $\vec{a}\times\vec{b} = 0$ b) $\vec{a}.\vec{b} = 0$ c) $\vert\vec{a}\times\vec{b}\vert = \vert\vec{a}\vert\vert\vec{b}\vert$ d) None of the above **Solution: a)** $\vec{a}\times\vec{b} = \vert\vec{a}\vert\vert\vec{b}\vert\ \boldsymbol{sin}\Theta$ if $\Theta = 0$ then, $\vec{a}\times\vec{b} = \vert\vec{a}\vert\vert\vec{b}\vert$ sin0 $\therefore\ \vec{a}\times\vec{b} = 0$
3	The unit vector in the direction of $\vec{a}$= 3$\hat{\imath}$ - 2$\hat{\jmath}$ + 6$\hat{k}$ a) $\frac{1}{49}$ (3$\hat{\imath}$ - 2$\hat{\jmath}$ + 6$\hat{k}$) b) $\frac{1}{7}$ (3$\hat{\imath}$ - 2$\hat{\jmath}$ + 6$\hat{k}$) c) 7(3$\hat{\imath}$ - 2$\hat{\jmath}$ + 6$\hat{k}$) d)49(3$\hat{\imath}$ - 2$\hat{\jmath}$ + 6$\hat{k}$) **Solution:** b)

	if $\vec{a}$= $3\hat{\imath}$ - $2\hat{\jmath}$ + $6\hat{k}$ $then\ \hat{a} = \frac{\vec{a}}{\lvert\vec{a}\rvert} = \frac{3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}}{\sqrt{3^2+(-2)^2+6^2}}$ $= \frac{1}{7}$ ($3\hat{\imath}$ - $2\hat{\jmath}$ + $6\hat{k}$)
4	If $\vec{a}$ is unit vector and if ($\vec{x}$ - $\vec{a}$). ($\vec{x}$+$\vec{a}$) = 15 then $\lvert\vec{x}\rvert$ is a) 16 b) 6 c) 4 d)1 **Solution:** c) ($\vec{x}$ - $\vec{a}$). ($\vec{x}$+$\vec{a}$) = 15 $\therefore$ $\lvert\vec{x}\rvert^2$ - $\lvert\vec{a}\rvert^2$=15 $\lvert\vec{x}\rvert^2$ – 1 =15 { since $\vec{a}$ is unit vector} $\lvert\vec{x}\rvert^2$=16 $\lvert\vec{x}\rvert$ = 4
5	The direction cosines of the vector -$2\hat{\imath}$+ $\hat{\jmath}$-$5\hat{k}$ are a) $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$ b) $\frac{-2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$ c) $\frac{2}{-\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$ d) $\frac{-2}{\sqrt{30}}, \frac{1}{-\sqrt{30}}, \frac{5}{-\sqrt{30}}$ **Solution:** c) The direction cosines of a vector $\vec{r}$ =x$\hat{\imath}$ + y$\hat{\jmath}$ + z$\hat{k}$ are : $l = \frac{x}{\lvert\vec{r}\rvert}$, m= $\frac{y}{\lvert\vec{r}\rvert}, n = \frac{z}{\lvert\vec{r}\rvert}$ So the DC's of -$2\hat{\imath}$+ $\hat{\jmath}$-$5\hat{k}$ are l = $\frac{-2}{\sqrt{(-2)^2+1^2+(-5)^2}}$,m= $\frac{1}{\sqrt{(-2)^2+1^2+(-5)^2}}, n =$ $\frac{-5}{\sqrt{(-2)^2+1^2+(-5)^2}}$ $= \mathbf{\frac{2}{-\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}}$
6	If the vectors $2\hat{\imath}$-3 $\hat{\jmath}$+ $4\hat{k}$ and a$\hat{\imath}$+ 6 $\hat{\jmath}$- $8\hat{k}$ are collinear then the value of a is a) -4 b) 4 c) 2 d) -2 **Solution:** a) if the vectors $2\hat{\imath}$-3 $\hat{\jmath}$+ $4\hat{k}$ and a$\hat{\imath}$+ 6 $\hat{\jmath}$- $8\hat{k}$ are collinear then $\frac{2}{a} = \frac{-3}{6} = \frac{4}{-8}$ Or $\frac{2}{a} = \frac{-3}{6}$ => $a = -4$
7	The position vector of the midpoint of the vector joining the points $P(2,3,4)\ and\ Q(4,1,-2)$ is a) $6\hat{\imath}$+ 4 $\hat{\jmath}$- $6\hat{k}$ b) $3\hat{\imath}$+ $2\hat{\jmath}$- $2\hat{k}$ c) $6\hat{\imath}$+ $4\hat{\jmath}$+ $6\hat{k}$ d) $3\hat{\imath}$+ $2\hat{\jmath}$+ $\hat{k}$ **Solution:** d) The position vector of the midpoint of the vector joining the points $P(2,3,4)$ and Q(4,1,-2) is $\frac{2+4}{2}\hat{\imath} + \frac{3+1}{2}\hat{\jmath} + \frac{4-2}{2}\hat{k}$ = $3\hat{\imath}$+ $2\hat{\jmath}$+ $\hat{k}$
8	The scalar components of the vector $\overrightarrow{AB}$ with initial point $A(2,1)$ and terminal point $B(1,0)$ are a) $1 , 1$ b) $-1 , -1$ c) $1 , -1$ d) $-1 , 1$ **Solution:** b) The vector $\overrightarrow{AB}$ with initial point A(2,1) and terminal point B(1,0) is $\overrightarrow{AB} = (1-2)\ \hat{\imath} + (0-1)\hat{\jmath}$ = -1$\hat{\imath}$ -1$\hat{\jmath}$ so the scalar components of $\overrightarrow{AB}$ are $-1, -1$

9	If $\vec{a} = 6\hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\jmath} - \hat{k}$ then $\vec{a} + \vec{b} - \vec{c}$ is a) $6\hat{\imath} + 4\hat{\jmath} - \hat{k}$ b) $7\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ c) $7\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ d) $7\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ **Solution:** c) if $\vec{a} = 6\hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\jmath} - \hat{k}$ $\therefore \vec{a} + \vec{b} - \vec{c} = 6\hat{\imath} + \hat{\jmath} + \hat{\imath} + 2\hat{\jmath} + \hat{k} - \hat{\jmath} + \hat{k}$ $= 7\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
10	The value of $(\hat{\imath} \times \hat{\jmath}).\hat{k} + \hat{\imath}.\hat{\jmath} - \hat{k}.(\hat{\jmath} \times \hat{\imath})$ is a) 0 b) 1 c) -1 d) 2 **Solution:** d) $(\hat{\imath} \times \hat{\jmath}).\hat{k} + \hat{\imath}.\hat{\jmath} - \hat{k}.(\hat{\jmath} \times \hat{\imath})$ $= \hat{k}.\hat{k} + 0 + \hat{k}.\hat{k}$ $\{\because \hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{\imath}.\hat{\jmath} = 0\}$ $= 1 + 1$ $=2$

CHAPTER	VIDEO LINK FOR MCQs	SCAN QR CODE FOR VIDEO
VECTORS	https://youtu.be/t4jGZmgSilc	

EXERCISE

1	if $\vec{a} = 4\hat{\imath} - 2\hat{\jmath} - \hat{k}$ then the direction cosines of $\vec{a}$ are a) $\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{1}{\sqrt{21}}$ b) $\frac{4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}, \frac{-1}{\sqrt{21}}$ c) $\frac{4}{\sqrt{7}}, \frac{-2}{\sqrt{7}}, \frac{-1}{\sqrt{7}}$ d) None of the above **Answer: (b)** $\frac{4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}, \frac{-1}{\sqrt{21}}$
2	If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 4\hat{\imath} - 8\hat{\jmath} + 12\hat{k}$ the vectors $\vec{a}$ *and* $\vec{b}$ are a) Parallel b) Perpendicular c) Co initial vectors d) None of the above **Answer: (a)** Parallel
3	The magnitude of $\vec{b} = \hat{\imath} - 8\hat{\jmath} + 2\hat{k}$ is a) $\sqrt{69}$ b) $\sqrt{63}$ c) $\sqrt{70}$ d) $\sqrt{11}$ **Answer:** ($\boldsymbol{a}$) $\sqrt{69}$

4	The vector components of $\vec{b} = 3\hat{\imath} + \hat{\jmath} + 7\hat{k}$ are a) 3,1,7 b) 3i , j , 7k c) $3\hat{\imath}$, $\hat{\jmath}$, $7\hat{k}$ d) $\frac{3\hat{\imath}}{\sqrt{59}}, \frac{\hat{\jmath}}{\sqrt{59}}, \frac{7\hat{k}}{\sqrt{59}}$ **Answer:**(**c**) $3\hat{\imath}$, $\hat{\jmath}$, $7\hat{k}$
5	Which of the following measure is vector? a) Distance b) Time period c) Volume d) Force **Answer: (d)** Force

ASSERTION AND REASONING QUESTIONS

The following questions are of one mark each, two statements are given, one labelled Assertion(A) and the other labelled Reason(R). Select the correct answer from the codes (a),(b),(c),(d) as given below

a) Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).
b) Both Assertion(A) and Reason(R) are true but Reason(R) is the not the correct explanation of the Assertion(A).
c) Assertion(A) is true and Reason(R) is false
d) Assertion(A) is false and Reason(R) is true

1	Assertion(A): the magnitude of vector $\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ is 7 Reason(R) : if $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $\lvert\vec{r}\rvert = x^2 + y^2 + r^2$ **Answers:** c) Assertion(A) is true and Reason(R) is false
2	Assertion(A): the vector in the direction of $\vec{a} = \hat{\imath} - 2\hat{\jmath}$ with magnitude 7 units is $\frac{7}{\sqrt{5}}\hat{\imath} - \frac{14}{\sqrt{5}}\hat{\jmath}$ Reason(R) : the vector in the direction of $\vec{r}$,which has magnitude *d* units is $d.\hat{r}$ **Answers:** a) Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).
3	Assertion(A) : the direction cosines of $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ Reason(R) : $for\ any\ vector\ \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $\lvert\vec{r}\rvert = \sqrt{x^2+y^2+z^2}$ **Answers:** b) Both Assertion(A) and Reason(R) are true but Reason(R) is not the correct explanation of the Assertion(A)
4	Assertion(A) : the angle between $\vec{a}\ and\ \vec{b}$ is 60^0 .if $\lvert\vec{a}\rvert = \sqrt{3}$, $\lvert\vec{b}\rvert$ =2, $\vec{a}.\vec{b} = \sqrt{3}$ Reason(R) : $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\ \lvert\vec{b}\rvert \sin\theta$ **Answers:** c) Assertion(A) is true and Reason(R) is false
5	Assertion(A) : If either $\vec{a} = 0$ or $\vec{b} = 0$ then $\vec{a}.\vec{b} = 0$ Reason(R) : if a,b,c are the direction ratios of the vector then $a^2+b^2+c^2 = 1$ **Answers:** c) Assertion(A) is true and Reason(R) is false
6	Assertion(A): for any two vectors $\vec{a}$ and $\vec{b}$ with $\lvert\vec{a}\rvert \neq 0 \neq \lvert\vec{b}\rvert$ we always have $\lvert\vec{a}+\vec{b}\rvert \leq \lvert\vec{a}\rvert\ \lvert\vec{b}\rvert$ Reason(R): $\vec{a}.\vec{a} = \lvert\vec{a}\rvert^2$

	Answers: d) Assertion(A) is false and Reason(R) is true
7	Assertion(A): the direction cosines of a vector equally inclined to the axes OX, OY, OZ are 1,1,1 Reason(R): the direction cosines of a vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is $l = \frac{x}{\lvert\vec{r}\rvert}$, $m = \frac{y}{\lvert\vec{r}\rvert}$, $n = \frac{z}{\lvert\vec{r}\rvert}$ **Answers:** d) Assertion(A) is false and Reason(R) is true
8	Assertion(A): $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2$,if and only if $\vec{a},\vec{b}$ are perpendicular, $\vec{a} \neq 0, \vec{b} \neq 0$. Reason(R) : $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ **Answers:** b) Both Assertion(A) and Reason(R) are true but Reason(R) is not the correct explanation of the Assertion(A).
9	Assertion(A): the vector $\vec{r}$ of magnitude $3\sqrt{2}$ units, which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y- and z- axes respectively is $\pm 3\hat{\imath} + 3\hat{\jmath}$ Reason(R): $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\,\lvert\vec{b}\rvert\, cos\Theta$ **Answers:** b) Both Assertion(A) and Reason(R) are true but Reason(R) is not the correct explanation of the Assertion(A).
10	Assertion(A): the unit vector in XY-Plane, making an angle of 30^0 with positive direction of x-axis is $\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$ Reason(R): $\hat{r} = \cos30^0\hat{\imath} + \sin30^0\hat{\jmath}$ **Answers:** a) Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).

EXERCISE

1	Assertion(A) : If $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 7\,\hat{k}$ and $\vec{b} = 4\hat{\imath} + 2\hat{\jmath} - 2\,\hat{k}$ then $\vec{a}.\vec{b} = 0$ Reason(R) :$\hat{\imath}.\hat{\imath} = 1$, $\hat{\jmath}.\hat{\jmath} = 1$, $\hat{k}.\hat{k} = 1$ **Answer:** Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).
2	Assertion(A) : the unit vector of $\hat{\imath} + \hat{\jmath} + \hat{k}$ is $\frac{\mathbf{1}}{\mathbf{1}}(\hat{\imath} + \hat{\jmath} + \hat{k})$ Reason (R) :$\hat{a} = \frac{\vec{a}}{\lvert\vec{a}\rvert}$ **Answer:** Assertion(A) is false and Reason(R) is true
3	Assertion(A) :$\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $3\hat{\imath} + 9\hat{\jmath} - 3\hat{k}$ are collinear vectors Reason (R) : two collinear vectors are always equal in magnitude **Answer:** Assertion(A) is true and Reason(R) is false
4	Assertion(A) : the unit vector in YZ-Plane, making an angle of 45^0 with positive direction of x-axis is $\frac{1}{\sqrt{2}}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$ Reason(R) :$\hat{r} = \sin45^0\hat{\jmath} + \cos45^0\hat{\imath}$ **Answer:** Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).
5	Assertion(A) : the vector joining the points A(1,0,-1) and B(2,1,0) is directed from

	B to A is $\hat{\imath} + \hat{\jmath} - \hat{k}$ Reason (R): $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ **Answer:** Assertion(A) is false and Reason(R) is true

2 MARK QUESTIONS

1	If $\vec{a} = 4\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, then find the unit vector along the vector $\vec{a} \times \vec{b}$ **Solution:** $\vec{a} = 4\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$ $= \hat{\imath}(-1+2) - \hat{\jmath}(4-2) + \hat{k}(-8+2)$ $= \hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ the unit vector along the vector $\vec{a} \times \vec{b} = \frac{\hat{\imath} - 2\hat{\jmath} - 6\hat{k}}{\sqrt{1^2+(-2)^2+(-6)^2}} = \frac{\hat{\imath} - 2\hat{\jmath} - 6\hat{k}}{\sqrt{41}}$
2	If the vectors $\vec{a}$ *and* $\vec{b}$ are such that $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is unit vector then find the angle between $\vec{a}$ *and* $\vec{b}$ **Solution:** Angle between $\vec{a}$ *and* $\vec{b}$ is Sin $\Theta = \frac{\lvert\vec{a}\times\vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ Sin $\Theta = \frac{1}{3.\frac{2}{3}}$ $\{\because \lvert\vec{a}\times\vec{b}\rvert = 1\}$ Sin $\Theta = \frac{1}{2}$ $\Theta = \frac{\pi}{6}$
3	Find the position vector of a point which divides the join of points with position vectors $(\vec{a} - 2\vec{b})$ and $(2\vec{a} + \vec{b})$ externally in the ratio 2:1 **Solution:** The required vector $= \frac{1(\vec{a}-2\vec{b})-2(2\vec{a}+\vec{b})}{1-2}$ $= \frac{\vec{a}-2\vec{b}-4\vec{a}-2\vec{b}}{1-2}$ $= \frac{-3\vec{a}-4\vec{b}}{-1}$ $= 3\vec{a} + 4\vec{b}$
4	*If* $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, then find the angle between $\vec{a}$ *and* $\vec{b} - \vec{c}$ **Solution:** Given $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ $\vec{a}.\vec{b} - \vec{a}.\vec{c} = 0$ $\vec{a}.(\vec{b} - \vec{c}) = 0$ $\therefore\ \vec{a}$ *is perpendicular to* $(\vec{b} - \vec{c})$ So the angle between $\vec{a}$ *and* $\vec{b} - \vec{c}$ is $\frac{\pi}{2}$
5	If $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$, then find $\lvert\vec{a}\times\vec{b}\rvert$ **Solution:** $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 1 & -2 \end{vmatrix} = \hat{\imath}(-2-3) - \hat{\jmath}(-4-9) + \hat{k}(2-3)$ $\vec{a} \times \vec{b} = -5\hat{\imath} + 13\hat{\jmath} - \hat{k}$ $\lvert\vec{a}\times\vec{b}\rvert = \sqrt{(-5)^2 + (13)^2 + (-1)^2}$ $= \sqrt{195}$

6	Find $\lvert\vec{a}\rvert$ and $\lvert\vec{b}\rvert$ if $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 8$ and $\lvert\vec{a}\rvert = 8\lvert\vec{b}\rvert$ **Solution:** $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = \lvert\vec{a}\rvert^2 - \lvert\vec{b}\rvert^2$ $8 = [8\lvert\vec{b}\rvert]^2 - \lvert\vec{b}\rvert^2$ $8 = 64\lvert\vec{b}\rvert^2 - \lvert\vec{b}\rvert^2$ Or $63\lvert\vec{b}\rvert^2 = 8$ $\lvert\vec{b}\rvert = \sqrt{\frac{8}{63}}$
7	The position vectors of points A,B,C are $\lambda\hat{\imath}+3\hat{\jmath}$, $12\hat{\imath}+\mu\hat{\jmath}$ and $11\hat{\imath}-3\hat{\jmath}$ respectively. If C divides the line segment joining A and B in the ratio 3:1, find the values of λ and μ **Solution:** $11\hat{\imath}-3\hat{\jmath} = \frac{3(12\hat{\imath}+\mu\hat{\jmath})+1(\lambda\hat{\imath}+3\hat{\jmath})}{4}$ $44 = 36+\lambda \Rightarrow \lambda = 8$, and $-12 = 3\mu+3 \Rightarrow \mu = -5$
8	If $\lvert\vec{a}+\vec{b}\rvert = 60$ and $\lvert\vec{a}-\vec{b}\rvert = 40$ and $\lvert\vec{a}\rvert = 22$,then find $\lvert\vec{b}\rvert$ **Solution:** $\lvert\vec{a}+\vec{b}\rvert^2 + \lvert\vec{a}-\vec{b}\rvert^2 = 2[\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2]$ $60^2 + 40^2 = 2[22^2 + \lvert\vec{b}\rvert^2]$ $\lvert\vec{b}\rvert^2 = 2600 - 484$ $\lvert\vec{b}\rvert = 46$
9	If $\vec{a}, \vec{b}$ *are* two vectors such that $\lvert\vec{a}+\vec{b}\rvert = \lvert\vec{a}\rvert$, then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$ **Solution:** Given $\lvert\vec{a}+\vec{b}\rvert = \lvert\vec{a}\rvert$ or $\lvert\vec{a}+\vec{b}\rvert^2 = \lvert\vec{a}\rvert^2$ $\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\vec{a}.\vec{b} = \lvert\vec{a}\rvert^2$ $\lvert\vec{b}\rvert^2 + 2\vec{a}.\vec{b} = 0$ $\vec{b}.\vec{b} + 2\vec{a}.\vec{b} = 0$ $\vec{b}(\vec{b}+2\vec{a}) = 0$ $\therefore\ \vec{b}$ *is perpendicular to* $(\vec{b}+2\vec{a})$
10	If $\vec{a}$ *and* $\vec{b}$ are perpendicular vectors, $\lvert\vec{a}+\vec{b}\rvert = 13$ and $\lvert\vec{a}\rvert = 5$, find the value of $\lvert\vec{a}\rvert$ **Solution:** $\lvert\vec{a}+\vec{b}\rvert^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\vec{a}.\vec{b}$ $13^2 = 5^2 + \lvert\vec{b}\rvert^2 + 0$ $169 - 25 = \lvert\vec{b}\rvert^2$ $\lvert\vec{b}\rvert = 12$

EXERCISE

1	Find a vector in the direction of vector $\vec{a} = -2\hat{\imath}+\hat{\jmath}+2\hat{k}$ that has magnitude 9 units. **Answer:** $-6\hat{\imath}+3\hat{\jmath}+6\hat{k}$
2	Find a unit vector in the direction of $\vec{a}+\vec{b}$ if $\vec{a} = \hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\vec{b} = \hat{\imath}+3\hat{\jmath}+2\hat{k}$ **Answer:** $2\hat{\imath}+4\hat{\jmath}$

3	The position vectors of A,B,and C are $\lambda\hat{\imath}+3\hat{\jmath}$, $12\hat{\imath}+\mu\hat{\jmath}$and $11\hat{\imath}-3\hat{\jmath}$respectively. If C divides the line segment joining A&B in the ratio 3:1,find the values of λ and μ **Answer:** λ =8 and μ = -5
4	If two vectors $\vec{a}$ and$\vec{b}$ are such that $\lvert\vec{a}\rvert=3$,$\lvert\vec{b}\rvert=1$, $\vec{a}.\vec{b}=2$ find $(3\vec{a}+\vec{b}).(2\vec{a}-3\vec{b})$ **Answer:** 37
5	If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $(\vec{a}-\vec{d})$ is parallel to $\vec{b}-\vec{c}$, it is Given that $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$ **Answer:** To show $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$

3 MARK QUESTIONS

1	Let $\vec{a}=4\hat{\imath}+5\hat{\jmath}-\hat{k}$, $\vec{b}=\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both vectors $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a}=21$ **Solution**: Since $\vec{d}$ is perpendicular to both vectors $\vec{c}$ and $\vec{b}$ so $\vec{d}=\lambda(\vec{c}\times\vec{b})=\lambda\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & 1 & -1\\ 1 & -4 & 5\end{vmatrix}$ { where λ is scalar} $=\lambda[\hat{\imath}(5-4)-\hat{\jmath}(15+1)+\hat{k}(-12-1)]$ $=\lambda[\hat{\imath}-16\hat{\jmath}-13\hat{k}]$ $\vec{d}=\lambda\hat{\imath}-\lambda16\hat{\jmath}-\lambda13\hat{k}$ given $\vec{d}.\vec{a}=21$ $\therefore(\lambda\hat{\imath}-\lambda16\hat{\jmath}-\lambda13\hat{k}).(4\hat{\imath}+5\hat{\jmath}-\hat{k})=21$ ⇨ $4\lambda-80\lambda+13\lambda=21$ ⇨ $\lambda=-\frac{1}{3}$ $\therefore\vec{d}=-\frac{1}{3}(\hat{\imath}-16\hat{\jmath}-13\hat{k})$
2	If $\vec{a}=\hat{\imath}+2\hat{\jmath}+\hat{k}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}$ and $\vec{c}=3\hat{\imath}-4\hat{\jmath}-5\hat{k}$. Find a unit vector perpendicular to both vectors$(\vec{a}-\vec{b})$ *and* $(\vec{c}-\vec{b})$ **Solution**: Given $\vec{a}=\hat{\imath}+2\hat{\jmath}+\hat{k}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}$ and $\vec{c}=3\hat{\imath}-4\hat{\jmath}-5\hat{k}$. $\vec{a}-\vec{b}=\hat{\imath}+2\hat{\jmath}+\hat{k}-2\hat{\imath}-\hat{\jmath}=-\hat{\imath}+\hat{\jmath}+\hat{k}$ $\vec{c}-\vec{b}=3\hat{\imath}-4\hat{\jmath}-5\hat{k}-2\hat{\imath}-\hat{\jmath}=\hat{\imath}-5\hat{\jmath}-5\hat{k}$ $(\vec{a}-\vec{b})\times(\vec{c}-\vec{b})=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 1 & 1\\ 1 & -5 & -5\end{vmatrix}$ $=-4\hat{\jmath}+4\hat{k}$

	$\lvert(\vec{a}-\vec{b}) \times (\vec{c}-\vec{b})\rvert = \sqrt{(-4)^2+4^2} = 4\sqrt{2}$ The required vector is = $\pm \frac{-4\hat{j}+4\hat{k}}{4\sqrt{2}} = \pm \frac{-\hat{j}+\hat{k}}{\sqrt{2}}$
3	If $\vec{r} = x\hat{\imath} + y\hat{j} + z\hat{k}$, find $(\vec{r}\times\hat{\imath}).(\vec{r}\times\hat{j}) + xy$ **Solution**: $(\vec{r}\times\hat{\imath}).(\vec{r}\times\hat{j}) + xy$ $\vec{r}\times\hat{\imath} = (x\hat{\imath} + y\hat{j} + z\hat{k})\times\hat{\imath} = 0 - y\hat{k} + z\hat{j} = z\hat{j} - y\hat{k}$ $\vec{r}\times\hat{j} = (x\hat{\imath} + y\hat{j} + z\hat{k})\times\hat{j} = x\hat{k} + 0 - z\hat{\imath} = x\hat{k} - z\hat{\imath}$ Now $(\vec{r}\times\hat{\imath}).(\vec{r}\times\hat{j}) + xy = (z\hat{j} - y\hat{k}).(x\hat{k} - z\hat{\imath})$ $= -xy + xy = 0$
4	The magnitude of the vector product of the vector $\hat{\imath} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{j} + 3\hat{k}$ is equal to $\sqrt{2}$. find the value of λ . **Solution**: Let $\vec{a} = \hat{\imath} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{\imath} + 4\hat{j} - 5\hat{k}$, $\vec{c} = \lambda\hat{\imath} + 2\hat{j} + 3\hat{k}$ Given $\vec{b} + \vec{c} = 2\hat{\imath} + 4\hat{j} - 5\hat{k} + \lambda\hat{\imath} + 2\hat{j} + 3\hat{k} = (2+\lambda)\hat{\imath} + 6\hat{j} - 2\hat{k} = \vec{d}$ (say) $\lvert\vec{d}\rvert = \sqrt{(2+\lambda)^2+6^2+(-2)^2} = \sqrt{(2+\lambda)^2+40}$ Now $\vec{a}\times\vec{d} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2+\lambda & 6 & -2 \end{vmatrix} = -8\hat{\imath} + (4+\lambda)\hat{j} + (4-\lambda)\hat{k}$ Given $\left\lvert\frac{\vec{a}\times\vec{d}}{\lvert\vec{d}\rvert}\right\rvert = \sqrt{2}$ => $\left\lvert\frac{-8\hat{\imath} + (4+\lambda)\hat{j} + (4-\lambda)\hat{k}}{\sqrt{(2+\lambda)^2+40}}\right\rvert = \sqrt{2}$ => $\frac{\sqrt{(-8)^2+(4+\lambda)^2+(4-\lambda)^2}}{\sqrt{(2+\lambda)^2+40}} = \sqrt{2}$ => $\frac{64+16+8\lambda+\lambda^2+16-8\lambda+\lambda^2}{\lambda^2+4\lambda+44} = 2$ => $\frac{96+2\lambda^2}{\lambda^2+4\lambda+44} = 2$ => $8\lambda = 8$ => $\lambda = 1$
5	let $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ be three vectors such that $\lvert\vec{a}\rvert=3$, $\lvert\vec{b}\rvert = 4$ and $\lvert\vec{c}\rvert=5$ and each one of them being perpendicular to the sum of the other two, find $\lvert\vec{a}+\vec{b}+\vec{c}\rvert$ **Solution**:Given $\vec{a}.(\vec{b}+\vec{c}) = 0$, $\vec{b}.(\vec{a}+\vec{c}) = 0$, $\vec{c}.(\vec{a}+\vec{b}) = 0$ Now $\lvert\vec{a}+\vec{b}+\vec{c}\rvert^2 = (\vec{a}+\vec{b}+\vec{c})^2$ $=(\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c})$ $= \vec{a}.\vec{a} + \vec{a}.(\vec{b}+\vec{c}) + \vec{b}.\vec{b} + \vec{b}.(\vec{a}+\vec{c}) + \vec{c}.\vec{c} +$ $\vec{c}.(\vec{a}+\vec{b})$ Put $\vec{a}.(\vec{b}+\vec{c}) = 0$, $\vec{b}.(\vec{a}+\vec{c}) = 0$, $\vec{c}.(\vec{a}+\vec{b}) = 0$ $\therefore \lvert\vec{a}+\vec{b}+\vec{c}\rvert^2 = \lvert\vec{a}\rvert^2 + 0 + \lvert\vec{b}\rvert^2 + 0 + \lvert\vec{c}\rvert^2 + 0$ $\lvert\vec{a}+\vec{b}+\vec{c}\rvert^2 = 3^2 + 4^2 + 5^2$ {$\because \lvert\vec{a}\rvert=3$, $\lvert\vec{b}\rvert = 4$ and $\lvert\vec{c}\rvert=5$} $\lvert\vec{a}+\vec{b}+\vec{c}\rvert^2 = 50$ $\lvert\vec{a}+\vec{b}+\vec{c}\rvert = \sqrt{50} = 5\sqrt{2}$
6	show that the points $A(-2\hat{\imath} + 3\hat{j} + 5\hat{k})$, $B(\hat{\imath} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{\imath} - \hat{k})$ are collinear **Solution**:Given the points are $A(-2\hat{\imath} + 3\hat{j} + 5\hat{k})$, $B(\hat{\imath} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{\imath} - \hat{k})$ $\overrightarrow{AB} = (\hat{\imath} + 2\hat{j} + 3\hat{k}) - (-2\hat{\imath} + 3\hat{j} + 5\hat{k}) = 3\hat{\imath} - \hat{j} - 2\hat{k}$

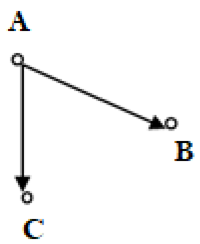
	$\overrightarrow{BC} = 7\hat{\imath} - \hat{k} - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 6\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$ $\overrightarrow{AC} = (7\hat{\imath} - \hat{k}) - (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) = 9\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{3^2+(-1)^2+(-2)^2} = \sqrt{14}$ $\lvert\overrightarrow{BC}\rvert = \sqrt{6+(-2)^2+(-4)^2} = 2\sqrt{14}$ $\lvert\overrightarrow{AC}\rvert = \sqrt{9^2+(-3)^2+(-6)^2} = 3\sqrt{14}$ Here $\lvert\overrightarrow{AC}\rvert = \lvert\overrightarrow{BC}\rvert + \lvert\overrightarrow{AB}\rvert$ Hence the points A , B, C are collinear
7	if $\vec{a}, \vec{b}$ *and* $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ **Solution**:Given $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 1$ and also $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ Or $(\vec{a} + \vec{b} + \vec{c})^2 = 0$ ⇨ $\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a} = 0$ ⇨ $1^2+1^2+1^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ ⇨ $2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -3$ ⇨ $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = \frac{-3}{2}$
8	if $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$,find the value of λ **Solution**: $Given\ \vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ $\vec{a} + \lambda\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(-\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ $= (2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+3\lambda)\hat{k}$ Given $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ $\therefore (\vec{a} + \lambda\vec{b}).\vec{c} = 0$ i.e $[(2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+3\lambda)\hat{k}].(3\hat{\imath} + \hat{\jmath}) = 0$ => $3(2-\lambda) + (2+2\lambda) = 0$ => $\lambda = 8$
9	if $\hat{a}, \hat{b}, \hat{c}$ are mutually perpendicular vectors, then find the value of $\lvert 2\hat{a} + \hat{b} + \hat{c}\rvert$ **Solution**: Given $\hat{a}, \hat{b}, \hat{c}$ are mutually perpendicular vectors $\therefore\ \hat{a}.\hat{b} = 0,\quad \hat{b}.\hat{c} = 0,\quad \hat{a}.\hat{c} = 0$ $\lvert\hat{a}\rvert = \lvert\hat{b}\rvert = \lvert\hat{c}\rvert = 1$ $\lvert 2\hat{a} + \hat{b} + \hat{c}\rvert^2 = 4\lvert\hat{a}\rvert^2 + \lvert\hat{b}\rvert^2 + \lvert\hat{c}\rvert^2 + 4\hat{a}.\hat{b} + \hat{b}.\hat{c} + \hat{a}.\hat{c}$ $= 4.1^2 + 1^2 + 1^2 + 0+0+0$ $= 4+1+1$ $= 6$ $\therefore\ \lvert 2\hat{a} + \hat{b} + \hat{c}\rvert = \sqrt{6}$
10	If $\vec{a}$ *and* $\vec{b}$ are two vectors such that $\lvert\vec{a} + \vec{b}\rvert = \lvert\vec{a}\rvert$, then prove that $2\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$. **Solution**: Given $\lvert\vec{a} + \vec{b}\rvert = \lvert\vec{a}\rvert$ or $\lvert\vec{a} + \vec{b}\rvert^2 = \lvert\vec{a}\rvert^2$

	⇨ $\|\vec{a}\|^2 + \|\vec{b}\|^2 + 2\,\vec{a}\,.\,\vec{b} = \|\vec{a}\|^2$ ⇨ $\|\vec{a}\|^2 - \|\vec{a}\|^2 + \|\vec{b}\|^2 + 2\,\vec{a}\,.\,\vec{b} = 0$ ⇨ $\|\vec{b}\|^2 + 2\,\vec{a}\,.\,\vec{b} = 0$ ⇨ $.\vec{b}(.\vec{b} + 2\,\vec{a}\,) =$ Hence $2\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$.

EXERCISE

1	If A, B, C, D are the points with position vectors $4\hat{\imath} + 5\hat{\jmath} + \hat{k}$, $-\hat{\jmath} - \hat{k}$, $3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ respectively, then find $\overrightarrow{AB}.(\overrightarrow{AC} \times \overrightarrow{AD})$ **Answers: 0**
2	Find $\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$ If A(1,2,3) , B(2,-1,4) , C(4,5,-1) are three points in space. **Answers:** $\sqrt{274}$
3	Find a unit vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ **Answers:** $\frac{-\hat{\imath}}{\sqrt{6}} + \frac{2\hat{\jmath}}{\sqrt{6}} - \frac{\hat{k}}{\sqrt{6}}$
4	If $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\|\vec{a}\|=5$, $\|\vec{b}\| = 12$, $\|\vec{c}\|=13$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ **Answers:** $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -169$
5	If $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, show that $\vec{a}+\vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ *and* $\vec{c}$. also find the angle.

CASE BASED QUESTIONS

I	Two boys Amit and Raju are playing in a park, they saw three points A(2,-1,3) , B(1,1,3) , C(3,2,0) and they decided to play a game Amit is moving from A to B and Raju is moving from A to C  Based on the above information answer the following

1) The vector $\overrightarrow{AB}$ is

a) $\hat{\imath}$ +5$\hat{\jmath} + \hat{k}$ b) $-\hat{\imath}$ +2$\hat{\jmath} + \hat{k}$

c) $-\hat{\imath}$ +2$\hat{\jmath}$ d) $\hat{\imath}$ + 2$\hat{\jmath}$

2) The vector $\overrightarrow{AC}$ is

a) $\hat{\imath}$ +3$\hat{\jmath} + 3\hat{k}$ b) $\hat{\imath}$ + 2$\hat{\jmath} + \hat{k}$

c) $\hat{\imath}$ +3$\hat{\jmath} - 3\hat{k}$ d) $\hat{\imath}$ + 3$\hat{k}$

3) The position vector of point A is

a) 2$\hat{\imath}$ - $\hat{\jmath} + 3\hat{k}$ b) -2$\hat{\imath}$ - $\hat{\jmath} + 3\hat{k}$

c) 2$\hat{\imath}$ -3$\hat{\jmath} - 3\hat{k}$ d) 2$\hat{\imath}$ + 3$\hat{\jmath} - 3\hat{k}$

4) The magnitude of $\overrightarrow{AC}$ is

a) $\sqrt{9}$ b)$\sqrt{29}$ c) $\sqrt{18}$ d) $\sqrt{19}$

Answer:

1. **c) $-\hat{\imath}$ +2$\hat{\jmath}$** { $\because \overrightarrow{AB}$ = (1$-$**2**)$\hat{\imath}$ +**(1+1)**$\hat{\jmath}$ + (3-3) $\hat{k}$ }
2. **c) $\hat{\imath}$ +3$\hat{\jmath} - 3\hat{k}$** { $\because \overrightarrow{AC}$ = (3$-$**2**)$\hat{\imath}$ +**(2+1)**$\hat{\jmath}$ + (0-3) $\hat{k}$ }
3. **a) 2$\hat{\imath}$ - $\hat{\jmath} + 3\hat{k}$** **{ The position vector of A** is **2$\hat{\imath}$ - $\hat{\jmath} + 3\hat{k}$}**
4. **d) $\sqrt{19}$** { $\because \left|\overrightarrow{AC}\right| = \sqrt{1^2 + 3^2 + (-3)^2}$ }

II The roof of a shop is in the shape of a trapezium with four points P, Q, R, S

The position vectors of P, Q, R and S are $\hat{\imath}$ +$\hat{\jmath} + 3\hat{k}$, 2$\hat{\imath}$ +3$\hat{\jmath} + \hat{k}$, $\hat{\imath}$ -3$\hat{\jmath} + 2\hat{k}$, 4 $\hat{\imath}$ +3$\hat{\jmath} - 4\hat{k}$ respectively, some ants moved on the boundary of the roof.

Based on the given information answer the following

1) The coordinates of Q are

a) (1,1,3) b) (2,3,1) c) (2,3,0) d) (1,2,0)

2) Two ants moved from the point P, one ant from P to Q and other ant from P to S and the vectors made by the given condition are

a) Collinear vectors b) Negative vectors

c) Coinitial vectors d) Two collinear vectors with unequal magnitudes

3) The direction ratios of $\overrightarrow{SR}$ are

a) 3,6,$-$3 b) 3,3,-2 c) 2,6,-3 d) - 3, - 6, 6

4) $\overrightarrow{PS}$ and $\overrightarrow{QR}$ are

a) Equal vectors b) Collinear vectors

c) With equal magnitudes d) None of the above

Answers:

1. b) (2,3,1) { $\because$ the position vector of Q is 2$\hat{\imath}$ +3$\hat{\jmath} + \hat{k}$ }
2. c) coinitial vectors
3. d) - 3, - 6, 6 { $\overrightarrow{SR}$ = $\hat{\imath}$-3$\hat{\jmath} + 2\hat{k}$ – (4$\hat{\imath}$ +3$\hat{\jmath} - 4\hat{k}$) = $-3\hat{\imath}$ - 6$\hat{\jmath} + 6\hat{k}$ }
4. d) none of the above

III Carrom or Karom is a game that has long been played throughout India and South East Asia but the game has become increasingly popular throughout much of the rest of the world during the last century.

A player is playing the carrom game, in the above picture suppose the striker is at point A(1,1,3) , a white coin is at the point B(2,3,5) and the black coin at C(4,5,7

Based on the above information answer the following

1) If the striker hit the white coin then the vector is
   a) $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ b) $\hat{\imath}+\hat{\jmath}+2\hat{k}$
   c) $-\hat{\imath}-2\hat{\jmath}-2\hat{k}$ d) $\hat{\imath}+2\hat{\jmath}+2\hat{k}$

2) The distance covered by the striker to the white coin is
   a) $\sqrt{5}$ b) 3 c) 5 d) $\sqrt{3}$

3) The direction cosines of $\overrightarrow{AB}$ are
   a) $\frac{1}{3},\frac{2}{3},\frac{1}{3}$ b) $\frac{-1}{3},\frac{-2}{3},\frac{-2}{3}$
   c) $\mathbf{\frac{1}{3},\frac{2}{3},\frac{2}{3}}$ d) $\frac{-1}{3},\frac{1}{3},\frac{2}{3}$

4) The direction ratios formed by the striker to the black coin is
   a) 3,4,4 b) 3,3,4 c)4,4,3 d) -3,-3, -4

Answer:

1. d) $\hat{\imath}+2\hat{\jmath}+2\hat{k}$ { $\because \overrightarrow{AB} = (2-1)\hat{\imath}+(3-1)\hat{\jmath}+(5-3)\hat{k}$ }
2. b) 3 { $|\overrightarrow{AC}| = \sqrt{1^2+2^2+(2)^2} = \sqrt{9} = 3$}
3. c) $\frac{1}{3},\frac{2}{3},\frac{2}{3}$ { Direction cosines of $\overrightarrow{AB}$ are $\frac{1}{3},\frac{2}{3},\frac{2}{3}$ }
4. a) 3,4,4 { $\overrightarrow{AC} = (4-1)\hat{\imath}+(5-1)\hat{\jmath}+(7-3)\hat{k} = 3\hat{\imath}+4\hat{\jmath}+4\hat{k}$ } so the direction ratios are 3,4,4

EXERCISE

I Three photos are placed on a wall at the points A, B, C

	and the coordinates of these points are (2,1,3) , (-2,1,-3) and (-3,3,2) respectively based on the given information answer the following 1) The direction ratios of $\overrightarrow{BC}$ is a) $\hat{\imath}$ +3$\hat{\jmath} - \hat{k}$ b) $4\hat{\imath} - 6\hat{k}$ c) $-\hat{\imath}$ +2$\hat{\jmath} + 5\hat{k}$ d) $\hat{\imath}$- 2$\hat{\jmath} - 5\hat{k}$ 2) The direction ratios of $\overrightarrow{BC}$ are a) $-1,2,5$ b) $1,-2,-5$ c) $4,0,-6$ d) $1,3,-1$ 3) The magnitude of $\overrightarrow{BA}$ are **a)** $\sqrt{44}$ b) $\sqrt{42}$ c) $\sqrt{54}$ d)$\sqrt{\mathbf{52}}$ 4) The direction cosines of $\overrightarrow{BA}$ are a) $\frac{4}{\sqrt{52}}$, $\frac{6}{\sqrt{52}}$,0 b) $\frac{4}{\sqrt{52}}$,0,$\frac{6}{\sqrt{52}}$ **c)** $\frac{-4}{\sqrt{52}}$, $\frac{6}{\sqrt{52}}$, 0 d) $\frac{4}{\sqrt{52}}$, $\frac{-6}{\sqrt{52}}$, 0 **Answer:** **1**) $-\hat{\imath}$ +2$\hat{\jmath} + 5\hat{k}$ 1) -1,2,5 2) $\sqrt{52}$ 3) $\frac{4}{\sqrt{52}}$,0,$\frac{6}{\sqrt{52}}$
II	A class XII student Ravi is going to write the board examination and he was asked to attempt the following question. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors Based on the above information answer the following 1) If $\vec{a}$ is perpendicular to$\vec{b}$ then a) $\vec{a}.\vec{b}$ = 1 b) $\vec{a}$. $\vec{b}$ = 0 c) $\vec{a} \times \vec{b}$ = 0 d) $\vec{a} \times \vec{b}$ = 1 2) If $\vec{a}$= $\hat{\imath}$ +3$\hat{\jmath} - \hat{k}$ and $\vec{b}$ = 2$\hat{\imath}$ +3$\hat{\jmath} + \hat{k}$ then $\vec{a}$. $\vec{b}$ is a) 11 b) 10 c) 9 d) 4 3) If $\vec{a}$ = $\hat{\imath}$ +3$\hat{\jmath} + \hat{k}$ and $\vec{b}$ = 2$\hat{\imath} - \hat{k}$ then $\left\|\vec{a} + \vec{b}\right\|$ is a) $3\sqrt{2}$ b) $\sqrt{19}$ c) $2\sqrt{5}$ d) $2\sqrt{2}$ 4) If vectors $\vec{a}$and$\vec{b}$ are such that $\left\|\vec{a} + \vec{b}\right\| = \left\|\vec{a} - \vec{b}\right\|$ then a) $\vec{a}$=$\vec{b}$ b) $\vec{a}$ is perpendicular to$\vec{b}$ c) $\vec{a}$ is parallel to $\vec{b}$ d) $\|\vec{a}\| = \|\vec{b}\|$ **Answer:** 1) $\vec{a}$. $\vec{b}$ = 0 2) 10 2) $3\sqrt{2}$ 3) $\vec{a}$ is perpendicular to$\vec{b}$

CHAPTER: THREE - DIMENSIONAL GEOMETRY

SYLLABUS: *Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.*

Definitions and Formulae:

- Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.
- Let a line makes the angles α, β, γ with x, y, z axis respectively, then the Direction Cosines of the line are $l = cos\alpha, m = cos\beta,\ n = cos\gamma$
- If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.
- Direction ratios of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$a = x_2 - x_1, \qquad b = y_2 - y_1, \qquad a = y_2 - y_1$$

- If l, m, n are the direction cosines and $a, b, c$are the direction ratios of a line then

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}},\ \ m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}},\ \ n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

- Direction cosines of a line joining two points $\mathrm{P}(\mathrm{x}_1, \mathrm{y}_1, \mathrm{z}_1)$ and $\mathrm{Q}(\mathrm{x}_2, \mathrm{y}_2, \mathrm{z}_2)$ are

$\frac{\mathrm{x}_2-\mathrm{x}_1}{\mathrm{PQ}}, \frac{\mathrm{y}_2-\mathrm{y}_1}{\mathrm{PQ}}, \frac{\mathrm{z}_2-\mathrm{z}_1}{\mathrm{PQ}}$ (where, PQ= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$)

- Direction ratios of a line are the numbers which are proportional to the direction cosines of a line.
- Skew lines are lines in space which are neither parallel nor intersecting. They lie in different planes.
- Angle between skew lines is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.
- If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then $cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$
- If $a_1, b_1, c_1 and\ a_2, b_2, c_2$ are the direction ratios of two lines and θ is the acute angle between the two lines; then $cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$
- Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector$\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.
- Equation of a line through a point (x_1, y_1, z_1) and having direction cosines l, m, n is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

- The vector equation of a line which passes through two points whose position vectors are $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
- Cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.
- If θ is the acute angle between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, then
$$cos\theta = \left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$$
- If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are the equations of two lines, then the acute angle between the two lines is given by $cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$.
- Shortest distance between two skew lines is the length of the line segment perpendicular to both the lines.
- Shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is $\left|\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$
- Shortest distance between the lines: $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is
$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$
- Distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is $\left|\frac{\vec{b}\times(\vec{a_2}-\vec{a_1})}{|\vec{b}|}\right|$
- Two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are coplanar if $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0$

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND SOLUTIONS
1	What is the distance of point (a, b, c) from x –axis? $A. \sqrt{b^2 + c^2}$ B. $\sqrt{a^2 + c^2}$ C. $\sqrt{a^2 + b^2}$ D. a **Solution:** Distance between (a, b, c) and (a, 0, 0) is $\sqrt{b^2 + c^2}$. **Correct Option: A**
2	What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$? A. 0° B. 30° C. 45° D. 90° **Solution:** DRs of first line are $\frac{1}{2}, \frac{1}{3}$ and -1. DRs of second line are $\frac{1}{6}, -1$, and $\frac{1}{-4}$ $a_1a_2 + b_1b_2 + c_1c_2 = \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = \frac{1-4+3}{12} = 0.$ So the lines are perpendicular.

	Correct Option: D
3	The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form A. $\vec{r}$ = $5\hat{\imath} + 4\hat{\jmath}$ +$6\hat{k}$ + $\lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$ B. $\vec{r}$=$1\hat{\imath} - 2\hat{\jmath}$ +$4\hat{k}$ + $\lambda(3\hat{\imath} - 7\hat{\jmath} - 2\hat{k})$ C. $\vec{r}$=$4\hat{\imath} - 5\hat{\jmath}$ +$6\hat{k}$+ $\lambda(3\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$ D. $\vec{r}$=$5\hat{\imath} - 4\hat{\jmath}$ +$6\hat{k}$ + $\lambda(3\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ **Solution:** Vector equation of the above is $\vec{r}$=$5\hat{\imath} - 4\hat{\jmath}$ +$6\hat{k}$ + $\lambda(3\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ **Correct Option:** D
4	Write the equation of a line passing through $(2, -3, 5)$ and parallel to line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$. A. $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-1}{1}$ B. $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-5}{-1}$. C. $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{5}$. D. $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{-1}$. **Solution:** Drs of $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$ are 3, 4, and -1. So the required equation is $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-5}{-1}$. **Correct Option:** B
5	What is the value of λ for which the lines $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$ are perpendicular to each other? A. +2 B. – 2 C. ±2 D. 0 **Solution:** $a_1a_2 + b_1b_2 + c_1c_2$ = 0 $\Rightarrow$ 6 $-$ 10 + 2 λ =0. So λ = 2 **Correct Option:** A
6	Write line $\vec{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(2\hat{\jmath} - \hat{k})$ into cartesian form. A. $\frac{x-1}{2} = \frac{y-1}{0} = \frac{z-1}{1}$ B. $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z-0}{-1}$ C. $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}$ D. $\frac{x+1}{0} = \frac{y-1}{2} = \frac{z}{1}$ **Solution:** The above line is passing through (1, -1, 0) and is with DRs 0, 2, -1. So, the cartesian form the line is $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z-0}{-1}$ **Correct Option:** B
7	If the direction ratios of a line are $1, -2, 2$ then what are the direction cosines of the line? A. $\frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}$ B. $\frac{-1}{\sqrt{8}}, \frac{2}{\sqrt{8}}, \frac{-2}{\sqrt{8}}$ C. $\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}$ D. $\frac{1}{\sqrt{8}}, \frac{-2}{\sqrt{8}}, \frac{2}{\sqrt{8}}$ **Solution**: If DRs are a, b and c then DCs are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$ So DCs are $\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}$. **Correct Option:** C
8	Write equation of a line passing through $(0, 1, 2)$ and equally inclined to co-ordinate axes.

	A. x = y − 1 = z − 2 B. x = y = z C. x = y + 1 = z + 2 D. x – 1 = y + 2 = z + 3 **Solution:** If a line is inclined equally then its DRs are 1, 1, 1. So the equation of a line passing through $(0, 1, 2)$ is $x = y - 1 = z - 2$ **Correct Option:** A
9	A line is defined by 5x − 3 = 15y + 7 = 1 − 10z. Its direction cosines are ____ A. $\frac{6}{-7}, \frac{2}{-7}, \frac{-3}{-7}$ B. $\frac{\pm 6}{7}, \frac{\pm 2}{7}, \frac{\pm 3}{7}$ C. $\frac{-6}{7}, \frac{-2}{7}, \frac{-3}{7}$ D. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ **Solution:** The equation of the given line is $\frac{x-\frac{3}{5}}{\frac{1}{5}} = \frac{y+\frac{7}{15}}{\frac{1}{15}} = \frac{z-\frac{1}{10}}{-\frac{1}{10}}$ Drs are $\frac{1}{5}, \frac{1}{15}, \frac{-1}{10}$. So DCs are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ **Correct Option: D**
10	Find the direction cosines of the normal to YZ plane? A. 0, 0, 0 B. 1, 0, 0 C. 0, 1, 0 D. 0, 0, 1 **Solution:** The direction cosines of the normal to YZ plane are 1, 0, 0 **Correct Option: B**

CHAPTER	VIDEO LINK	SCAN QR CODE FOR VIDEO
THREE DIMENSIONAL GEOMETRY	https://youtu.be/XimWUG-c_J0	

EXERCISE

1	The co-ordinates of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the YZ plane are _____. A. (0, 11, 9) B. (9, 11, 0) C. (0, 0, 0) D. (0, 3, 5)
2	What is the X coordinate of the point where the line $\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5}$ crosses ZX-plane **A.** – 2 B. 5 C. $\frac{17}{3}$ D. 3
3	What are the direction cosines of the Y axis?

	A. 0, 0, 0 B. 0, b, 0 C. 0,1,0 D. 1, 0, 1
4	What is the cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + 2\hat{k}$ makes with y – axis. **A.** 0° B. cos $\frac{1}{\sqrt{7}}$ C. $\frac{1}{\sqrt{7}}$ D. –1
5	For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other? A. $-\frac{5}{2}$ B. $\frac{2}{5}$ C. $\frac{5}{2}$ D. $\frac{1}{2}$
Answers: 1. (0, 11,9) 2. $\frac{17}{3}$ 3. 0, 1, 0 4. $\frac{1}{\sqrt{7}}$ 5. $\frac{5}{2}$	

ASSERTION REASONING QUESTIONS

The following questions consist of two statements, one labelled as 'Assertion (A)' and the other labelled as 'Reason (R)'. Select your answer to these items using the codes given below and then select the correct option.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

1	*Assertion:* If a line makes an angle of $\frac{\pi}{4}$ with each of y and z- axes, then it makes a right angle with X axis. *Reason:* The sum of the angles made by a line with the coordinate axes is 180^0 Solution: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ So $\cos^2\alpha = 1 - (\frac{1}{\sqrt{2}})^2 - (\frac{1}{\sqrt{2}})^2$; Hence $\cos\alpha = 0$, $\alpha = 90^0$ A is True and R is False So Answer is (c)
2	*Assertion:* The acute angle between the line $\vec{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - \hat{\jmath})$ and X- axis is $\frac{\pi}{4}$ *Reason:* If θ is the acute angle between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, then $cos\theta = \left\|\frac{\vec{b_1}.\vec{b_2}}{\|\vec{b_1}\|\|\vec{b_2}\|}\right\|$ Solution: Angle between vectors with DRs 1, -1, 0 (Given line) and 1, 0, 0 (X-axis) is $cos\theta = \left\|\frac{\vec{b_1}.\vec{b_2}}{\|\vec{b_1}\|\|\vec{b_2}\|}\right\|$ i.e $\theta = \frac{\pi}{4}$ So A is true and R is the right reason Answer: (a)
3	*Assertion:* The distance of a point P (a, b, c) from Y axis is b *Reason:* Point on Y axis is (0, b, 0)

	Solution: The distance of a point P (a, b, c) from Y axis is $\sqrt{(a-0)^2+(b-b)^2+(c-0)^2} = \sqrt{(a)^2+(c)^2}$ A is false and R is true. So the answer is (d)
4	*Assertion:* The vector form of the line through the point (5, 2, −4) and which is parallel to the vector $2\hat{i}+\hat{j}-6\hat{k}$ is $2\hat{i}+\hat{j}-6\hat{k}+\lambda(5\hat{i}+2\hat{j}-4\hat{k})$ *Reason:* Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ *is* $\vec{r}=\vec{a}+\lambda\vec{b}$. Solution: The vector form of the line through the point (5, 2, −4) and which is parallel to the vector $2\hat{i}+\hat{j}-6\hat{k}$ is $5\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+\hat{j}-6\hat{k})$ A is false and R is true. So the answer is (d)
5	If $l_1: \frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ $l_2: \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ *Assertion:* If $l_1 \perp l_2$ then $p = \frac{70}{11}$ *Reason:* If two lines with DRs a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular then $a_1a_2+b_1b_2+c_1c_2=0$ Solution: The Drs of the given lines are $-3, \frac{2p}{7}$, and 2 ; $\frac{-3p}{7}$, 7, –5 And $a_1a_2+b_1b_2+c_1c_2=0 \Rightarrow p=\frac{70}{11}$ Hence A is true and R is the right reason. So the answer is (a)

EXERCISE

1	Assertion : The lines $\frac{x+1}{1}=\frac{y+4}{0}=\frac{z-7}{0}$ and $\frac{2x+4}{2}=\frac{y-5}{0}=\frac{2z-7}{0}$ are parallel Reason : Two lines are parallel if their DRS are proportional
2	Assertion : The angle between the lines whose DRs are given by 2l − m + 2n = 0 and mn + nl + lm = 0 is 90^0 Reason : Two lines with DRs a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$
3	Assertion : Skew lines are non - intersecting non - parallel lines Reason : They exsist in 3D space only.
4	Assertion : The angle between the diagonals of a cube is $\cos^{-1}(\frac{1}{3})$ Reason : The DRs of the diagonals of the cube are proportional to a, a, a *and* $-a, a, a$
5	Assertion : The image of (0, 2, 0) in X axis is (0, -2, 0) Reason : X axis is perpendicular to Y axis and with reference to (0, 2, 0), (0, 0, 0) is foot of the perpendicular on X axis
ANSWERS: Q1. (a) Q2. (a) Q3. (b) Q4. (a) Q5. (a)	

2 MARK QUESTIONS

1	**Find the equation of a line parallel to x-axis and passing through the origin.** **Solution:** Since the line is parallel to the x-axis, The direction ratio of a line is given by (a, 0, 0). $\therefore$ The equation of a line is $\frac{x-x_1}{a} = \frac{y-y_1}{b}$ Here $(x_1, y_1, z_1) = (0, 0, 0)$, $(a, b, c) = (a, 0, 0)$ Required equation of a line is $\frac{x}{a} = \frac{y}{0} = \frac{z}{0}$ or $\frac{x}{1} = \frac{y}{0} = \frac{z}{0} = k$ or $x = k$.
2	**Find the equation of a line passing though $(2, 0, 5)$ and which is parallel to line $6x - 2 = 3y + 1 = 2z - 2$** **Solution:** Drs of $6x - 2 = 3y + 1 = 2z - 2$ are $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ As $6x - 2 = 3y + 1 = 2z - 2$ can be written as $\frac{x-\frac{2}{6}}{\frac{1}{6}} = \frac{y+\frac{1}{3}}{\frac{1}{3}} = \frac{z-\frac{2}{2}}{\frac{1}{2}}$ $\therefore$ Required equation of the line passing through the point (2, 0, 5) with DRs $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ is $\frac{x-2}{\frac{1}{6}} = \frac{y-0}{\frac{1}{3}} = \frac{z-5}{\frac{1}{2}}$
3	**Find the equation of the line passing through the points $(2, 3, -4)$ and $(1, -1, 3)$ and parallel to the x -axis.** **Solution:** The direction ratios of the two points $(2, 3, -4)$ *and* $(1, -1, 3)$ are $(-1, -4, 7)$ Hence, the equation of line is $\frac{x+1}{1} = \frac{y+4}{0} = \frac{z-7}{0}$
4	**Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (– 1, – 2, 1), (1, 2, 5)** **Solution:** Drs of line through the points (4, 7, 8), (2, 3, 4) are 2, 4, 4 Drs of line through the points (– 1, –2, 1), (1, 2, 5) are also 2, 4, 4. So the lines are parallel
5	**If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.** Solution: If the lines are perpendicular then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ $\therefore -3(3k) + 2k(1) + 2(-5) = 0 \implies -9k + 2k - 10 = 0.$ $\therefore k = \frac{-10}{7}$
6	**The cartesian equation of a line is 3x + 1 = 6y−2 = 1 – z. Find a point on the line, its DRs and also its vector equation.** Solution: The point on the line is $(\frac{-1}{3}, \frac{1}{3}, 1)$ Direction Ratios are 2, 1, −6 Vector Equation is $\vec{r} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + \lambda\ (2\hat{i} + \hat{j} - 6\hat{k})$
7	**Find the cartesian equation of the line passing through the point (2, -1, 3) and parallel to the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda\ (2\hat{i} + \hat{j} - 2\hat{k})$**

	Solution: The DRs of required line are 2, 1, –2 Cartesian Equation of the required line is $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z}{-2}$
8	**If the coordinates of the points A, B, C, D are (1, 2, 3), (4, 5, 7), (−4, 3, −6) and (2, 9, 2), then find the angle between AB and CD.** Solution: The DRs of line AB are 3, 3, 4 DRs of line CD are 6, 6, 8 As DRs are proportional, The angle between the lines is 0.
9	**Find the value of k so that the lines x =−y = kz and x – 2 = 2y + 1 = – z + 1 are perpendicular to each other.** Solution: The DRs of the line 1 are 1, –1, $\frac{1}{k}$ DRs of line 2 are 1, $\frac{1}{2}$, –1 Now that the lines are perpendicular $a_1a_2 + b_1b_2 + c_1 c_2 = 0$, So k = 2.
10	**Find vector equation of the line parallel to X axis and passing through origin.** Solution: DRs of X axis: 1, 0, 0 and given point is (0, 0, 0) So the required equation is $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

EXERCISE

1	Find the projection of the line segment joining the points (−1, 0, 3) and (2, 5, 1) on the line whose direction ratios are (6, 2, 3)
2	Find the direction ratios of a line perpendicular to the lines having direction ratios (1, 3, 2) and (–2, 2, 4) respectively.
3	Find the vector equation of the line passing through the point (1, 2, –3) and parallel to the vector 2i + 3j–4k.
4	Find the vector equation of the straight line passing through the Points (2, 1, –3) and (5, –4, 1).
5	Find the angle between the lines: $\frac{x-1}{2} = \frac{2-y}{2} = \frac{z}{-4}$ and $\frac{x-3}{1} = \frac{y-4}{3} = \frac{2-z}{1}$
6	Find the angle between the lines with direction ratios (2, 2, 1) and the line joining (3, 1, 4) to (7, 2, 12).
7	Find the vector equation of the line passing through the point A (1, 2, –1) and parallel to 5x – 25 = 14 – 7y = 35z
8	Show that the lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

9	Find the value of λ, if the lines $\frac{x-1}{-3}=\frac{7y-14}{2\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda},\ \frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.
10	If A, B are points (2, 3, -6) and (3, −4, 5), find the angle that OA makes with OB where O is origin.

Answers:

Q1. $\frac{22}{7}$ Q2. 8, –8, 8 (Hint Find $\vec{a}$ X $\vec{b}$)

Q3. $\vec{r} = 1\hat{i}+2\hat{j}-3\hat{k}+\lambda\ (2\hat{i}+3\hat{j}-4\hat{k})$

Q4. $\vec{r} = 2\hat{i}+1\hat{j}-3\hat{k} + \lambda\ (3i-5j+4k)$ Q5. 0 Q6. $\cos^{-1}(\frac{2}{3})$

Q7. $\vec{r} = 1\hat{i}+2\hat{j}-\hat{k}+\lambda\ (7\hat{i}-5\hat{j}+\hat{k})$ Q8. Hint: $a_1a_2+b_1b_2+c_1c_2=0$

Q9. $\lambda = \frac{70}{11}$ Q 10. $\cos\theta = \frac{18\sqrt{2}}{35}$

3 MARK QUESTIONS

1	**If a line makes angle α,β,γ with co-ordinate axes then what is the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$** Solution: $\sin^2\alpha+\sin^2\beta+\sin^2\gamma = (1-\cos^2\alpha)+(1-\cos^2\beta)+(1-\cos^2\gamma)$ $= 3-(\cos^2\alpha+\cos^2\beta)+\cos^2\gamma)$ $= 3-1 = 2$
2	**Find the equation of a line passing through the point $(2,0,1)$ and parallel to the line whose equation is $\vec{r}=(2\lambda+3)\hat{\imath}+(7\lambda-1)\hat{\jmath}+(3\lambda+2)\hat{k}$** **Solution:** The DRs of the line $\vec{r}=(2\lambda+3)\hat{\imath}+(7\lambda-1)\hat{\jmath}+(3\lambda+2)\hat{k}$ are 2, 7, 3 ∴ Required equation of the line passing through the point (2, 0, 1) with DRs 2, 7, −3 is $\frac{x-2}{2}=\frac{y-0}{7}=\frac{z-1}{3}$
3	**Find the vector equation of a line passing through the point (1, 2, -4) and perpendicular to two lines $L_1=\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $L_2=\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$** **Solution:** DRs of the required lines can be obtained by calculating the determinant $\begin{vmatrix} i & j & k \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ which is equal to $24\hat{\imath}+36\hat{\jmath}+72\hat{k}$ ∴ DRs are 24, 36, 72 i.e DRs 2, 3, 6 equation of line is $\hat{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\boldsymbol{\lambda}(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$

4	**Find the angle between the lines whose direction ratios are a, b, c and b – c, c – a, a – b.** **Solution:** $\cos\theta = \frac{a(b-c)+b(c-a)+c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}$ $= \frac{ab-ac+bc-ab+ac-bc}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}} = 0 \Rightarrow \theta = \pi/2$
5	**If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (– 4, 3, – 6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.** Solution: DRs of AB are 3, 3, 4 and DRs of CD are 6, 6, 8 as DRs of both the lines are proportional the angle between them is 0. i.e. the lines are parallel.
6	**The points A (4, 5,10), B (2, 3, 4) and C (1, 2, –1) are the three vertices of a parallelogram ABCD. Find vector equation for the diagonal BD.** Solution: The coordinates of D be (x, y, z). Midpoint of AC is same as midpoint of BD. So $\left(\frac{5}{2},\frac{7}{2},\frac{9}{2}\right) = \left(\frac{2+x}{2},\frac{3+y}{2},\frac{4+z}{2}\right)$ So, D (x, y, z) is (3, 4, 5) and $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ ∴ Equation of BD is $\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$
7	**Find the point on the line $\frac{2+x}{3} = \frac{1+y}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ from the point** **P (1, 2, 3).** Solution : The general point Q on the given line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ is $x = 3\lambda - 2,\ y = 2\lambda - 1\ \ z = 2\lambda + 3$ Given PQ = $3\sqrt{2}$. ∴ $\sqrt{(3\lambda-2-1)^2 + (2\lambda-1-2)^2 + (2\lambda+3-3)^2} = 3\sqrt{2}$ So $\lambda = \frac{30}{7}$, And the required point P is $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$
8	Find the distance from the point P(3, -8, 1) to the line $\frac{x-3}{3} = \frac{y+7}{-1} = \frac{z+2}{5}$ Solution: Q (3, -7, -2) is a point on the given line. $\vec{b} = 3\hat{i} - \hat{j} + 5\hat{k}$ are the DRs of the above line. Then the distance 'd' from a point P(3, -8, 1) from the line is given by d = $\frac{\lvert\vec{b} \times \overrightarrow{PQ}\rvert}{\lvert\vec{b}\rvert}$ $\vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} i & j & k \\ 3 & -1 & 5 \\ 0 & 1 & -3 \end{vmatrix} = 2\hat{i} - 9\hat{j} - 3\hat{k}$ and $\lvert\vec{b} \times \overrightarrow{PQ}\rvert = \sqrt{94}$, $\lvert\vec{b}\rvert = \sqrt{35}$ ∴ d = $\sqrt{\frac{94}{35}}$
9	**Find the DRs of the line perpendicular to the lines passing through A (2 ,3, −4), B (−3, 3, 2) and C(−1, 4, 2) D (3, 5, 1)** Solution: DRs of the line passing through A (2 ,3 , −4), B (−3, 3, 2) are −5, 0, 6 DRs of the line passing through C(−1, 4, 2) D (3, 5, 1) are 4, 1, −1 DRs of the line perpendicular to AB and CD are $\begin{vmatrix} i & j & k \\ -5 & 0 & 6 \\ 4 & 1 & -1 \end{vmatrix}$ = –6i +19j –5k

10	Find the angle between the lines whose direction cosines are given by $l + m + n = 0$ and $l^2+m^2-n^2 = 0$ Solution: Given $l^2 + m^2 = n^2$ and $l + m + n=0$ and we know that $l^2 + m^2 + n^2 = 1$ By solving above equations, we get $n = \pm\frac{1}{\sqrt{2}}$ For $n = \frac{1}{\sqrt{2}}$, then we get $l = \pm\frac{1}{\sqrt{2}}$, m=0 or l=0, $m = \pm\frac{1}{\sqrt{2}}$ So the one possible set of DRs are $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$ and $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ Let the angle between them be θ We have $\cos\theta = \left\lvert \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \right\rvert = \frac{1}{2}$ So we get $\theta = \frac{\pi}{3}$

EXERCISE

1	Find the equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to z –axis.
2	Find the coordinates of the point where the line through the points A (3, 4, 10) and B (5, 1, 6) crosses XY plane
3	Find the equation of a line which passes through (5, –7, –3) and is parallel to the line $\frac{x-2}{3} = \frac{y+1}{1} = \frac{z-7}{9}$
4	Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Hint : Show $(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$
5	Find the equation of a straight line through (1, –2, 3) and equally inclined to the axes.
6	Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, –1), (4, 3, –1).
7	Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$
8	Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units form the point P (1, 3, 3)
9	Find the points on the line through A(1, 2, 3) B (3, 5, 9) at a distance of 14 units form the midpoint of line segment AB.
10	Find the equation of the line passing through A (−2, 4, 7), B (3, −6, −8). Hence show A, B and C (1, −2, −2) are collinear.

Answers:

Q1. Hint: DRs of line joining (1, 2, 3) and (–3, 4, 3) are –4, 2, 0. DRs of Z axis are 0, 0, 1
So the line and Z axis are perpendicular.

Q2. $(8, \frac{-7}{2}, 0)$ Q3. $\frac{x-5}{3} = \frac{y+7}{1} = \frac{z+3}{9}$

Q5. $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$ Q6. Hint: $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Q7. $2\sqrt{6}$ Q8. (-2, -1, 3) and (4, 3, 7)

Q9. (7,11,21) and (-1,-1,-3). Q10. AB = $-2\hat{i} + 4\hat{j} + 7\hat{k} + \lambda(5\hat{i} - 10\hat{j} - 15\hat{k})$

5 MARK QUESTIONS

1 **Find the shortest distance between lines**

$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ **and** $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

Solution: Let $\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$ $\vec{a}_2 = -4\hat{i} - \hat{k}$

$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$ $\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

Shortest distance, d = $\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

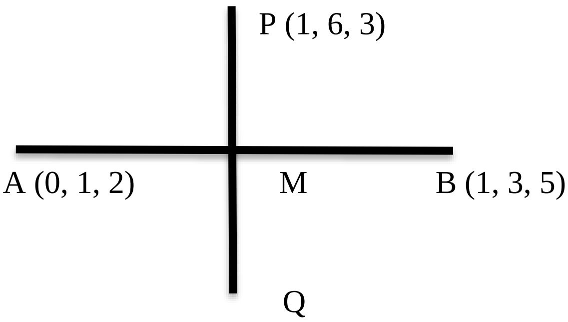
$(\vec{a}_2 - \vec{a}_1) = -10\hat{i} - 2\hat{j} - 3\hat{k}$

$(\vec{b}_1 \text{X} \vec{b}_2) . (\vec{a}_2 - \vec{a}_1) = -80 - 16 - 12 \quad \Rightarrow -108$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{64 + 64 + 16} = \sqrt{144} = 12. \Rightarrow$ d = $\frac{108}{12}$ = 9 unit.

2 **Find the image of a point P (1, 6, 3) with respect to the line** $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Solution:



Let M be the foot of the perpendicular.

Let $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$ (say)

General point on the line AB is x = λ , y = 2 λ + 1, z = 3 λ + 2

DRs of PM = $\lambda - 1$, $2\lambda - 5$, $3\lambda - 1$

PM is perpendicular to AB. So $a_1 a_2 + b_1b_2 + c_1 c_2 = 0$

⇨ $1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

⇨ $\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

So $14\lambda = 14$, and $\lambda = 1$

∴ Point M is (1, 3, 5)

Let the image be Q (x_3, y_3, z_3)

Now , using M as mid point of PQ

$\frac{x_1+x_3}{2} = x_2 \quad \Rightarrow \frac{1+x_3}{2} = 1$ and $x_3 = 1$

$\frac{y_1+y_3}{2} = y_2 \quad \Rightarrow \frac{6+y_3}{2} = 3$ and $y_3 = 0$

$\frac{z_1+z_3}{2} = z_2 \quad \Rightarrow \frac{3+z_3}{2} = 5$ and $z_3 = 7$

∴ Image Q is (1, 0, 7)

3	**Show that the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect.** **Also find their point of intersection.** **Solution:** Part I : (Hint : Show that the shortest distance between the lines is 0 using shortest distance formula.) Part II : For finding their point of intersection for first line. ⇨ $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ ⇨ x = $2\lambda + 1$, y = $3\lambda + 2$, z = $4\lambda + 3$ Since, the lines are intersecting. So, Let's put these values in equation of another line. Thus, $\frac{2\lambda+1-4}{5}=\frac{3\lambda+2-1}{2}=\frac{4\lambda+3}{1}$ ⇨ $\frac{2\lambda-3}{5}=\frac{3\lambda+1}{2}=\frac{4\lambda+3}{1}$ ⇨ $\frac{2\lambda-3}{5}=\frac{4\lambda+3}{1}$ $\Rightarrow 2\lambda-3 = 20\lambda+15 \Rightarrow 18\lambda = -18 = -1$ So, the required point of intersection is x = 2(−1) + 1 = −1, y = 3(−1) + 2 = −1, z = 4(−1) + 3 = −1 Thus, the lines intersect at (-1, -1,-1).
4	**If the points (-1, 3 , 2), (-4, 2 , -2) and (5, 5, λ) are collinear, Use the concept of lines to find the value of 'λ'.** **Solution:** Let the position vectors of the points be $\vec{a}$, $\vec{b}$ & $\vec{c}$ $\therefore \vec{a} = -\hat{i} + 3\hat{j} + 2\hat{k}$ $\vec{b} = -4\hat{\imath} + 2\hat{j} - 2\hat{k}$ $\vec{c} = 5\hat{i} + 5\hat{j} + \lambda\hat{k}$ The equation of a line passing through the points where position vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{r} = \vec{a} + \mu(\vec{b} - \vec{a})$ $\vec{r} = -\hat{i} + 3\hat{j} + 2\hat{k} + \mu(-4\hat{\imath} + 2\hat{j} - 2\hat{k} + \hat{i} - 3\hat{j} - 2\hat{k})$ $= -\hat{i} + 3\hat{j} + 2\hat{k} + \mu(-3\hat{i} - \hat{j} - 4\hat{k})$ As it passes through $\vec{c}$, $5\hat{i} + 5\hat{j} + \lambda\hat{k} = -\hat{i} + 3\hat{j} + 2\hat{k} + \mu(-3\hat{i} - \hat{j} - 4\hat{k})$ $6\hat{i} + 2\hat{j} + (\lambda - 2)\hat{k} = \mu(-3\hat{i} - \hat{j} - 4\hat{k})$ $6 = -3\mu \Rightarrow \mu = -2.$ $\lambda - 2 = -4\mu \Rightarrow \lambda = -4(-2) + 2 \Rightarrow \lambda = 8+2 \Rightarrow$ $\lambda = 10.$

EXERCISE

1	Find the shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$
2	Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ & $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect each other.

3	Find the foot of the perpendicular drawn from the point A(1, 0, 3) to the join of the points B(4, 7, 1) and C(3, 5, 3).
4	Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}\ \frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect and find their point of intersection.
5	Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
	Find the distance between lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$.

Answers:

Q1. $\frac{8}{\sqrt{29}}$ Q2. Hint: Show that shortest distance is not '0'. Q3. $(\frac{5}{3}, \frac{7}{3}, \frac{17}{3})$

Q4. $(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2})$ Q5. (1, 0, 7) Q6. $\frac{\sqrt{293}}{7}$

CASE BASED QUESTIONS

I The equation of motion of a rocket are: x = 2t, y = -4t, z = 4t, where the time 't' is given in seconds, and the distance measured is in kilometers.

Based on the above information, answer the following questions.

(i) What is the path of the rocket?

(a) Straight line (b) Circle (c) Parabola (d) None of these

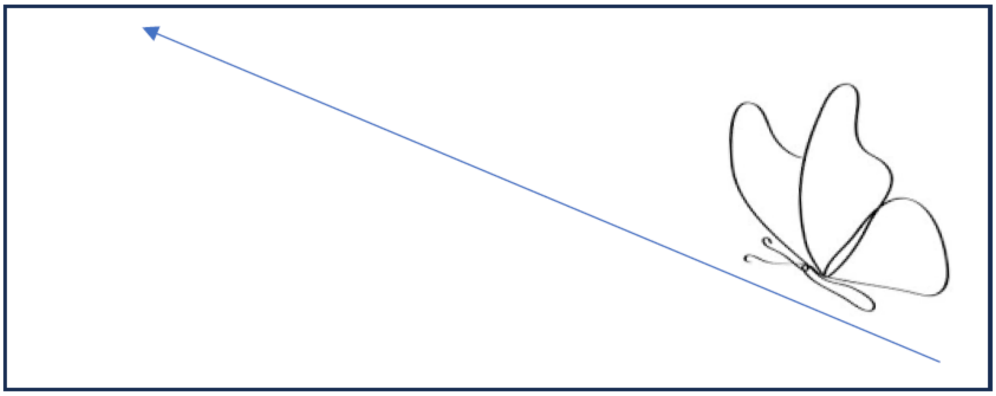
(ii) Which of the following points lie on the path of the rocket?

(a) (0, 1, 2) (b) (1, -2 , 2) (c) (2 , -2, 2) (d) None of these

(iii) At what distance will the rocket be from the starting point (0, 0, 0) in 10 seconds?

(a) 40 km (b) 60 km (c) 30 km (d) 80 km

(iv) If the position of rocket at certain instant of time is (3, -6, 6), then what will be the height of the rocket from the ground, which is along the xy-plane?

	(a) 3 km (b) 2km (c) 4 km (d) 6 km **Answers:** i) a ii) b iii) b iv) d
2	A butterfly is moving in a straight path in the space. Let this path be denoted by a line l whose equation is $\frac{x-1}{2} = \frac{2y+2}{4} = \frac{3z-3}{-6}$ say.  Using the information given above, answer the following with reference to the line l : (i) What are the direction ratios of the line? (a) 2, 4, –6 (b) –2, 2, 2 (c) 2, –3, 4 (d) 2, 2, –2 (ii) If the z-coordinate of a point on this line is 7, then the x-coordinate of the same point on this line, is (a) –5 (b) 5 (c) 0 (d) 1 (iii) The vector equation of the given line is (a) $\vec{r} = (2\hat{i} + 4\hat{j} - 6\hat{k}) + \lambda\ (\hat{i} + 2\hat{j} + 3\hat{k})$ (b) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda\ (2\hat{i} + 4\hat{j} - 6\hat{k})$ (c) $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda\ (2\hat{i} + 3\hat{j} + 6\hat{k})$ (d) $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda\ (2\hat{i} + 2\hat{j} - 2\hat{k})$ (iv) The unit vector in the direction of the vector parallel to the given line, is (a) $\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ (b) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$ (c) $\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$ (d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ Answers: i) d ii) a iii) d iv) b

CHAPTER : LINEAR PROGRAMMING

SYLLABUS: *Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints.)*

Definitions and Formulae:

1) Let R be the feasible region (convex polygon) for a linear programming problem and let Z = $ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point* (vertex) of the feasible region.
2) Let R be the feasible region for a linear programming problem, and let Z = $ax + by$ be the objective function. If R is **bounded****, then the objective function Z has both a **maximum** and a **minimum** value on R and each of these occurs at a corner point (vertex) of R

Remark: If R is **unbounded**, then a maximum or a minimum value of the objective function may not exist. However, if it exists, it must occur at a corner point of R.

➢ Solving linear programming problem using Corner Point Method.

The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points(vertices) either by inspection or by solving the two equations of the lines intersecting at the point.
2. Evaluate the objective function Z=ax+ by at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is bounded, M and m are the maximum and minimum value of Z.

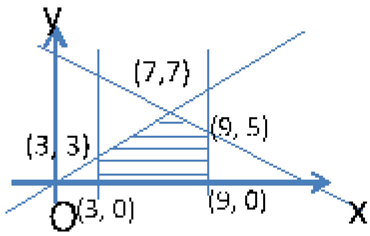
   (ii) In case, the feasible region is unbounded, we have:

   (a) M is the maximum value of Z , if the open half plane determined by ax+ by >M has no point in common with the feasible region. Otherwise, Z has no maximum value.

   (b) Similarly, m is the minimum value of Z, if the open half plane determined by ax+ by<m has no point in common with the feasible region. Otherwise, Z has no minimum value.

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND ANSWERS
1	Solution set of the inequality $2x + y > 5$ is a) Half-plane containing origin b) Half plane not containing origin c) xy-plane except the points on the line $2x + y = 5$ d) No solution **Solution:** Origin does not satisfy this inequality. Ans: **b**
2	Objective function of a LPP is a) constant graph b) A function to be optimized c) Inequality d) Quadratic function **Solution:** Objective function is a function to be optimized. Ans: b
3	In an LPP, if the objective function $Z = ax + by$ has same maximum at two corner points of the feasible region, then the number of points at which maximum value of Z occurs is a) 0 b) 1 c) 2 d) infinite **Solution:** every point on the line joining these two corner points gives same maximum. Ans: d
4	The corner points of the feasible region determined by a system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $Z = ax + by$ where $a,b > 0$ occurs at both (2, 4) and (4, 0), then a) $a = 2b$ b) $2a = b$ c) $a = b$ d) $3a = b$ **solution:** $Z_{(2, 4)} = Z_{(4, 0)} \Rightarrow 2a + 4b = 4a \Rightarrow a = 2b$ Ans: a
5	A linear programming problem is as follows: Maximize /minimize objective function $Z = 2x – y + 5$ subject to constraints $3x + 4y \leq 60,\ x + 3y \leq 30,\ x \geq 0,\ y \geq 0.$ If the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0, 0), then which of the following are true a) Maximum value of Z is 40

<table>
<tr><td></td><td>b) Minimum value of Z is -5
c) Difference between maximum and minimum values of Z is 35
d) At two corner points the values of Z are equal
Solution: $Z_{(0,\ 10)} = 2x0 - 10 + 5 = -5$ is minimum value of Z   Ans: b</td></tr>
<tr><td>6</td><td>A linear programming problem is as follows:
Minimize $Z = 2x + y$ subject to constraints $x \geq 3$, $x \leq 9$, $y \geq 0$, $x - y \geq 0$, $x + y \leq 14$
The feasible region has
a) 5 corner points including (0, 0) and (9, 5)
b) 5 corner points including (7, 7) and (3, 3)
c) 5 corner points including (14, 0) and (9, 0)
d) 5 corner points including (3, 6) and (9, 5)
Solution: Five corner points as shown in the figure.


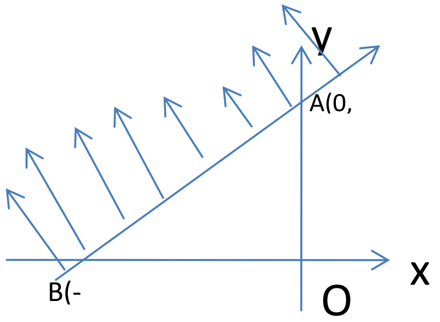
It includes (3,3) and (7, 7)
Ans : b</td></tr>
<tr><td>7</td><td>The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true
a) a = 9, b = 1   b) a = 5, b = 2
c) a = 3, b = 5   d) a = 5, b = 3
solution: $Z_{(4,\ 6)} = 42 \Rightarrow 4a + 6b = 42$ $Z_{(3,\ 2)} = 19 \Rightarrow 3a + 2b = 19$ solving we get a = 3, b = 5
Ans:c</td></tr>
<tr><td>8</td><td>The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and (20/3, 4/3). If $Z = 30x + 20y$ is the objective function, then (Maximum value of Z – Minimum value of Z) is equal to
a) 40   b) 96   c) 160   d) 136
Solution: Max. = Z(8, 0) =240 and Min. = Z(0, 4) = 80
Ans: c</td></tr>
<tr><td>9</td><td>The position of points O(0, 0) and P(2, -1) is-------, in the solution region of the inequality $2y - 3x < 5$
a) O is inside the region and P is outside the region</td></tr>
</table>

	b) O and P both are inside the region c) O and P both are outside the region d) O is outside and P is inside the region **Solution:** Both O and P satisfy the inequality. Ans: b
10	If the corner points of the feasible region of an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5), then the minimum value of the objective function Z = 4x + 6y occurs at a) (0, 2) only b) (3, 0) only c) The midpoint of the line segment joining the points (0, 2) and (3, 0) only d) Every point on the line segment joining the points (0, 2) and (3, 0) **Solution:** If Z has same min.value at two points, then Z has same min. value at every point on the line segment joining the two points. Ans: d

CHAPTER	VIDEO LINK	SCAN QR CODE FOR VIDEO
LINEAR PROGRAMMING	https://youtu.be/Zx0yeGYW0kw	

EXERCISE

1	The feasible region of constraints x+ y $\leq$ 4, 3x+ 3y $\geq$18, x $\geq$ 0, y $\geq$ 0 defines on .... a) bounded feasible region b) unbounded feasible region c) feasible region in first and second quadrants **d)** does not exist
2	The maximum value of Z = 3x+ 4y subject to constraints x+ y $\leq$ 4, x $\geq$ 0, y $\geq$ 0 is

	a) 16 b) 12 c) 0 d) not possible
3	Which of the following statement is correct? (a) Every L.P.P. admits an optimal solution (b) An L.P.P. admits a unique optimal solution (c) If an L.P.P. admits two optimal solutions, it has an infinite number of optimal solutions (d) The set of all feasible solutions of a L.P.P. is not a convex set.
4	The shaded region in the given figure is the graph of.........  (a) $4x - 2y \leq 3$ (b) $4x - 2y \leq -3$ (c) $2x - 4y \geq 3$ (d) $2x - 4y \leq -3$
5	Under the constraints $x - 2y \leq 6$, $x + 2y \geq 0$, $x \leq 6$, the maximum value of $Z = 3x + 4y$ is (a) 16 b) 17 c) 18 d) 19
ANSWERS: 1) d 2) a 3) c 4) a 5) c	

ASSERTION AND REASONING QUESTIONS

The following questions consists of two statements-Assertion(A) and Reason(R). Answer these questions selecting appropriate option given below.

a) Both A and R are true and R is the correct explanation for A

b) Both A and R are true and R is not the correct explanation for A

c) A is true but R is false

d) A is false but R is true

1	**Assertion (A):**The maximum value of $Z = 5x + 3y$, satisfying the conditions $x \geq 0$, $y \geq 0$ and $5x + 2y \leq 10$, is 15

	Reason(R): A feasible region may be bounded or unbounded **Solution:** corner points are (0, 0), (2, 0) and (0, 5) Z_{max} =5x0 + 3x5 =15 at (0, 5) So, both A and R are true but R is not the correct explanation for A Ans: b
2	**Assertion (A):**The max. value of Z = x + 3y subject to $2x + y \leq 20$, $x + 2y \leq 20$ $x \geq 0, y \geq 0$ is 30 **Reason(R):** The variables that are present in the problem are called decision variables. **Solution:** corner points are (0, 0), (10, 0), (20/3, 20/3) and (0, 10) $Z_{max.}$ =x + 3y = 0 + 3x10 = 30 both A and R are true but R is not the correct explanation for A Ans: b
3	**Assertion (A):** The feasible region represented by $2x + 5y \geq 80$, $x + y \leq 20$, $x \geq 0$, $y \geq 0$ is bounded. **Reason(R):** A region is said to be convex if the line joining any two of its points lies completely in the region. **Solution:** There is no feasible region => A is false R is true Ans: d
4	**Assertion (A):** The maximum value of Z = 11x + 7y subjected to $2x + y \leq 6$, $x \leq 2$, $x ,y \geq 0$, occurs at (0, 6) **Reason(R):** If the feasible region of an LPP is bounded, then maximum and minimum Value of the objective function occurs at corner points. **Solution:** corner points are (0, 6), (3, 2), (3, 0) => Z is max. at (3, 2) A is false , clearly R is true Ans: d
5	The corner points of the feasible region for an LPP are (60, 0), (120, 0), (60, 30) and (40, 20). The objective function Z = ax + by, a, b > 0 has maximum value 600 at points (120, 0) and (60, 30) **Assertion (A):** Minimum value of Z is 300 **Reason(R):** a = 5, b = 10 **Solution:** Z = ax + by maximum value 600 at points (120, 0) and (60, 30)

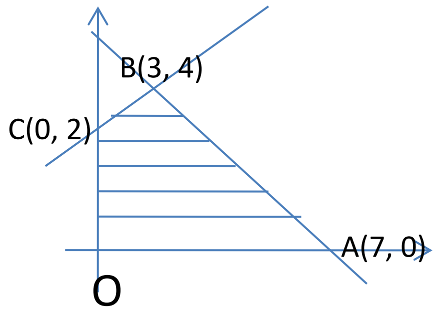
	$120a + 0 = 600 \Rightarrow a = 5$ Also, $60a + 30b = 600 \Rightarrow 60x5 + 30b = 600 \Rightarrow b = 10$ Z at (60, 0) = 5 x 60 + 0= 300 is min. Ans: a
6	**Assertion (A):** If the feasible region of an LPP is bounded, then the objective function Z = ax + by has both maximum and minimum values. **Reason(R):** A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle. **Solution:** conceptual /theory Ans; b
7	Corner points of feasible region are (0, 0), (3, 0) and (0, 3) and the objective function is Z = 4x + 3y **Assertion (A):** Minimum value of Z is 9 **Reason(R):** Maximum value of Z is 21 **Solution:** Z is min. at (0, 3) => $Z_{min.}$ = 4x0 +3x 3 =9 Cleary, A is true and R is false Ans: c
8	**Assertion (A):** The point (4, 2) does not lies in the half plane $4x + 6y – 24 < 0$ **Reason(R):** The point (1, 2) lies in the half plane $4x + 6y – 24 < 0$ **Solution:** Clearly A is true and R is false Ans: c
9	**Assertion (A):** If the corner points of the feasible region for an LPP are (0, 4), (1, 4), (4, 1) and (12, -1), then the minimum value of the objective function Z = 2x + 4y is at (4, 1) **Reason(R):** If the corner points of the feasible region for an LPP are (0, 4), (1, 4), (4, 1) and (12, -1), then the maximum value of the objective function Z = 2x + 4y is 20. **Solution: min =** $Z_{(4, 1)}$ = 2x4 + 4x1 = 12**, max.=** $Z_{(12, -1)}$= 2x12 + 4x(-1) = 20 Hence A is true and R is true but R is not correct explanation Ans: b
10	The corner points of the feasible region for an LPP are (4, 0),(5, 0), (5, 3), (3, 5), (0, 5) and (0, 4). The objective function Z = ax - by + 1900, a, b > 0 has maximum value 1950 at (5, 0) and minimum 1550 at (0, 5). **Assertion (A):** The value of Z at the point (5, 3) is 1740 **Reason(R):** a = 10, b = 70

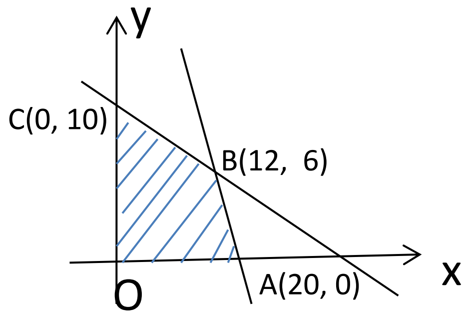
	Solution: $Z_{max.}$at (5, 0) = 1950 => 5a – bx0 + 1900 = 1950 => a = 10 $Z_{min.}$at (0,5) = 1550 => ax0 – bx5 + 1900 =1550 => b = 70 $Z_{(5, 3)}$ = 10x5 – 70x3 + 1900 = 1740 Ans : a

EXERCISE

1	**Assertion:** All feasible regions are convex sets **Reason:** A set is said to be convex set if the line segment joining any two points of the set, is completely within the set
2	**Assertion:** Graphical method is not suitable for solving all Linear programming problems **Reason:** Graphical method is applicable only in case of LPP having two variables
3	**Assertion:** The objective function describes the purpose of formulating LPP **Reason:** The objective function can be maximized or minimized
4	**Assertion:** The objective function is always non-negative **Reason:** The variables involved in the objective function are non-negative due to constraints
	ANSWERS: **1)** d 2) a 3) a 4) a

2 MARKS QUESTIONS

1	Minimise Z = 13x – 15y subject to the constraints x + y ≤ 7, 2x – 3y + 6 ≥ 0, x ≥ 0, y ≥ 0 **Solution:**clearly Z is min. at C(0, 2) and $Z_{min.}$= 13x0 -15x2 = 30 
2	Maximise Z = 80x + 120y subject to the constraints 3x + 4y ≤ 60, x + 3y ≤ 30, $x, y \geq 0$ **Solution:** The corner points are (0, 0), (20, 0), (12, 6), (0, 10)

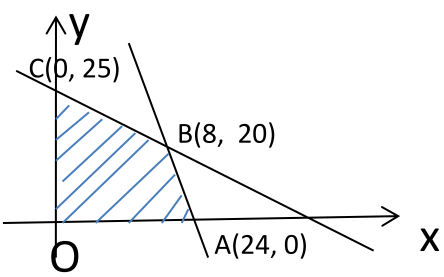


Corner points	Z= 80x + 120y
(0,0)	0
(20, 0)	1600
(12, 6)	1680 maximum
(0, 10)	1200

3 Maximise Z = 100x + 120y subject to the constraints $5x + 8y \leq 200$, $5x + 4y \leq 120$, $x, y \geq 0$

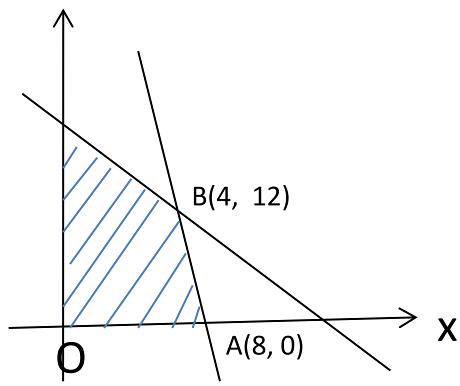
Solution:

Corner points	Z = 100x + 120y
(0, 0)	0
(0, 25)	3000
(24, 0)	2400
(8, 20)	3200



4 Maximise Z = 20x + 10y subject to the constraints $1.5x + 3y \leq 42$, $3x + y \leq 24$, $x, y \geq 0$

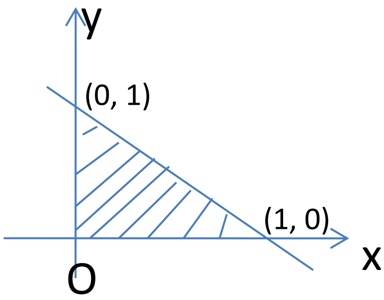
Solution:



Corner points	Z = 20x + 10y
(0, 0)	0
(8, 0)	160
(0, 14)	140
(4, 12)	200 maximum

5 Find max. value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$

Solution:

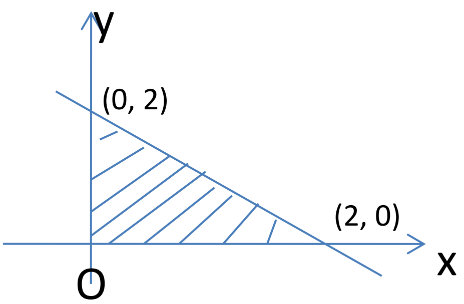


$Z(1, 0) = 3 + 0 = 3$

$Z(0, 1) = 0 + 4 = 4$ maximum

6 Find $Z_{max.} = 3x + 2y$ subject to the constraints $x + y \leq 2$, $x, y \geq 0$

Solution:



Corner points	Z = 3x + 2y
(0, 0)	0
(2, 0)	6 Max.
(0, 2)	4

7 Let $Z = ax + by$ has optimal value at two points (2, 3) and (5, 7), then find the relation between a and b

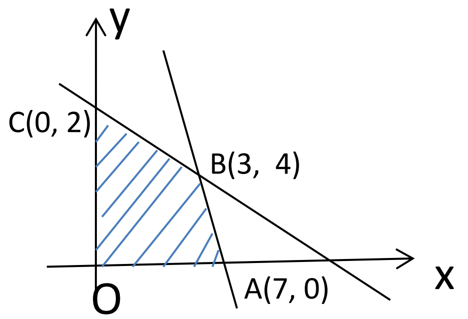
Solution: $Z(2, 3) = Z(5, 7)$

$\Rightarrow 2a + 3b = 5a + 7b$

$\Rightarrow 3a + 4b = 0$

8 Feasible region for an LPP is shown in the figure. Maximize $Z = 5x + 7y$

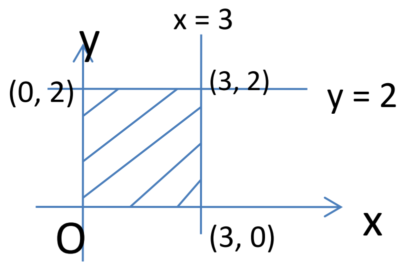
	Solution:



Corner points	Z = 5x + 7y
(0, 0) (7, 0) (3, 4) (0, 2)	0 35 43 Maximum 14

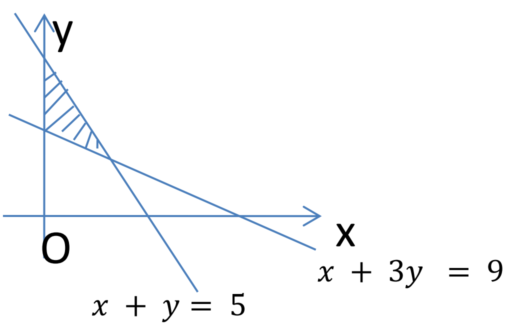
9 Maximise the function Z = 9x + 11y, subject to $x \le 3$, $y \le 2$, $x, y \ge 0$

Solution:



Corner points	Z =9x +11y
(0, 0)	0
(3, 0)	27 Maximum
(0, 2)	22

10 The feasible region for an LPP is shown in the figure. Find the min. value of Z = 11x +7y



Solution:

		Corner points	Z = 11x + 7y
		(0, 3) (3, 2) (0, 5)	21 Minimum 44 35

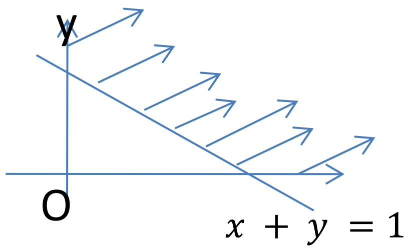
EXERCISE

1	Find the maximum of Z = 6x + 16y subject to $x + y \geq 2$, $x, y \geq 0$
2	Find the maximum value of Z = 3x + 2y where the corner points of the feasible region are (0, 0), (0, 8), (2, 7), (5, 4) and (6, 0)
3	Solve the linear inequation $-3x + 2y \geq 6$ graphically
4	Find the maximum value of Z = 4x + 3y subject to $x + y \leq 10$, $x, y \geq 0$
5	Is the feasible region represented by $x + y \geq 1$, $x, y \geq 0$ bounded? Justify your answer

ANSWERS:

1) 32   2) 23   4) 40

5) Unbounded from the graph

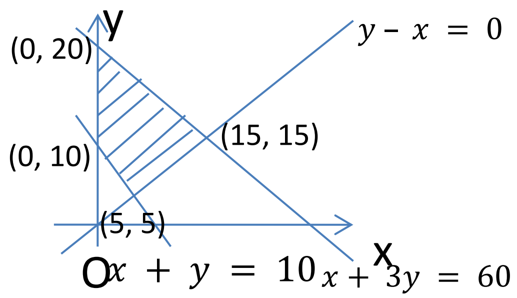


3 MARK QUESTIONS

1	Maximise: Z = 3x + 9y subject to $x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x, y \geq 0$

Solution: On solving x + y = 10 and y – x = 0 we get (5, 5)

Similarly solving y – x = 0 and x + 3y = 60 we get (15, 15)



Corner points	$Z = 3x + 9y$
(0, 20)	180 Maximum
(0, 10)	90
(5, 5)	60
(15, 15)	180 Maximum

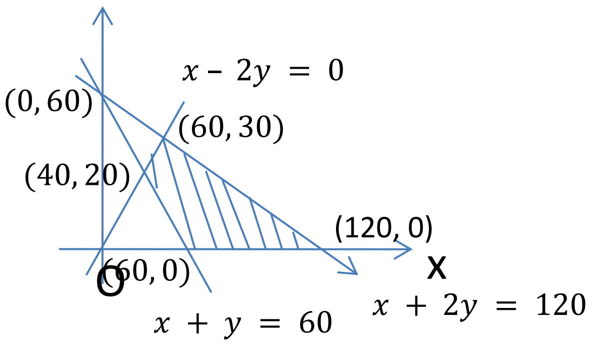
$Z_{max.} = 180$ at infinitely many points on the line segment joining (0, 20) and (15, 15)

2 Maximise: $Z = 5x + 10y$ subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$

Solution:

On solving $x + y = 60$ and $x - 2y = 0$ we get (40, 20)

On solving $x + 2y = 120$ and $x - 2y = 0$ we get (60, 30)



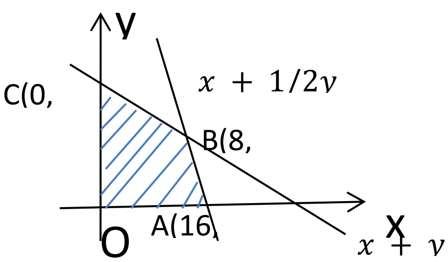
Corner points	$Z = 5x + 10y$
(40, 20)	400
(60, 0)	300 Minimum
(120, 0)	600
(60,30)	600

$Z_{min.} = 300$ at (60, 0)

3 Maximise : $Z = 300x + 190y$ subject to $x + y \leq 24$, $x + \frac{1}{2} y \leq 16$, $x, y \geq 0$

Solution:

On solving $x + y = 24$, $x + \frac{1}{2} y = 16$ we get (8, 16)



Maximum value of Z is 5440 at (8, 16)

EXERCISE

1	Maximise :Z = 6x + 3y subject to 4x + y ≥80, 3x + 2y ≤ 150, x + 5y ≥ 15, x, y ≥ 0
2	Minimise: Z = 200x + 500y subject to x + 2y ≥ 10, 3x + 4 y ≤ 24, x, y ≥ 0
3	Maximise: Z = 20x + 10y subject to 1.5x + 3y ≤ 42, 3x + y ≤ 24 , x, y ≥0

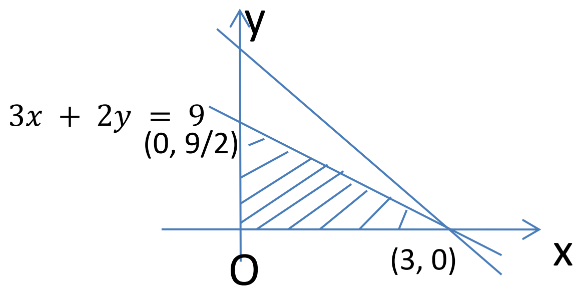
ANSWERS:

1) $Z_{max.}$ = 285 at (40, 15)
2) $Z_{min.}$ = 2300 at (4, 3)
3) $Z_{max.}$ = 200 at (4, 12)

5 MARK QUESTIONS

1 Maximise : Z = 70x + 40y subject to 3x + 2y ≤ 9, 3x + y ≤ 9, x, y ≥ 0

Solution:



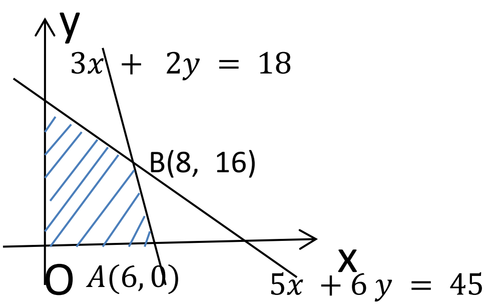
Corner points	Z =70x + 40y
(0, 0)	0
(3, 0)	210 maximum
(0, 9/2)	180

Maximum value of Z is 210 at (3, 0)

2 Maximise : Z = 60x + 40y subject to 5x + 6y ≤ 45, 3x + 2y ≤ 18 , x, y ≥ 0

Solution:

On solving 5x + 6y = 45, 3x + 2y = 18 we get (9/4, 45/8)



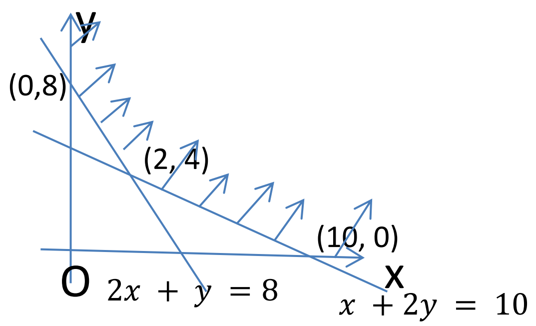
Corner points	Z =60x + 40 y
(0, 0)	0
(0, 15/2)	300
(9/4, 45/8)	360 Maximum
(6, 0)	360Maximum

Maximum value of Z is 360 at any point on the line segment joining (6, 0) and (9/4, 45/8)

3 Minimise: $Z = 5x + 7y$ subject to $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

Solution:

On solving $2x + y = 8$, $x + 2y =10$ we get (2, 4)



Corner points	Z = 5x + 7y
(10, 0)	50
(2, 4)	38 Minimum
(0, 8)	56

The minimum value of Z is 38 at (2, 4)

EXERCISE

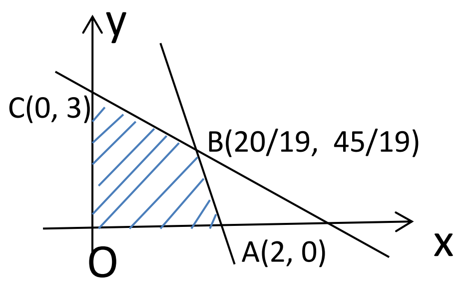
1	Minimise : $Z = 5x + 2y$ subject to $x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x, y \geq 0$
2	Minimise : $Z =x +2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$
3	Minimise : $Z = 3x + 5y$ subject to $x + 2y \geq 10$, $x + y \geq 6$, $3x + y \geq 8$, $x, y \geq 0$

1) $Z_{min} = 0$ at (0 , 0)

2) $Z_{min} = 100$ at all points on the lin segment joining (0, 50) and (20, 40)

3) $Z_{min} = 26$ at (2, 4)

CASE BASED QUESTIONS

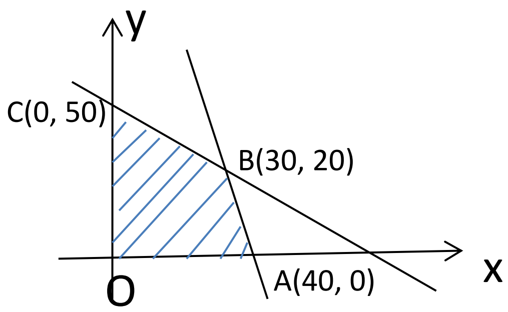
1	A dealer Ram Singh residing in a rural area opens a shop to start his business with an investment of Rs.5760. He wishes to purchase ceiling fans and table fans. A ceiling fan costs him Rs. 360 and table fan costs Rs.240 Based on the above information answer the following questions. i) Ram Singh purchases x ceiling fans and y table fans. He has space in his store for at most 20 items. Write its constraint ii) If he sells ceiling fan at a profit of Rs. 22 and table fan for a profit of Rs. 18, then express the profit Z in terms of x and y iii) What is the maximum profit of selling all the fans **Solution:** i) He has space in store for at most 20 items => $x + y \leq 20$ ii) profit on ceiling fans = Rs.22, Profit on table fans = Rs18 Hence $Z = 22x + 18y$ iii)$360x + 240y \leq 5760$ => $3x + 2y \leq 48$ also $x + y \leq 20$ and $x, y \geq 0$ on solving we get the corner points(0, 0), (16,0), (8, 12), (0, 20) Maximum value of Z occurs at (8, 12) $Z_{max.} = 22 \times 8 + 18 \times 12 = 392$
2	The students of class XII are asked to write linear inequalities in two variables. They have written : $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$ and $y \geq 0$ Based on the above information answer the following questions. i) Draw the feasible region of above system of inequalities ii) Find the corner points of the solution region **Solution:** i)  **(ii)** corner points are (0, 0), (2, 0), (20/19, 45/19) and (0, 3)
3	Corner points of a feasible region of an LPP are (0, 0), (7, 0), (6, 2), (0, 5). Let $Z = 3x + 4y$ be the objective function Based on the above information answer the following questions. i) The minimum value of Z occurs at

	a) (7, 0) b) (6, 2) c) (0, 5) d) (0, 0) ii) The maximum value of Z occurs at a) (7, 0) b) (6, 2) c) (0, 5) d) (0, 0) iii) Maximum value of Z – Minimum value of Z is equal to a) 26 b) 28 c) 21 d) 20 iv) The feasible solution of LPP belongs to a) First and second quadrants b) First and third quadrants **c)** Only second quadrant d) Only first quadrant **Solution:** <table><tr><th>Corner point</th><th>Z = 3x + 4y</th></tr><tr><td>(0, 0)</td><td>**0 Minimum**</td></tr><tr><td>(7, 0)</td><td>21</td></tr><tr><td>(6, 2)</td><td>**26 Maximum**</td></tr><tr><td>(0, 5)</td><td>20</td></tr></table> **i)** d ii) b iii) a iv) d

EXERCISE

1	In an LPP, the objective function Z =3x + 4y + 370 is to be optimized subjected to the constraints : $x + y \geq 10$, $x + y \leq 60$, $x \leq 40$, $x, y \geq 0$ Based on the above information answer the following questions. i) The maximum value of Z occurs at a) (40, 0) b) (40, 20) c) (20, 40) d) (0, 40) ii) The minimum value of Z is a) 300 b) 400 c) 500 d) 600 iii) The value of Z at (40, 20) is a) 490 b) 530 c) 550 d) 570 iv) Max. Z – Min. Z = **a)** 190 b) 210 c) 230 d) 250

2	The feasible region of an LPP is shown in the figure.



i) Equation of AB is

a) $2x + y = 80$ b) $x + y = 50$ c) $x + 2y = 50$ d) $x + y = 40$

ii) Equation of BC is

a) $2x + y = 80$ b) $x + y = 50$ c) $x + 2y = 50$ d) $x + y = 40$

iii) Constraints are

a) $x + y \leq 50,\ 2x + y \leq 80,\ x \geq 0,\ y \geq 0$

b) $x + y \leq 50,\ 2x + y \geq 80,\ x \geq 0,\ y \geq 0$

c) $x + y \geq 50,\ 2x + y \leq 80,\ x \geq 0,\ y \geq 0$

d) None of these

iv) The objective function $Z = 10500x + 9000y$ is maximum at the point

a) A b) B c) C d) O

ANSWERS:

1) i) a ii) b iii) a iv) d

2) i) a ii) b iii) a iv) b

CHAPTER:PROBABILITY

SYLLABUS:

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

Definitions and Formulae:

Conditional Probability: If A and B are two events associated with any random experiment, then $P(A/B)$ represents the probability of occurrence of event A knowing that the event B has already occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0.$$

$P(B) \neq 0,$, means that the event B should not be impossible.

Multiplication Theorem on Probability: If the event A and B are associated with any random experiment and the occurrence of one depends on the other, then

$$P(A \cap B) = P(A).P(B/A), where P(A) \neq 0$$

Independent Events:
When the probability of occurrence of one event does not depend on the occurrence /non-occurrence of the other event then those events are said to be independent events.
Then $P(A/B) = P(A)\ and\ P(B/A) = P(B)$
So, for any two independent events A and B ,$P(A \cap B) = P(A).P(B)$.

Theorem on total probability:
$If E_i(i = 1,2,3, \dots n)$ be a partition of sample space and all E_i havenon-zero roabability.Abeanyeventassociatedwiththesamplespace, which occurs with $E_1 or E_2 or E_3 or \dots or E_n$ then

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + \dots + P(E_n)P(A/E_n)$$

Bayes' Theorem:
"Let " S "be the sample space and " $E_1, E_2, \dots, E_n$ " be " n" mutuallyexclusive and "exhaustive events associated with " a" random experiment." If A is any event which occurs with $E_1\ or\ E_2\ or, .. or\ E_n\ then$

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

Random Variable: It is a real valued function whose domain is the sample space of random experiment.

Probability Distribution: It is a system of number of random variable (X) such that

X	X_1	X_2	….	X_n
P(X)	P_1	P_2	….	P_n

where $P(X_i) > 0$ and $\sum_{i=1}^{n} P(E_i) = 1$

Mean: Mean or Expectation of a random variable X is denoted by E(X)

$$E(X)=\sum_{i=1}^{n} x_i P_i$$

MULTIPLE CHOICE QUESTIONS:

Q. No	QUESTIONS AND SOLUTIONS
1.	If P(A/B)=0.3, P(A)= 0.4 and P(B)=0.8, then P(B/A) is equal to (a) 0.6 (b) 0.3 (c) 0.06 (d) 0.4 **Solution:** $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B)}{0.8}=0.3$ $P(A\cap B)=0.24$ $P(B/A)=0.24/0.4=0.6$ **Ans: (a)**
2.	Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that the she missed the target exactly once is (a) 2/3 (b) 1/3 (c) 4/9 (d) 1/9 **Solution:** $P(A)=\frac{2}{3}, P(A')=\frac{1}{3}\ \ (A-hit, A'-nothit)$ $P(onlyoncehit)=\frac{2}{3}.\frac{1}{3}+\frac{1}{3}.\frac{2}{3}=4/9.$ **Ans: (c)**
3.	For any two events A and B, $P(A)=\frac{4}{5}$ and $P(A\cap B)=\frac{7}{10}$, then $P(B/A)$is (a) 1/10 (b) 1/8 (c) 17/20 (d) 7/8 **Solution:** $P(B/A)=\frac{7/10}{4/5}=7/8.$ **Ans: (d)**
4.	Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is (a) 27/32 (b) 5/32 (c) 31/32 (d) 1/32 **Solution:** $P(atleastoneH)=1-P(noneisH)$ $=1-1/32=31/32.$ **Ans: (c)**

5.	If A and B are two independent events such that P(A)=1/2 and P(B)=1/4, then $P(B'/A)$ is (a) 1/4 (b) 3/4 (c) 1/8 (d) 1 **Solution:** $P(B'/A) = \frac{P(B' \cap A)}{P(A)} = \frac{P(B')P(A)}{P(A)} = P(B') = 3/4$ **Ans: (b)**
6.	If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is (a) 1/9 (b) 4/9 (c) 1/18 (d) ½ **Solution:** $A = Sum9 = \{(3,6),(4,5),(5,4),(6,3)\}$ $B = onedieshows4 = \{(1,4),(2,4),(3,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5),(4,6)\}$ $P(A) = 4/36 P(B) = 10/36$ $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{2/36}{4/36} = 1/2$ **Ans: (d)**
7.	If A and B are two events such that P(A/B)=2.P(B/A) and P(A)+P(B)=2/3, then P(B) is (a) 2/9 (b) 7/9 (c) 4/9 (d) 5/9. **Solution:** $\frac{P(A \cap B)}{P(B)} = 2\frac{P(A \cap B)}{P(A)}$ $P(A) = 2P(B)$ $3P(B) = 2/3$ $P(B) = 2/9$ **Ans: (a)**
8.	If two events A and B, P(A-B)=1/5 and P(A)= 3/5, then P(B/A) is equal to (a) ½ (b) 3/5 (c) 2/5 (d) 2/3 **Solution:** $P(A - B) = 1/5, P(A) = 3/5$ $P(A \cap B) = 3/5 - 1/5 = 2/5$ $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{2/5}{3/5} = 2/3.$ **Ans: (d)**
9.	If $P(A \cap B) = 1/8$ and $P(\bar{A}) = 3/4$, then $P(B/A)$ is equal to

	(a) 1/2 (b) 1/3 (c) 1/6 (d) 2/3 **Solution:** $P(A) = 1 - 3/4 = 1/4$ $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/8}{1/4} = 1/2$ **Ans: (a)**
10.	For any two events A and B, $P(A') = 1/2, P(B') = 2/3$ and $P(A \cap B) = 1/4$, then $P\left(\frac{A'}{B'}\right)$ equals (a) 8/9 (b) 5/8 (c) 1/8 (d) ¼ **Solution:** $(A) = 1/2, P(B) = 1/3$ $P(AUB) = 1/2 + 1/3 - 1/4 = 7/12$ $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(AUB)}{P(B')} = \frac{1 - 7/12}{2/3} = \frac{5/12}{2/3} = 5/8$ **Ans: (b)**

CHAPTER	VIDEO LINK	SCAN QR CODE FOR VIDEO
PROBABILITY	https://youtu.be/YNifzbxLS5M	

EXERCISE

	MCQ -PRACTICE QUESTION
1.	X and Y are independent events such that $P(X \cap \bar{Y}) = 2/5$ and $P(X) = 3/5$. Then P(Y) is equal to: (a) 2/3 (b) 2/5 (c) 1/3 (d) 1/5 **Answer:c**
2.	The probability that A speaks the truth is 4/5 and that of B speaking the truth is ¾. The probability that they contradict each other in stating the same fact is (a) 7/20 (b) 1/5 (c) 3/20 (d) 4/5

	Answer:a
3.	For any two events A and B, if P(A)=0.4 and P(B)=0.8 and P(B/A)=0.6, the $P(A \cup B)$ is: (a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96 **Answer:d**
4.	The events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F)=0.5$ then P(E/F)-P(F/E) equals: (a) 1/7 (b) 2/7 (c) 3/35 (d) 1/70 **Answer:d**
5.	If A and B are independent events such that P(A)=0.4 , P(B) =x and $P(A \cup B) = 0.5$ then x is (a)4/5 (b) 0.1 (c) 1/6 (d) None of these **Answer: c**

ASSERTION-REASONING QUESTIONS

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is **not** the correct explanation of the Assertion (A)
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

1.	**Assertion (A):** Two coins are tossed simultaneously. The probability of getting two heads, if it is know that at least one head comes up, is 1/3. **Reason (R) :** Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$. **Ans:** (a) For Assertion $F = \{HH\}$ $E = \{HH, HT, TH\}$ $P(F/E) = \frac{1/4}{3/4} = 1/3$ A is true and R is a correct formula and correct explanation.
2.	Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A).P(B)$ Assertion (A): P(A/B)=P(A) and P(B/A)=P(B) Reason (R): $P(A \cup B)=P(A)+P(B)$ **Ans:** (c) as A is correct but R is false.

3.	For any two events A and B. P(A)=p and P(B)=q **Assertion (A):** The probability that exactly one of the events A and B occurs is p+q-2pq Reason (R): $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ **Ans:** (b) A is correct but R is not the correct explanation of A.
4.	**Assertion (A):** Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely. If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then P(E/F)=1/4 and P(F/E)=1/13. **Reason (R):** E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events. **Ans:** (b) as A is correct but R is not the correct explanation of A.
5.	Consider that following statements: **Assertion (A):** Let A and B be two independent events. Then P(A and B)= P(A)+P(B) **Reason (R):** Three events A,B and C are said to be independent if $P(A\cap B\cap C)=P(A).P(B).P(C)$ **Ans:** (d) as $P(A \text{ and } B)=P(A)+P(B)-P(A).P(B)$, hence A is false and R is True.
6.	**Assertion (A):** In rolling a die, event A= {1, 3, 5} and event B = {2, 4} are mutually exclusive events. **Reason (R):** in a sample space two events are mutually exclusive if they do not occur at the same time. **Ans:** (a) A is true as $P(A\cap B)=\phi$ and R is the correct explanation of A.
7.	Let A and B be two independent events. Assertion (A): If P(A)=0.3 and $P(A\cup \bar{B})$=0.8 then P(B) is 2/7 Reason (R) : $P(\bar{E})$=1-P(E), for any event E. Ans : (a) as $P(AUB')=P(A)+P(B')-P(A).P(B')$ $P(B')=5/7$ $P(B)=2/7$ Hence A is true and R is the correct explanation for A.
8.	**Assertion (A) :** Let A and B be two events such that P (A) =1/5 and P (A or B) =1/2 then P(B)=3/8 for A and B are independent events. **Reason (R) :** For independent events P (A or B) = P (A) + P(B) - P(A).P(B). **Ans:** (a) as A is true and R is the correct explanation for A. For Assertion : $P(AUB)=1/2$ $P(A)+P(B)-P(A).P(B)=1/2$ $P(B)=3/8\ (\because P(A)=1-1/5=4/5)$ R is correct exxplanation.
9.	**Assertion (A) :** If A and B are mutually exclusive events with P (A') =5/6 and P (B)=1/3. Then P (A/B')=1/4. **Reason (R) :** If A and B are two events such that P (A)=0.2, P(B)=0.6 and P(A/B)=0.2 then the value of P(A/B') is 0.2. **Ans:** (b) as A is true and R is not the correct explanation for A.

	$$P(A/B') = \frac{P(A\cap B')}{P(B')} = \frac{P(A) - P(A\cap B)}{1 - P(B)} = \frac{1/6 - 0}{1 - 1/3} = 1/4$$ *Reason:* $P(A\cap B) = 0.12$ $$P(A/B') = \frac{P(A\cap B')}{P(B')} = \frac{P(A) - P(A\cap B)}{1 - P(B)} = \frac{0.08}{0.4} = 0.2$$
10	**Assertion (A):** If A and B are two independent events with P(A)=1/5 and P(B)=1/5, then P(A'/B) is 1/5. Reason (R) : $P(A'/B) = \frac{P(A'\cap B)}{P(B)}$ **Ans: (d)** $P(A'/B) = \frac{P(A'\cap B)}{P(B)} = \frac{P(A').P(B)}{P(B)} = P(A')$ $= 1 - P(A) = 4/5$ So A is false and R is True.

EXERCISE

1.	**Assertion A**: Two cards are drawn from a well shuffled pack of 52 playing cards without replacement. Probability of getting 02 jacks is 16/169 **Reason (R):** For independent events A and B, P(A and B)=P(A).P(B) **Answer: d**
2.	**Assertion (A):** A can solve 80% of the problems in a book and b can solve 60%. then probability that at least one of them will solve a problem is 0.92 **Reason (R):** P (at least one solve a problem) = 1- P(none of them solve it) **Answer: a**
3.	**Assertion A:** P(A) = 0.6 and P(B) = 0.4 then P(AUB) = 1 when A and B are mutually exclusive events Reason (R) : $P(A/B) = \frac{P(A\cap B)}{P(B)}$ **Answer: b**
4.	The given below is a probability distribution table: X: 0, 1, 2, 3 P: k, k/2, k/4, k/8 **Assertion A:** The value of k is 8/15 **Reason R: Mean of X=** $\sum px$ **Answer: b**
5.	**Assertion A:** Three numbers are selected from first six natural numbers at random without replacement. If X denotes the greatest of three numbers selected, then X={2,3,4,5,6} **Reason R:** Random variable is a real valued function whose domain is a sample space of a random experiment. **Answer: d**

2 MARK QUESTIONS

1.	A pair of dice is thrown. If the two numbers appearing on them are different, find the probability that the sum of the numbers is 6.
	Ans: A: Number appearing are different n(A)=30 (except (1,1),(2,2),(3,3),(4,4),(5,5) and (6,6)) B: Sum of the numbers is 6.

	P(A)=30/36 A and B={(1,5),(2,4),(4,2),(5,1)} P(A and B)=4/36 $P\left(B/A\right)=\frac{P(A\cap B)}{P(A)}=\frac{4/36}{30/36}=4/30$
2.	In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is girl?
	Ans: A: Student of Class XII B: The student is a girl. n(A & B)=10% of 430=43. $P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}=\frac{43}{430}=\frac{1}{10}$
3.	Two balls are drawn from a bag containing 2white,3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?
	Ans: P(at lest one red ball)=1-P(none of the ball is red) (that is 1^{st} ball is non red and 2^{nd} ball is non red.) $=1-\frac{6}{9}.\frac{5}{8}=\frac{42}{72}=\frac{7}{12}$
4.	If P(A)=3/8, P(B)=1/2 and P(A and B)=1/4, find P(A'/B')
	Ans: $P(A'/B')=\frac{P(A'\cap B')}{P(B')}=\frac{P(AUB)'}{1-P(B)}=\frac{1-P(AUB)}{1-P(B)}$ $P(AUB)=P(A)+P(B)-P(A\cap B)$ $=3/8+1/2-1/4=\frac{5}{8}$ $P(A'/B')=\frac{1-P(AUB)}{1-P(B)}=\frac{1-\frac{5}{8}}{1-\frac{1}{2}}=\frac{3/8}{1/2}=3/4$
5.	A committee of 4 students is selected at random from a group of 8 boys and 4 girls. Given that there is at least one girl in the committee, calculate the probability that there are exactly 2 girls in the committee.
	Ans: A: at least one girl in the committee. B: exactly 02 girls in the committee. $P(B/A)=\frac{P(A\cap B)}{P(A)}$ $P(A)=1-P(\text{none is girl})=1-\frac{{}^{8}C_{4}}{{}^{12}C_{4}}=1-\frac{70}{495}=\frac{85}{99}$ $P(A\cap B)=P(\text{2G and 2 B})=\frac{{}^{8}C_{2}.{}^{4}C_{2}}{{}^{12}C_{4}}=\frac{28.6}{495}=\frac{56}{165}$
6.	Events E and F are independent. Find P(F), if P(E) =0.35 and P(EUF)=0.6.
	Ans:

	$P(EUF) = P(E) + P(F) - P(E \cap F)$ $= P(E) + P(F) - P(E).P(F)$ $0.6 = 0.35 + x - 0.35x$ $0.25 = 0.65x$ $x = \frac{25}{65} = \frac{5}{13}$
7.	A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them selected is 0.6. Find the probability that B is selected.
	Ans: E: Selecting A F: Selecting B E and F are independent events. $P(E) = 0.7,$ $P\left[(E \cap F')U(E' \cap F)\right] = 0.6$ $P(E).P(F') + P(E').P(F) - P\left[(E \cap F') \cap (E' \cap F)\right] = 0.6$ $P(E)(1 - P(F)) + (1 - P(E))P(F) = 0.6$ $P(E) + P(F) - 2.P(E).P(F) = 0.6$ $0.7 + x - 2(0.7).x = 0.6$ $0.1 = 0.4x$ $x = \frac{1}{4} = P(F).$
8.	A bag contains 3 white, 4 red and 5 black balls. Two balls are drawn at random. Find the probability that both balls are of different colours.
	Ans: $P(\text{both balls are of different colours}) = 1 - P(\text{both balls of same colour})$ $P(\text{both balls of same colour}) = \frac{^3C_2}{^{12}C_2} + \frac{^4C_2}{^{12}C_2} + \frac{^5C_2}{^{12}C_2} = \frac{3}{66} + \frac{6}{66} + \frac{10}{66} = \frac{19}{66}$ $P(\text{both balls are of different colours}) = 1 - \frac{19}{66} = \frac{47}{66}$
9.	An unbiased die is thrown thrice. Find the probability of getting at least 2 sixes.
	Ans: $P(\text{at least 2 sixes}) = P(\text{02 sixes}) + P(\text{03 Sixes})$ $= 3.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \frac{1}{6}.\frac{1}{6}.\frac{1}{6} = \frac{16}{216} = \frac{2}{27}$
10.	A problem is given to A, B and C. The probabilities that they solve the problem correctly are 1/3, 2/7 and 3/8 respectively. If they try to solve the problem simultaneously, find the probability that exactly one of them solve the problem.
	Ans: P(Exactly one solve)=P(AB'C')+P(A'BC')+P(A'B'C) $= \frac{1}{3}.\frac{5}{7}.\frac{5}{8} + \frac{2}{3}.\frac{2}{7}.\frac{5}{8} + \frac{2}{3}.\frac{5}{7}.\frac{3}{8}$ $= \frac{25 + 20 + 30}{168} = \frac{75}{168} = \frac{25}{56}$

EXERCISE

1.	A die is tossed once. If the random variable X is defined as $X = \begin{cases} 1, \text{ if the die shows an even number} \\ 0, \text{otherwise} \end{cases}$ Find the mean of X.
2.	There are 5 bags, each containing 5 white and 3 black balls. Also, there are 6 bags, each containing 2 white balls and 4 black balls. A ball is taken at random from a bag. Find the probability that it is a white ball.
3.	The odds against a man who is 45 year old, living till he is 70 are 7:5 and the odds against his wife who is now 40, living till she is 65 are 5:3. Find the probability that the couple will be alive 25 years hence.
4.	A coin is tossed thrice. Let E be the event, 'first throw results in a head' , and the event F, 'the last throw results in a tail'. Find whether the events E and F are independent.
5.	In a class, 40% students study mathematics; 25% study biology and 15% study both mathematics and biology. One student is selected at random. Find the probability that he studies biology if it is known that he studies mathematics.
Answers: (1) 1/2 (2) 41/88 (3) 5/32 (4) Yes. E and F are independent (5) 3/8	

3 MARK QUESTIONS

1. The probability distribution of a random variable X is given below:

X	1	2	3
P(X)	k/2	k/3	k/6

(i) Find the value of k (ii) Find $P(1 \le X)$ (iii) Find E(X), the mean of X.

Ans:

$(i) \sum p = 1$

$$\frac{k}{2} + \frac{k}{3} + \frac{k}{6} = \frac{12k}{12} = k$$

$k = 1$

$(ii) P(1 \le x) = P(1) + P(2) + P(3) = 1/2 + 1/3 + 1/6 = 1$

$(iii) E(X) = \sum px = 1/2 + 2/3 + 3/6 = 5/3$

2. A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$

Find P(A) and P(B).

Ans:

	$P(A).P(B') = 1/4$ & $P(A').P(B) = 1/6$ $P(A) = x,\ P(B) = y$ $x(1-y) = 1/4$ and $(1-x)y = 1/6$ on solving we get, $x - y = 1/12$ $x = y + 1/12$ On substituting, we get $12y^2 - 11y + 2 = 0$ $y = 1/4$ or $y = 2/3$ Corresponding $x = 1/3$ or $3/4$ $P(A) = 1/3\ P(B) = 1/4$ (OR) $P(A) = 3/4\ P(B) = 2/3$
3.	Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.
	Ans: X={0,1,2}-No. of red Balls. P(0)=G and G =1/2.1/2=1/4 P(1)=GR +RG=1/2.1/2+1/2.1/2=1/2 P(2)=RR=1/2.1/2=1/4 <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>1/4</td><td>1/2</td><td>1/4</td></tr><tr><td>xP(x)</td><td>0</td><td>1/2</td><td>1/2</td></tr></table>Mean=0+1/2+1/2=1
4.	A and B throw a die alternately till one of them get a '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.
	Ans: S-Getting 6 F- not getting 6 P(S)=p=1/6 P(F)=q=5/6 P(A-Wins)=p+qqp+qqqqp+…… $=1/6+(5/6)^2 1/6+(5/6)^4 1/6+\ldots$ $= \dfrac{1/6}{1-\dfrac{25}{36}} = \dfrac{6}{11}$ $P(B - Wins) = 1 - \dfrac{6}{11} = \dfrac{5}{11}$
5.	Two number are selected from first six natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X.Also find the mean of X
	Ans: Total number of ways of selecting two numbers one by one without replacement=6x5=30 X=the greater number obtained={2,3,4,5,6} P(2)=2/30=1/15 (1,2),(2,1) P(3)=4/30=2/15 (1,3),(2,3),(3,1),(3,2) P(4)=6/30=3/15 (1,4),(2,4),(3,4),(4,1),(4,2),(4,3) P(5)=8/30=4/15 similarly as above P(6)=10/30=5/15 <table><tr><td>X</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>P(X)</td><td>1/15</td><td>2/15</td><td>3/15</td><td>4/15</td><td>5/15</td></tr><tr><td>X(P(X)</td><td>2/15</td><td>6/15</td><td>12/15</td><td>20/15</td><td>30/15</td></tr></table>

	$\text{Mean} = \sum XP(X) = \frac{2}{15} + \frac{6}{15} + \frac{12}{15} + \frac{20}{15} + \frac{30}{15} = \frac{70}{15} = \frac{14}{3}$
6.	A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B is the event, "3 comes on the die". Find whether A and B are independent events or not.
	Ans: S=((H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)} A: H appears B: 3 on die P(A)=6/12=1/2 P(B)=2/12=1/6 P(A and B)=1/12 P(A).P(B)=1/2.1/6=1/12=P(A and B) Hence A and B are independent events.
7.	A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.
	Ans: X={0,1,2,3,4,5} X: 0, 1, 2, 3, 4, 5 P(X): 6/36, 10/36, 8/36, 6/36, 4/36, 2/36 XP(X): 0, 10/36, 16/36, 18/36, 16/36, 10/36 Mean= $\sum PX = 70/36 = 35/18$
8.	There are two coins. One of them is a biased coin such that P(head): P(tail) is 1:3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.
	Ans: A: Selecting Biased Coin B; Selecting fair coin C: Getting H P(A)=P(B)=1/2, P(C/A)=1/4 (since ratio for head and tail is 1:3) P(C/B)=1/2 P(C) =P(A).P(C/A)+P(B).P(C/B)=1/2.1/4+1/2.1/2=1/8+1/4=3/8 $P(B/C) = \frac{P(B).P(C/B)}{P(A).P(C/A)+P(B).P(C/B)} = \frac{1/4}{3/8} = 2/3$
9.	From a log of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.
	Ans: X={0,1,2} p-getting good 24/30=4/5 q=getting defective=6/30=1/5 P(X=0)=both good=4/5.4/5=16/25 P(X=1)=1 good and 1 def=4/5.1/5+1/5.4/5=8/25 P(X=2)=both defective=1/5.1/5=1/25 X: 0, 1, 2 P(X): 16/25, 8/25, 1/25 XP(X): 0, 8/25, 2/25 $\text{Mean} = \sum xP(x) = 0 + 8/25 + 2/25 = 10/25 = 2/5$
10.	Two fair dice are thrown simultaneously. If X denotes the number of sixes, find the mean of X.
	Ans: X={0,1,2} P(0) =both non six=5/6.5/6=25/36 P(1)=one six and one non-six=2.(1/6.5/6)=10/36 P(2)=both six=1/6.1/6=1/36

X	0	1	2
P(X)	25/36	10/36	1/36
XP(X)	0	10/36	2/36

$\text{Mean} = \sum xP(x) = 0 + 10/36 + 2/36 = 12/36 = 1/3$

EXERCISE

1.	In a game, a man wins Rs.5 for getting a number greater than 4 and loses Rs.1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.
2.	A urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the mean of the distribution of the number of red balls drawn.
3.	A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.
4.	A coin is biased so that the head is 4 times as likely to occur as tail. If the coin is tossed thrice. Find the mean of the distribution of number of tails.
5.	If A and B are two independent events, then prove that the probability of occurrence of at least one of A and B is given by 1- P(A').P(B').

Answers: (1) Mean=57/27 (2) Mean= 8/3 (3) For A wins:12/23, B wins:11/23
(4) Mean=3/5

5 MARK QUESTIONS

1. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/5 is the probability that he knows the answer and 2/5 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/3. What is the probability that the student knows the answer, given that he answered it correctly?

Ans: A: Knows the answer B: Guesses the answer E: Answered Correctly
P(A)=3/5, P(B)=2/5 P(E/A)=1, P(E/B)=1/3
By Bayes theorem,

$$P(A/E) = \frac{P(A).P(E/A)}{P(A).P(E/A) + P(B).P(E/B)}$$

$$= \frac{\frac{3}{5}.1}{\frac{3}{5}.1 + \frac{2}{5}.\frac{1}{3}} = \frac{3}{11}$$

2. A box contains 10 tickets , 2 of which carry a prize of Rs.8 each, 5 of which carry a prize of Rs.4 each, and remaining 3 carry a prize of Rs.2 each. If one ticket is drawn at random, find the mean value of the prize.

Ans:

X	8	4	2
P	2/10	5/10	3/10
XP	16/10	20/10	6/10

Mean=42/10=4.2 Rs. 4.2

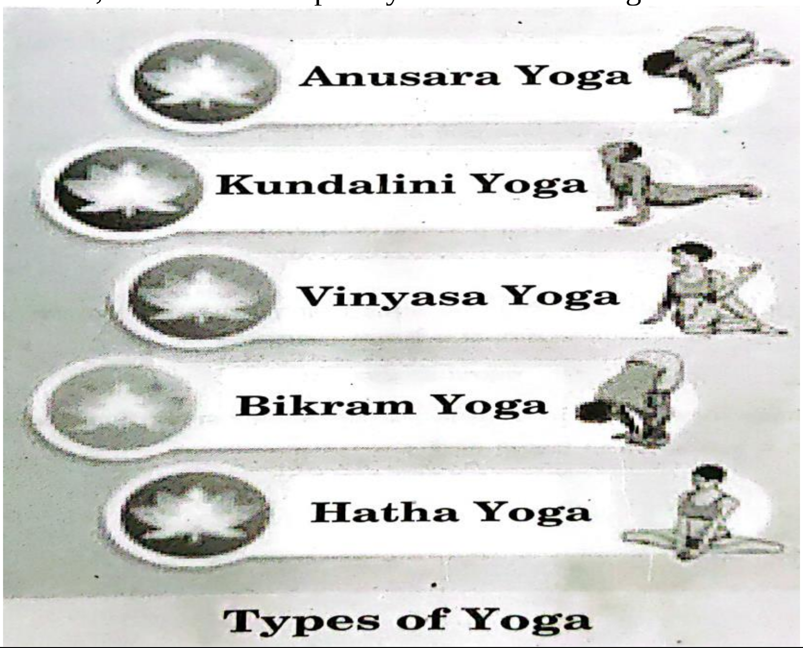
3. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

	Ans: A : two headed B : Biased(75%H) coin C : biased (40%T) $P(A) = P(B) = P(C) = 1/3$ H : getting H $P(H/A) = 1$, $P(H/B) = 75/100$, $P(H/C) = 60/100$ $P(A/H) = \frac{P(A).P(H/A)}{P(A).P(H/A) + P(B).P(H/B) + P(C).P(H/C)} = \frac{100}{235}$

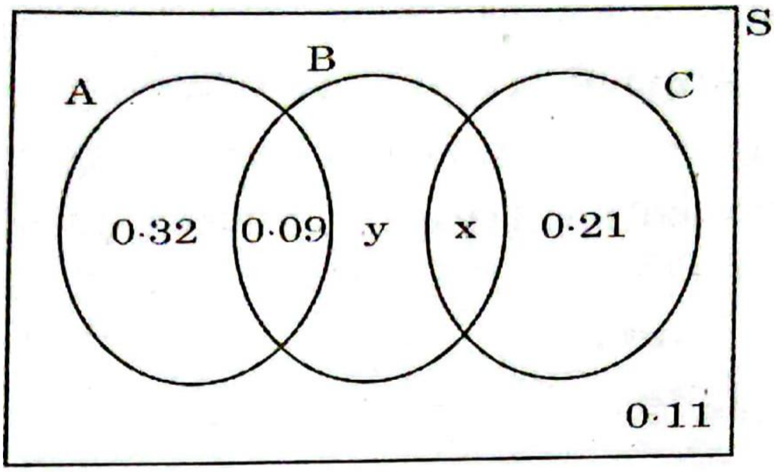
EXERCISE

	LONG ANSWER (LA)- PRACTICE QUESTIONS
1.	A man is known to speak truth 3 out of 4 times. He throws a die and report that it is a more than 4. Find the probability that it is actually more than 4.
2.	Bag A contains 3 red and 5 black balls, while bag b contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from A to B.
3.	In a factory which manufactures bolts, machine A,B and C manufacture respectively 30% , 50% and 20% of the bolts. Of their outputs, 3, 4 and 1 percent respectively are defective bolts. a bolts is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.
4.	An electronic assembly consists of two sub-systems say A and B. From previous testing procedures, the following probabilities are assumed to be known: P(A fails)=0.2, P(B fails alone)=0.15, P(A and B fail)=0.15. Evaluate the following probabilities. (i) P(B fails) (ii) P(A fails or B fails) (iii) P(A fails/B has failed) (iv) P(A fails alone)
Answers: (1) 6/10 (2) 18/133 (3) 11/31. (4) (i) 0.30 (ii) 0.55 (iii) 0.5 (iv)0.05	

CASE STUDY QUESTIONS

	Case Study-04 Marks
1.	There are different types of yoga which involve the usage of different poses of yoga Asanas, Meditation and pranayama as show in fig. below. 

The Venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society. Further, it is given that the probability of a member performing type C yoga is 0.44.



(i) Find the value of x. (ii) Find the value of y (iii) (a) Find P(C/B) (OR)

(a) Find the probability that a randomly selected person of the society does Yoga type A or B but not C.

Ans: (i) x=0.44-0.21=0.23 (ii) y=1-0.96=0.04

$$\text{(iii) } P\left(C/B\right)=\frac{P(C\cap B)}{P(B)}=\frac{0.23}{0.36}=\frac{23}{36}$$

(OR)

$$P(\text{A OR B notC})=0.32+0.09+y=0.41+0.04=0.45$$

2. Recent studies suggest that roughly 12% of the world population is left handed. Depending upon the parents, the chances of having a left handed child are as follows:

A: When both father and mother are left handed:
Chances of left handed child is 24%

B: When father is right handed and mother is left handed:
Chances of left handed child is 22%

C: When father is left handed and mother is right handed:
Chances of left handed child is 17%

D: when both father and mother are right handed:
Chances of left handed child is 9%

Assuming that P(A)=P(B)=P(C)=P(D)=1/4 and L denote the event that child is left handed. Based on the above information, answer the following questions:

(i) Find P(L/C) (ii) Find (L'/A)

(iii) (a) Find P(A/L) (OR) (b) Find the probability that a randomly selected child is left handed given that exactly one of the parent is left handed.

Ans:

(i) P(L/C) =17/100

(ii) P(L'/A) =1-24/100=76/100

(iii) $(a)P(A/L) \quad = \frac{P(A\cap L)}{P(L)}$

$$P(A/L) = \frac{P(A\cap L)}{P(L)} = \frac{6/100}{12/100} = 1/2$$

(OR)

$$(b)P(\frac{L}{BUC}) = \frac{P[(L)\cap(BUC)]}{P(BUC)}$$

$$= \frac{P[(L\cap B)U(L\cap C)]}{P(B)+P(C)-P(B).P(C)}$$

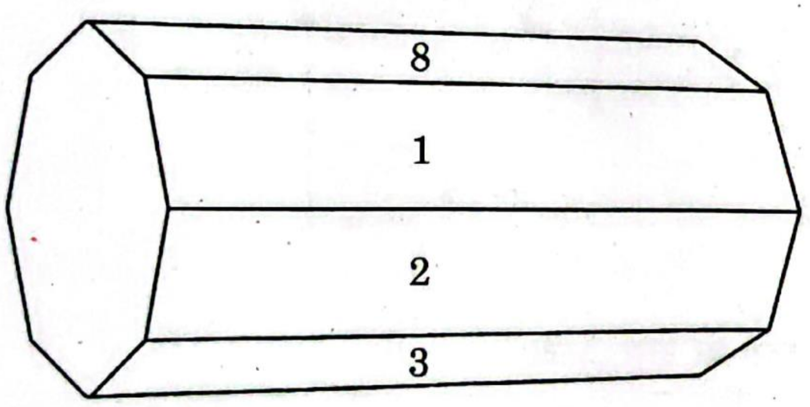
$$= \frac{P(L\cap B)+P(L\cap C)-P(L\cap B)P(L\cap C)}{P(B)+P(C)-P(B).P(C)}$$

$$= \frac{\frac{22}{100}.\frac{1}{4}+\frac{17}{100}.\frac{1}{4}-\frac{22}{400}.\frac{17}{400}}{\frac{1}{4}+\frac{1}{4}-\frac{1}{4}\frac{1}{4}}$$

$$= \frac{\frac{39}{400}-\frac{374}{160000}}{\frac{1}{2}-\frac{1}{16}}$$

$= 0.217$

3. An octagonal prism is a three-dim polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denotes the number obtained on the bottom face and the following table give the probability distribution of X.

X	1	2	3	4	5	6	7	8
P(X)	p	2p	2p	p	2p	P^2	$2\ P^2$	$7\ P^2+p$

Based on the above information, answer the following questions:

(i) Find the values of p.

(ii) Find P(X>6)

(iii) (a) Find P(X=3m), when m is a natural number

(OR)

(a) Find the Mean E(X).

Ans:

(i) Sum of P(X)=1 , $9p+10p^2=1$ on solving p=1/10 or p=-1<0 (Rej) . p=1/10

(ii) $P(x>6)=P(7)+P(8)=2p^2+7p^2+p=9/100+1/10=19/100$

(iii) (a) $P(X=3m)=P(x=3)+P(x=6)=2p+p^2=2/10+1/100=21/100$ (OR)

(b) $E(X) = \sum xP(x) = 33p + 76p^2 = \frac{33}{10} + \frac{76}{100} = \frac{406}{100}$

EXERCISE

<table>
<tr><td>1.</td><td>A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. From this situation answer the following:

(i) What is the probability that B,C will hit and A will lose?
(ii) What is the probability that none of them hit the target?
(iii) (a) What is the probability that only one of them hit the target? (OR)
(b) What is the probability that at least two of them hit the target?</td></tr>
<tr><td>2.</td><td>A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job s 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.
Let E1: represent the even when many workers were not present for the job
E2: represent the event when all workers were present
E3: represent completing the construction work on time
Based on the above information, answer the following questions:
(i) What is the probability that all the workers are present for the job?
(ii) What is the probability that construction will be completed on time?
(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time?
(OR)
(b) What is the probability that all workers were present given that the construction job was completed on time?</td></tr>
<tr><td colspan="2">Answers: (1) (i) 1/10 (ii) 1/60 (iii) (a) 9/60 (b) 50/60
(2) 35/100 (ii) 5075/10000 (iii) (a) 2275/5075 (b) 2800/5075</td></tr>
</table>

SAMPLE QUESTION PAPER - 1

BLUE PRINT

CLASS XII MATHEMATICS

CHAPTERS	MCQ	A & R	VSA(2M)	SA (3M)	LA(5M)	CSQ(4M)	TOTAL
Relations & Functions Inverse Trigonometric Functions	-	1	1	-	1	-	8
Matrices & Determinants	5	-	-	-	1	-	10
Continuity & Differentiability	2	-	1	-	-	-	4
Application Of Derivatives	-	-	1	-	-	2	10
Integrals	2	-	-	3	-	-	11
Application Of Integrals	-	-	-	-	1	-	5
Differential Equations	2	-	-	1	-	-	5
Vector Algebra	3	-	1	-	-	-	5
Three-Dimensional Geometry	1	1	1	-	1	-	9
Linear Programming Problem	2	-	-	1	-	-	5
Probability	1	-	-	1	-	1	8
	18(1M)	2(1M)	5(2M)	6(3M)	4(5M)	3(4M)	80 M

CLASS XII : MATHEMATICS
SAMPLE QUESTION PAPER - 1

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A
(Multiple Choice Questions)
Each question carries One Mark

1. Given a matrix A $=[a_{ij}]$ of order 3×3 whose elements $a_{ij} = \frac{(2i-j)^2}{i+j}$, then the element a_{32} of matrix A is :

a) 12 b) 18 c) $\frac{16}{5}$ d) $\frac{15}{4}$

2. If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then A^2 is equal to

a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

3. $If\ A = (1 \quad 2 \quad 3), then\ AA'\ is\ equal\ to$

a) $(1 \quad 4 \quad 9)$ $b)$ $\begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$ $c)$ (14) d) (6)

4. If $A = \begin{pmatrix} \sin 15° & \cos 15° \\ -\sin 75° & \cos 75° \end{pmatrix}$ then the value of $|A|$ is

a) 0 b) 1 c) -1 d) $-\frac{\sqrt{3}}{2}$

5. If $A = \begin{pmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{pmatrix}$,then A^{-1} exists if :

a) $\lambda = 2$ b) $\lambda \neq 2$ c) $\lambda \neq \frac{8}{5}$ d) $\lambda \neq -\frac{8}{5}$

6. For what value of λ, the function defined by $f(x) = \begin{cases} \lambda(x^2 - 2x), \; if \; x \le 0 \\ 4x + 1, \; if \; x > 0 \end{cases}$

is continuous at $x = 0$

a) $\lambda = 1$ b) $\lambda = 2$ c) No possible value exists d) $\lambda = -2$

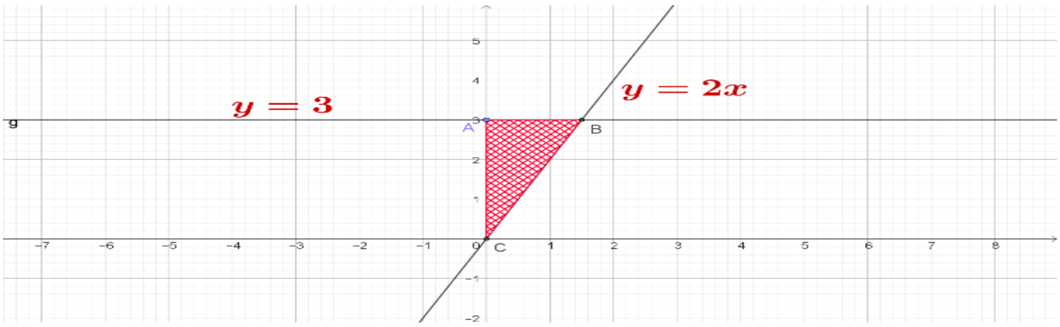
7. If $3x + 2y = sin\, y$, then $\frac{dy}{dx}$ is :

a) $\frac{3}{\cos y - 2}$ b) $\frac{\sin y - 3}{2}$ c) $\frac{2 - \sin y}{3}$ d) $\frac{2 - \cos y}{3}$

8. The value of $\int_0^{\frac{\pi}{2}} e^x(\sin x + \cos x)\, dx$ is

a) e b) $e^{\frac{\pi}{2}}$ c) $e^{\frac{\pi}{2} - 1}$ d) e^2

9.The area bounded by the shaded figure is :



a) $\frac{1}{4}$ b) $\frac{1}{4}$ c) $\frac{9}{4}$ d) $\frac{5}{4}$

10. If m and n are the order and degree of the differential equation

$\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then m + n =

a) 1 b) 9 c) 3 d) 4

11. The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

a) $\frac{e^x}{x}$ b) $\frac{e^{-x}}{x}$ c) xe^x d) x^2e^x

12. The vector in the direction of the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 9 is:

a) $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ b) $\frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$

c) $3(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ d) $9(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

13. If θ is the angle between the vectors, $\vec{a}$ and $\vec{b}$ then $\vec{a} \bullet \vec{b} \geq 0$, only if

a) $0 < \theta < \frac{\pi}{2}$ b) $0 \leq \theta \leq \frac{\pi}{2}$

c) $0 < \theta < \pi$ d) $0 \leq \theta \leq \pi$

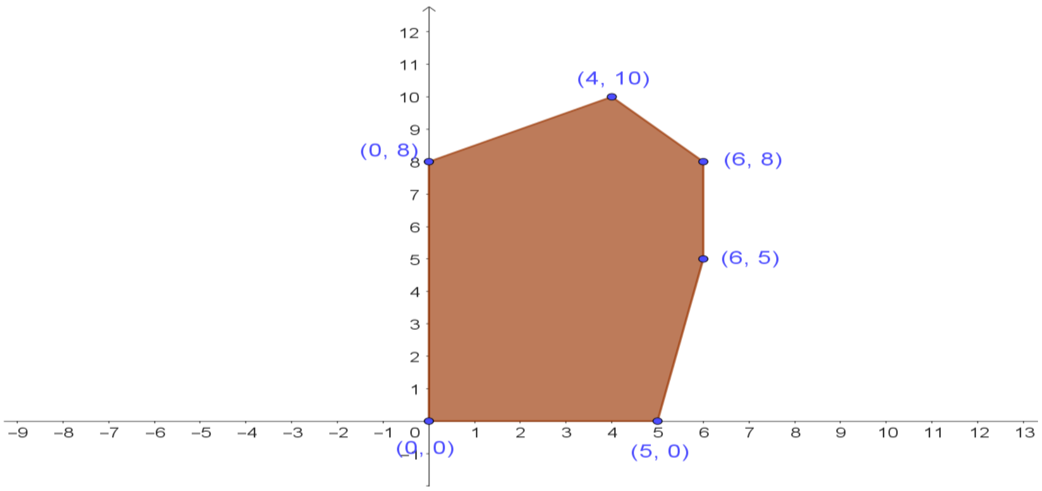
14. If $|\vec{a}| = 10$, $|\vec{b}| = 2$, $\vec{a} \bullet \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ *is*

a) 5 b) 10 c) 14 d) 16

15. If a line makes angles α, β, γ with the positive direction of the coordinate axes, then the value of $sin^2\alpha + sin^2\beta + sin^2\gamma$ is :

a) 0 b) 1 c) 2 d) 3

16. The feasible solution for a LPP is shown in the given figure.



Let Z = 3x – 4y. Minimum of Z occurs at:

a) (0,0) b) (0 ,8) c) (5 ,0) d) (4, 10)

17. The graph of the inequality $2x + 3y > 6$ is

a) Half plane that contains the origin
b) Half plane that neither contains the origin nor the points on the line $2x + 3y = 6$
c) Whole XOY plane except the points on the line $2x + 3y = 6$
d) Entire XOY plane

18. A die is thrown and a card is selected at random from a pack of 52 cards.

The probability of getting an even number and a spade card is:

a) $\frac{1}{4}$ b) $\frac{1}{8}$ c) $\frac{3}{4}$ d) $\frac{17}{52}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion**: The domain of the function $sec^{-1}(2x)$ is

$\left(-\infty \quad -\frac{1}{2}\right) \cup \left(\frac{1}{2} \quad \infty\right)$

Reason : $sec^{-1}(-2) = -\frac{\pi}{4}$

20. **Assertion:(A)** The lines $\frac{x+1}{2} = \frac{y}{5} = \frac{z+3}{4}$ and $\frac{x+3}{1} = \frac{y+5}{2} = \frac{z-4}{-3}$ are perpendicular

Reason:(R): The lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

SECTION B

(Each question carries 2 marks)

21. Given a relation R on the set R, the set of real numbers as

R = $\{(x, y), x^2 - 3xy + 2y^2 = 0\}$. Is R reflexive, symmetric?

22. Find the value of 'k' for which $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} & x \neq 2 \\ k & x = 2 \end{cases}$ is continuous at $x = 2$

23. Show that the function $f(x) = tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4} \quad \frac{\pi}{2}\right)$

(OR)

A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate

24. Find the area of the parallelogram whose one of the sides is $\hat{\imath} - \hat{\jmath} + \hat{k}$ and diagonal is $4\hat{\imath} + 5\hat{\jmath}$

25. Find the vector equation of the line passing through the point $A(1, 1, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$

(OR)

Find the direction ratios and direction cosines of the line whose equation is $6x - 12 = 3y + 9 = 2z - 2$

SECTION C

(Each question carries 3 marks)

26. Evaluate: $\int \frac{\sin x}{\sqrt{1+\sin x}} dx$

(OR)

Evaluate: $\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx$

27. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, log\left(\frac{1+x}{1-x}\right) dx$

28. Evaluate: $\int \frac{cos^{-1}x}{x^2} dx$

29. Solve : $x\frac{dy}{dx} = y - x \, tan\left(\frac{y}{x}\right)$

(OR)

Solve : $\frac{dy}{dx} = \frac{x - y \cos x}{1 + \sin x}$

30. Solve the Linear Programming problem graphically:

Min $Z = 16x + 20y$,

subject to the constraints: $x + 2y \geq 10, x + y \leq 6, 3x + y \geq 8, x \geq 0, y \geq 0$

31. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs are drawn at random. Find the probability of the number of defective bulbs.

SECTION D

(Each question carries 5 marks)

32. Let $f: W \to W$ be defined by $f(x) = \begin{cases} x+1, if\ x\ is\ even \\ x-1, is\ x\ is\ odd \end{cases}$. Show that f is bijective.

33. If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

34. Using integration find the area of the region

$\{(x\,, y): 9x^2 + y^2 \le 36, 3x + y \ge 6\}$

(OR)

Find the area bounded by the region $\{(x\,, y):\ x^2 +\ y^2 \le 1 \le x + y\}$

35. Find shortest distance between the following pairs of lines

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

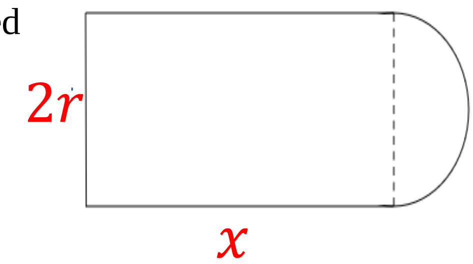
(OR)

Find the vector equation of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+3}{4}$ and $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines

SECTION – E

(This section consists of three case study questions of 4 marks each)

36. A company is interested in making a new complex. It is planned to make an open space the shape of rectangle and a restaurant in the form of semi-circular portion is attached along the breadth



The total perimeter of the complex is 400 m.

Based on the above information answer the following questions

a) Derive the function for the total area of open space and the restaurant

b) What is the value of r for which the area of the rectangular region is maximum

37. The Production function of a company is given by

$$P(x) = x^3 - 12x^2 + 36x + 17$$

where x is the years of production.

The company wants to decide the production rate for first 10 years

a) Find the range of years in which the production was increasing

b) Find the range of years in which the production was decreasing

38. In a hockey match both teams A and B scored same number of goals up to the end of the match.

To decide the winner, the referee asked both captains to throw a die alternatively and decided that the team, whose captain gets a six first, will be declared winner.

a) If the captain of the team A was asked to start, find their respective probabilities of winning the match

b) Show that the decision of the referee was not fair.

CLASS XII : MATHEMATICS - 1
SAMPLE PAPER
MARKING SCHEME

Q.No	Answer	Marks
1	**c)** $\frac{16}{5}$	1
2	**d)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1
3	**c)** (14)	1
4	**b) 1**	1
5	**d)** $\lambda \neq -\frac{8}{5}$	1
6	**c) No possible value exists**	1
7	**a)** $\frac{3}{\cos y - 2}$	1
8	**b)** $e^{\frac{\pi}{2}}$	1
9	**c)** $\frac{9}{4}$	1
10	**c) 3**	1
11	**a)** $\frac{e^x}{x}$	1
12	**c)** $3(\hat{i} - 2\hat{j} + 2\hat{k})$	1
13	**b)** $0 \leq \theta \leq \frac{\pi}{2}$	1
14	**d) 16**	1
15	**c) 2**	1
16	**b) (0 ,8)**	1
17	**b) Half plane that neither contains the origin nor the points on the line** $2x + 3y = 6$	1
18	**b)** $\frac{1}{8}$	1
19	**(c) A is true but R is false**	1
20	**b) Both A and R are true but R is not the correct explanation of A**	1
21	**Proving R is Reflexive**	1
	Proving R is not symmetric	1
22	**Using the condition for continuity x = 2**	1
	Getting the value of k = 7	1
23	**Find** $f'(x)$	1
	Checking the nature of $f'(x)$ **in the given interval and writing the conclusion**	1

Q.No	Answer	Marks
23	**Find the derivative and using the condition**	1
	Find the points as $\left(\mathbf{1} \quad \frac{\mathbf{5}}{\mathbf{3}}\right)$ **and** $\left(\mathbf{-1} \quad \frac{\mathbf{1}}{\mathbf{3}}\right)$	1
24	**Getting the other side as** $\vec{\boldsymbol{b}} = 3\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + 4\hat{\boldsymbol{k}}$	1
	Getting the area as $\sqrt{\mathbf{42}}$ **square units**	1
25	**Getting the direction ratios of the required line as** $\mathbf{7} : \ \mathbf{-5} : \ \mathbf{1}$	1
	Getting the required equation as $\vec{\boldsymbol{r}} = \ (\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{k}}) + \boldsymbol{\lambda}(7\hat{\boldsymbol{\imath}} - 5\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}})$	1
25	**Getting the direction ratios of the line as** $\frac{\mathbf{1}}{\mathbf{6}} : \frac{\mathbf{1}}{\mathbf{3}} : \frac{\mathbf{1}}{\mathbf{2}}$	1
	Getting the direction cosines as $\frac{\mathbf{1}}{\sqrt{\mathbf{14}}}, \frac{\mathbf{2}}{\sqrt{\mathbf{14}}}, \frac{\mathbf{3}}{\sqrt{\mathbf{14}}}$	1
26	**Splitting the integral**	1
	Getting the answer as $-2\sqrt{2}\sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + \frac{2}{\sqrt{2}}\,log\left\|\sec\left(\frac{\pi}{4} - \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\|$+c	1+1
26	**Converting into partial fractions using substitution**	1
	Getting the correct values of the constants at partial fractions	1
	Getting the answers as $\boldsymbol{log}(\mathbf{1} \ - \boldsymbol{cos}\ \boldsymbol{x}) - \boldsymbol{log}(\mathbf{2} \ - \boldsymbol{cos}\ \boldsymbol{x}) + \boldsymbol{C}$	1
27	**Proving f(x) is an odd function**	2
	Getting the answer as 0 by using the property	1
28	**Converting the problem to integration by parts using substitution**	1
	Integrating using integration by parts	1
	Getting the answer as $-\frac{\cos^{-1}x}{x} + \log\left\|\frac{1 + \sqrt{1 - x^2}}{x}\right\| + c$	1
29	**Converting the given Differential Equation by using substation y = vx**	1
	Applying the variable separable method and separating the variables	1
	Getting the solution : $\boldsymbol{log}\left(\boldsymbol{sin}\ \frac{\boldsymbol{y}}{\boldsymbol{x}}\right) = -\log \boldsymbol{x} + \boldsymbol{C}$	1
29	**Writing in general form of Linear Differential Equation**	1
	Find the Integrating factor as $\mathbf{1} + \boldsymbol{sin}\ \boldsymbol{x}$	1
	Getting the general solution as	1

	$y(1+\sin x)=\frac{x^2}{2}+C$	
Q.No	**Answer**	**Marks**
30	**Drawing the graph of lines**	1
	Getting the corner points as A($\frac{6}{5}$, $\frac{22}{5}$), **B(1 , 5) ; C(2 , 4), D(10 , 0)**	1
	Getting the minimum value as 107.2 at $\left(\frac{6}{5},\frac{22}{5}\right)$	1
31	**Getting the probability for x=0,1,2**	$2\frac{1}{2}$
	Expressing the distribution in tabular form	$\frac{1}{2}$
32	**Proving one-one**	2
	Proving onto	2
	Conclusion as bijection	1
33	**Getting the value of** $\|A\|=1200$	1
	Finding $A^{-1}=\frac{1}{1200}\begin{pmatrix}75 & 150 & 75\\ 110 & -100 & -30\\ 72 & 0 & -24\end{pmatrix}$	2
	Expressing the given system in matrix form	$\frac{1}{2}$
	Getting the values as $x=2$; $y=-3$; $z=5$	$1+\frac{1}{2}$
34	**For rough figure of the region**	$\frac{1}{2}$
	Finding the limits for integration	$\frac{1}{2}$
	Calculating the area using integration	3
	Getting the answer as $3(\pi-2)$	1
34	**For rough figure of the region**	$\frac{1}{2}$
	Finding the limits for integration	$\frac{1}{2}$
	Calculating the area using integration	3
	Getting the answer as $\left(\frac{\pi}{4}-\frac{1}{2}\right)$	1
35	**Identifying the vectors for the formula**	2
	Using the formula to get the shortest distance	2
	Getting the answer as $2\sqrt{29}$	1

Q.No	Answer	Marks
35	**Getting the direction ratios the given lines**	1
	Getting the direction ratios of the required line using the given condition	2
	Getting the equation of the line as $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(-4\hat{i} + 4\hat{j} - \hat{k})$	1
	Getting the angle as $$\cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$$	1
36 a)	**Getting the total area as** $$(400\,r - \pi r^2 - 2r^2) + \frac{\pi r^2}{2}$$	2
36 b)	**Finding the first and second derivative**	1
	Proving area is maximum when $r = \frac{200}{\pi+2}$	1
37 a)	**Finding years in which increasing**	2
37 b)	**Finding years in which decreasing**	2
38 a)	**Calculating the probabilities**	3
38 b)	**Drawing conclusion from the values of probabilities**	1

SAMPLE QUESTION PAPER (2023-24) - 02

CLASS:XII MAX MARKS:80

SUBJECT : MATHEMATICS TIME : 3 HRS

BLUE PRINT

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

CHAPTER	MCQ'S & ASSERTION-REASON (1 MARK)	VERY SHORT ANSWER (2 MARK)	SHORT ANSWER (3 MARK)	LONG ANSWER (5 MARK)	SOURCE BASED/CASE BASED (4 MARKS	TOTAL	UNIT TOTAL
RELATIONS & FUNCTIONS	---	1(2) *	---	1(5)*	---	2(7)	3(8)
INVERSE TRIGONOMETRIC FUNCTIONS	1(1) (A R)	---	---	---	---	1(1)	
MATRICES	1(1)	---	---	---	---	1(1)	6(10)
DETERMINANTS	4(4)	---	---	1(5)	---	5(9)	
CONTINUITY & DIFFERENTIABILITY	2(2)	1(2)	---	---	---	3(4)	15(35)
APPLICATION OF DERIVATIVES	---	1(2)	---	---	2(8)	3(10)	
INTEGRALS	2(2)	---	1(3)*+2(6)	---	---	5(11)	
APPLICATION OF INTEGRALS	---	---	---	1(5)	---	1(5)	
DIFFERENTIAL EQUATIONS	2(2)	---	1(3)*	---	---	3(5)	
VECTOR ALGEBRA	3(3)	1(2)	---	---	---	4(5)	8(14)
THREE DIMENSIONAL GEOMETRY	1(1)+1(1) (A R)	1(2) *	---	1(5)*	---	4(9)	
LINEAR PROGRAMMING	2(2)	---	1(3)	---	---	3(5)	3(5)
PROBABILITY	1(1)	---	1(3)*	---	1(4)	3(8)	3(8)
TOTAL	20(20)	5(10)	6(18)	4(20)	3(12)	38(80)	38(80)

* QUESTIONS WITH INTERNAL CHOICE

SAMPLE QUESTION PAPER (2023-24) - 02

CLASS:XII MAX MARKS:80

SUBJECT : MATHEMATICS TIME : 3 HRS

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 Mark

1. The number of all matrices of order 2×2 with each entry 2 or 3 is
   a) 4 b) 8 c)16 d)32
2. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is
   $a) \pm 1$ b) ± 2 c)$\pm\sqrt{5}$ d) $\pm\sqrt{7}$
3. The angle between $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is
   a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{5\pi}{6}$
4. If the function $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous , then the value of k is
   a) $\frac{2}{7}$ b) $\frac{7}{2}$ c) $\frac{3}{7}$ d) $\frac{4}{7}$
5. $\int e^x \left(\log\sqrt{x} + \frac{1}{2x}\right) dx =$
   $a) e^x logx + c$ b) $e^x \log\sqrt{x} + c$ c) $\frac{e^x}{2x} + c$
   d)$e^x \log x^2 + c$
6. The order and the degree of the differential equation
   $$\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^3 + 5x = 0 \text{ are}$$
   a) 3 ; 6 b) 3 ; 3 c) 3;9 d) 6;3
7. The graph of the inequality $2x + 3y > 6$ is
   a) Half plane that contains the origin

b) Half plane that neither contains the origin nor the points of the line $2x + 3y = 6$

c) Whole XOY-plane excluding the points on the line $2x + 3y = 6$

d) Entire XOY-plane

8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the points $A(2,3,-4)$, $B(3,-4,-5)$ and $C(3,2,-3)$ respectively then $\left|\vec{a} + \vec{b} + \vec{c}\right|$=

a) $\sqrt{113}$ b) $\sqrt{185}$ c) $\sqrt{209}$ d) $\sqrt{203}$

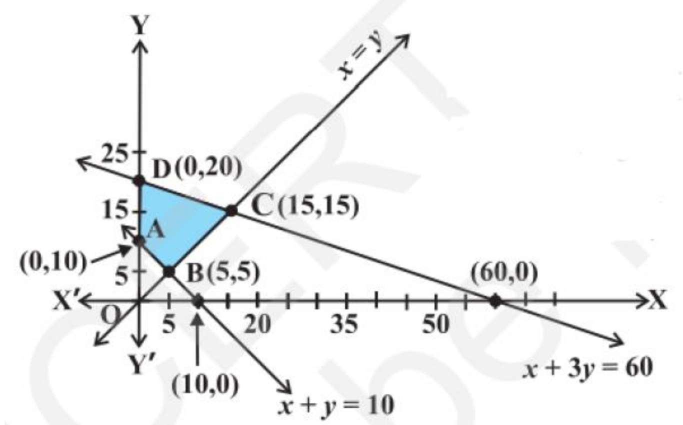
9. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\cot x}} dx =$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

10. For what value of x the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is a singular matrix

a) 1 b) -1 c)2 d)-2

11. Based on the given shaded region as the feasible region in the graph , at which point(s) is the objective function $z = 3x + 9y$ maximum



a) Point B b) Point C c) Point D

d) every point on the line segment CD

12. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then $x =$

a) ± 1 b) ± 2 c)$\pm\sqrt{2}$ d) $\pm\sqrt{3}$

13. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then the value of $|adj\ A|$ is

a) 11 b)-11 c)9 d)-9

14. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$, Find $P(A'/B')$

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{5}{6}$

15. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2 cosx$ is

a) $\log x$ b) $-\log x$ c) x d) $\frac{1}{x}$

16. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx} =$

a) e^{y-x} b) e^{x+y} c) $-e^{y-x}$ d) $2e^{x-y}$

17. The value of p for which $p(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector is

a) 0 b) $\frac{1}{\sqrt{3}}$ c) 1 d) $\sqrt{3}$

18.The coordinates of the foot of the perpendicular drawn from the point $(2,-3,4)$ on the y −axis is

a) $(2,3,4)$ b)$(-2,-3,-4)$ c)$(0,-3,0)$ d)$(2,0,4)$

ASSERTION-REASON BASED QUSETIONS

In the following questions , a statement of assertion (A) is followed by as statement of Reason (R) . Choose the correct answer out of the following choices .

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true .

19.**Assertion(A):** $\sin^{-1}(0.76)$ is defined

Reason (R) : $\sin^{-1}(0.76)$ is defined because it is defined for all real numbers.

20. **Assertion(A):** $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ is perpendicular to $\vec{b} = -\hat{\imath} + \hat{\jmath}$

Reason (R) : Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a}.\vec{b} = 0$ is

SECTION B

This section comprises of very short answer type questions (VSA) of 2 marks each

21. Show that the signum function $f: R \to R$ given by

$$f(x) = \begin{cases} 1 \ , if\ x > 0 \\ 0 \ , if\ x = 0 \\ -1 \ , if\ x < 0 \end{cases}$$

Is neither one-one nor onto

OR

Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$

22. If $x = at^2$, $y = 2at$ then find $\frac{d^2y}{dx^2}$

23. The radius of a right circular cylinder is increasing at the rate of 2cm/s and its height is decreasing at the rate of 8cm/s. Find the rate of change of its volume, when the radius is 3 cm and height is 6cm.

24. Using vectors, find the area of triangle ABC with vertices $A(1,1,1)\ B(1,2,3)\ and\ C(2,3,1)$.

25. Find the angle between the lines $\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$

OR

Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find $\int \sin^{-1} x\, dx$.

27. Evaluate $\int_{-4}^{4} |x+2|\, dx$.

OR

Evaluate $\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}} dx$

28. Find $\int \frac{2x}{x^2+3x+2}\, dx$

29. Find the general solution of the following differential equation

$2xe^{\frac{y}{x}}dy + \left(x - 2ye^{\frac{y}{x}}\right)dx = 0$

OR

Find the particular solution of the differential equation $(2x^2 + y)\frac{dx}{dy} = x$ given that $when\ x = 1\ , y = 2$

30. Solve the following linear programming problem graphically

Maximize $z = 3x + 5y$ subject to $x + y \leq 5$; $x \geq 3$; $x \leq 4$;$y \geq 0$

31. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket. Also find the mean of the probability distribution.

OR

Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as “number greater than 5”.Also find the mean of the probability distribution.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$; Find A^{-1}

Hence , solve the following system of equations

$$3x + 4y + 2z = 8$$
$$2y - 3z = 3$$
$$x - 2y + 6z = -2$$

33. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

OR

Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Rightarrow (x - y)$ $is\ divisible\ by$ 3 is an equivalence relation.

34. If the area between the curves $x = y^2$ and $x = 4$ divided into two equal parts by the line $x = a$, then find the value of a using integration .

35. Prove that the line through A (0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(– 4, 4, 4).

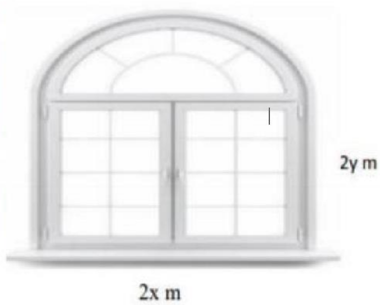
OR

Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1).

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub parts (i) , (ii) ,(iii) of marks 1,1,2 respectively . The third case study question has two sub parts of 2 marks each)

36.

Case-study 1 : Read the following passage and answer the questions given below ($2x$ and $2y$ are length and breadth of rectangular part)
The windows of a newly constructed building are in the form of a rectangle surmounted by a semi circle . The perimeter of each window is 40m.

(i) Find the relation between x and y

(ii) What is the area of the window in terms of x

(iii) Find the value of x for which area of window will be maximum ?

OR

Find the value of y for which area of window will be maximum ?

37. **Case Study -2** : The total profit function of a company is given by

$P(x) = -5x^2 + 125x + 37500$ where x is the production of the company

(i) Find the critical point of the function ?

(ii) Find the interval in which the function is strictly increasing ?

(iii) If $P(x) = -5x^2 + mx + 37500$ and 14 is the critical point , then find

the value of m

OR

Find the absolute maximum for this value of m in $[0,16]$

38. Case Study -3

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

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SAMPLE QUESTION PAPER (SESSION 2023-24) -02

CLASS:XII MAX MARKS:80

SUBJECT : MATHEMATICS TIME : 3 HRS

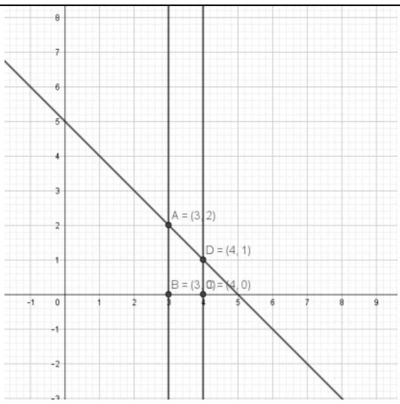
MARKING SCHEME

Question No	Value Points	Marks
1 to 20	1) c) 16 2) d) $\pm\sqrt{7}$ 3) b) $\frac{2\pi}{3}$ 4) b) $\frac{7}{2}$ 5) b) $e^x \log\sqrt{x} + c$ 6) b) $3\,;\,3$ 7) b) Half plane that neither contains the origin nor the points of the line 2x+3y=6 8) c) $\sqrt{209}$ 9) d) $\frac{\pi}{4}$ 10) c) 2 11) d) every point on the line segment CD 12) d) $\pm\sqrt{3}$ 13) b) -11 14) b)$\frac{2}{3}$ 15) d)$\frac{1}{x}$ 16) c) $-e^{y-x}$ 17) b) $\frac{1}{\sqrt{3}}$ 18) c) $(0,-3,0)$ 19) c) A is true but R is false. 20) a) Both A and R are true and R is the correct explanation of A.	1 Mark for each correct answer
21	$f(1) = f(2) = 1$, So $f\ is$ not one one as f(x) takes only 3 values (1, 0, or −1) there does not exist any x in domain R such that f(x) = –2. ∴ f is not onto. OR $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] = \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{7}\right)\right]$ $= \sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right] = -\frac{\pi}{7}$	1 ½ ½ 1 1
22	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$ $\frac{d^2y}{dx^2} = -\frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{t^2}.\frac{1}{2at} = -\frac{1}{2at^3}$	1 1
23	$\frac{dr}{dt} = 2$ cm/s and $\frac{dh}{dt}$ =−8cm/s $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = \pi r^2\frac{dh}{dt} + \pi h 2r\frac{dr}{dt}$	½ 1

	$\frac{dV}{dt}(at\ r = 3 \text{ and } h = 6) = 0$	½
24	$\overrightarrow{AB} = \hat{j} + 2\hat{k}$ $\overrightarrow{AC} = \hat{i} + 2\hat{j}$ $\overrightarrow{AB} \times \overrightarrow{AC} = -4\hat{i} + 2\hat{j} - \hat{k}$ Required area $= \frac{1}{2}\sqrt{21}$	½ ½ ½ ½
25.	$\cos\theta = \left\lvert \frac{(\hat{i} + 2\hat{j} + 2\hat{k})\cdot(2\hat{i} + 3\hat{j} - 6\hat{k})}{\sqrt{3}\cdot\sqrt{7}} \right\rvert$ $\theta = \cos^{-1}\frac{4}{\sqrt{21}}$ OR The lines, $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$ and $\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$ are perpendicular $\therefore\ 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow k = 2$	1 1 1 1
26	$I = \int \sin^{-1} x \cdot 1\ dx$ $= \sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x\ dx$ $= x \cdot \sin^{-1} x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\ dx$ $= x \cdot \sin^{-1} x + \frac{1}{2} \cdot 2\sqrt{1-x^2} + C$ or $x \sin^{-1}x + \sqrt{1-x^2} + C$	1 ½ ½ 1
27	$\int_{-4}^{4} \lvert x+2 \rvert\, dx = \int_{-4}^{-2} -(x+2)\, dx + \int_{-2}^{4} -(x+2)\, dx$ $= -\left[\frac{x^2}{2} + 2x\right]_{-4}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{4}$ $= 20$	1 ½ ½ 1

27(OR)	$I=\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}dx$ $=\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}dx$ (using property) $\Rightarrow 2I=\int_1^3 1\,dx = x]_1^3 = 2$ $\Rightarrow I=1$	1 1½ ½
28	$I=\int \frac{2x\cdot dx}{x^2+3x+2}=\int \frac{2x}{(x+1)(x+2)}dx$ $=\int\left(\frac{-2}{x+1}+\frac{4}{x+2}\right)dx$ using partial fraction $=-2\log\lvert x+1\rvert+4\log\lvert x+2\rvert+C$	½ 1 ½ ½
29	Given diff. equation is $\frac{dy}{dx}=\frac{2ye^{y/x}-x}{2xe^{y/x}}$ Put, $\frac{y}{x}=v \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore \quad v+x\frac{dv}{dx}=\frac{2ve^v-1}{2e^v}$ $\Rightarrow x\frac{dv}{dx}=\frac{2ve^v-1}{2e^v}-v=-\frac{1}{2e^v}$ $\Rightarrow 2e^v dv=-\frac{dx}{x}$ Integrating to get, $2e^v=-\log\lvert x\rvert+C$ $\Rightarrow 2e^{y/x}+\log\lvert x\rvert=C$ OR Given diff. equation can be written as $x\frac{dy}{dx}-y=2x^2 \quad or \quad \frac{dy}{dx}-\frac{1}{x}y=2x$ IF = $e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\frac{1}{x}}=\frac{1}{x}$ $\therefore$ Solution is $y\cdot\frac{1}{x}=\int 2x\cdot\frac{1}{x}dx=2x+C$ $\Rightarrow y=2x^2+Cx$ when $x=1,\ y=2 \Rightarrow 2=2+C \Rightarrow C=0$ $\Rightarrow$ Particular Solution is $y=2x^2$	½ ½ ½ 1 1 ½ ½ ½ ½

30



Correct graph 1

The corner points of the feasible region are $A(3,2)$,$B(3,0)$,$C(4,0)$ and $D(4,1)$ ½

Corner Points	$Z = 3x + 5y$
$A(3,2)$	19
$B(3,0)$	9
$C(4,0)$	12
$D(4,1)$	17

1

Maximum of Z occurs at (3,2) and maximum of Z =19 ½

31

Let X = No. of even tickets drawn ½

X	0	1	2
P(X)	$\frac{^{10}C_2}{^{19}C_2} = \frac{5}{19}$	$\frac{^{10}C_1 \cdot {^9C_1}}{^{19}C_2} = \frac{10}{19}$	$\frac{^9C_2}{^{19}C_2} = \frac{4}{19}$

½ +½+½

Mean of the distribution = $0 \times \frac{5}{19} + 1 \times \frac{10}{19} + 2 \times \frac{4}{19} = \frac{18}{19}$ 1

OR

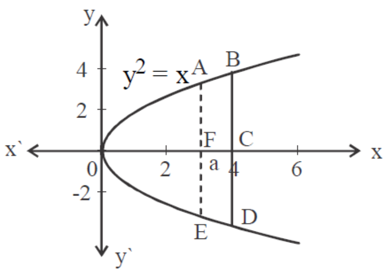
X =No of successes = No of times getting a number greater than 5 ½

X	0	1	2
P(X)	$\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$	$2 \times \frac{1}{6} \times \frac{5}{6} = \frac{10}{36}$	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

½+½+½

Mean of the distribution =$0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{12}{36} = \frac{1}{3}$ 1

32	$\|A\| = 2.$	½
	co-factors of the elements of the matrix. $\left.\begin{matrix} A_{11} = 6 & A_{12} = -3 & A_{13} = -2 \\ A_{21} = -28 & A_{22} = 16 & A_{23} = 10 \\ A_{31} = -16 & A_{32} = 9 & A_{33} = 6 \end{matrix}\right\}$	2 (1 mark for 4 correct cofactors)
	$\therefore\ A^{-1} = \dfrac{1}{\|A\|} \cdot \text{adj}(A) = \dfrac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$	1
	The given system of equations can be written as $A \cdot X = B$	½
	where, $X = A^{-1}.B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ $\therefore\ x = -2,\ y = 3,\ z = 1$	1
33	Reflexive: For any $(a, b) \in N \times N$ $a \cdot b = b \cdot a$ $\therefore$ (a, b) R (a, b) thus R is reflexive	1
	Symmetric: For $(a, b), (c, d) \in N \times N$ (a, b) R (c, d) $\Rightarrow a \cdot d = b \cdot c$ $\Rightarrow c \cdot b = d \cdot a$ $\Rightarrow$ (c, d) R (a, b) $\therefore$ R is symmetric	1 ½
	Transitive : For any $(a, b), (c, d), (e, f), \in N \times N$ (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow\ a \cdot d = b \cdot c$ and $c \cdot f = d \cdot e$ $\Rightarrow\ a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$ $\therefore$ (a, b) R (e, f), $\therefore$ R is transitive	2
	$\therefore$ R is an equivalance Relation	½
	OR	1

	$(x-x)=0$ is divisible by 3 for all $x \in z$. So, $(x, x) \in R$	1 ½
	$\therefore$ R is reflexive.	
	$(x-y)$ is divisible by 3 implies $(y-x)$ is divisible by 3.	
	So $(x, y) \in R$ implies $(y, x) \in R$, $x, y \in z$	2
	$\Rightarrow$ R is symmetric.	½
	$(x-y)$ is divisible by 3 and $(y-z)$ is divisble by 3.	
	So $(x-z)=(x-y)+(y-z)$ is divisible by 3.	
	Hence $(x, z) \in R \Rightarrow$ R is transitive	
	$\Rightarrow$ R is an equivalence relation	
34	 	Correct graph 1
	$ar(OAEO) = ar(ABDEA)$	
	$\Rightarrow\ 2 \cdot ar(OAFO) = 2 \cdot ar(ABCFA)$	1
	$\int_0^a \sqrt{x}\,dx = \int_a^4 \sqrt{x}\,dx$	1
	$\frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right)$	1
	$\Rightarrow \frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right)$	1
	$\Rightarrow a^{3/2} = 4, \ \therefore\ a = 4^{2/3}$	
35	Equation of line AB is $\frac{x}{4} = \frac{y+1}{6} = \frac{z+1}{2}$	1 ½
	Any point on this line is of the form $(4\lambda, 6\lambda - 1, 2\lambda - 1)$	1
	Equation of line CD is $\frac{x-3}{-7} = \frac{y-9}{-5} = \frac{z-4}{0}$	
	Any point on this line is of the form $(-7\mu + 3, -5\mu + 9, 4)$	½

	If line intersect then $(4\lambda, 6\lambda - 1, 2\lambda - 1) = (-7\mu + 3, -5\mu + 9, 4)$ $\Rightarrow \lambda = \frac{5}{2}$ and $\mu = -1$ Also these values of λ *and* μ gives the same set of points .Hence lines intersect OR Let equation of required line is $\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$ ....(i) Since this line is perpendicular to $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$, $a + 2b + 4c = 0$ ....(ii) $2a + 3b + 4c = 0$ ....(iii) Solving (ii) and (iii) , $\frac{a}{-4} = \frac{b}{4} = \frac{c}{-1}$ $\therefore$ DR's of line in cartesian form is : $-4, 4, -1$ Equation of line in Cartesian form is: $\frac{x-1}{-4} = \frac{y-1}{4} = \frac{z-1}{-1}$ Vector form of line is $\vec{r} = (\hat{i} + j + k) + \lambda(-4\hat{i} + 4j - k)$	1 ½ ½ 1 1 1 ½ ½ 1
36	(i) $2x + 4y + \pi x = 40$ (ii) $40x - 2x^2 - \frac{\pi x^2}{2}$ (iii) Area is maximum when $\frac{d}{dx}\left(40x - 2x^2 - \frac{\pi x^2}{2}\right) = 0$ That is $x = \frac{40}{(\pi+4)}$ Second derivative = $-4 - \pi < 0$ OR Using in the equation related to x and y $y = \frac{40}{2(\pi + 4)}$	1 1 ½ 1 ½ 1 1
37	(i) $x = 12.5$ (ii) $(0, 12.5)$ (iii) For differntiating and equation to 0 $m = 140$ OR For finding $P(0), P(14), P(16)$ Absolute maximum = 38480	1 1 1 1 1 ½ ½
38	Let E1 = The policy holder is accident prone. E2 = The policy holder is not accident prone. E = The new policy holder has an accident within a year of purchasing a policy. (i) $P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$ $= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10} = \frac{7}{25}$ (ii) By Bayes' Theorem, $P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$ $\frac{\frac{20}{100} \times \frac{6}{10}}{\frac{280}{1000}} = \frac{3}{7}$	1 1 1 1

SAMPLE QUESTION PAPER – 03

CLASS XII MATHEMATICS (041) 23—24

BLUE PRINT OF SAMPLE QUESTION PAPER (SQP)

UNITS	Unit wise marks	Chapters	1 Mark		2 marks		3 marks		5 marks		**Case based questions (4 marks)**
			Single questions	Either Or type questions	Single questions	Either Or type questions	Single questions	Either Or type questions	Single questions	Either Or type questions	
Relations and Functions	8	Relation and function	**1(A&R)**							**1/1**	
		Inverse Trigonometric Functions				**1/1**					
Algebra	10	Matrices	**1**								
		Determinants	**4**						**1**		
Calculus	35	Continuity and Differentiability	**2**				**1**				
		Applications of Derivatives	**1(A&R)**		**2**	**1/1**					**1**
		Integrals	**1**		**1**		**1**	**1/1**			
		Application of the Integrals							**1**		
		Differential Equations	**2**					**1/1**			
Vectors and Three-dimensional Geometry	14	Vectors	**3**								**1**
		Three dimensional Geometry	**2**							**1/1**	
Linear Programming	5	Linear Programming	**2**					**1/1**			
Probability	8	Probability	**1**				**1**				**1**
Total Marks	**80**	**Total Questions**	**18 + 2 Questions**		**5 Questions**		**6 Questions**		**4 Questions**		**3 Questions**
				Assertion Reason based questions							

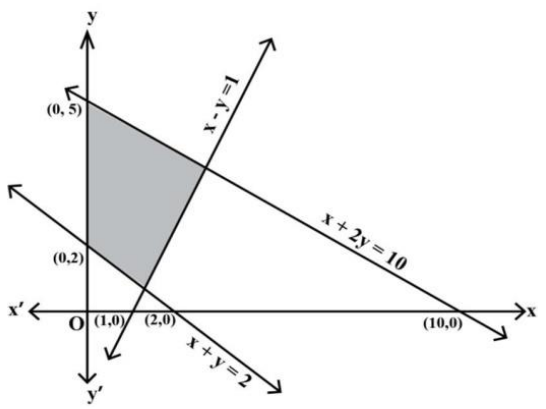
SAMPLE QUESTION PAPER - 03
MATHEMATICS (CODE – 041)
SESSION 2023-2024

Time: 3 hours **Maximum marks: 80**

<u>General Instructions</u>:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there areinternal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** of 4 marks each withsub-parts.

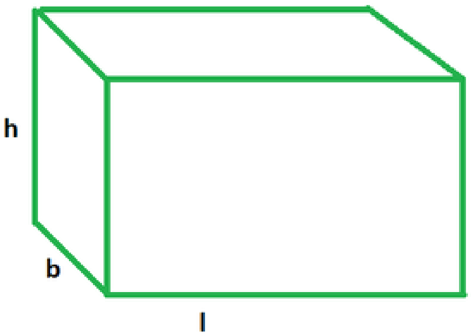
	SECTION A **(Multiple Choice Questions)** **Each question caries 1 mark**
Q1	If A = [a_{ij}] is a skew-symmetric matrix of order n, then a) $a_{ij} = \frac{1}{a_{ij}}$ $\forall\ i, j$ b) a_{ij} = 0 $\forall\ i, j$ c) a_{ij} = 0, where i=j d) $a_{ij} \neq 0$, where i = j
Q2	Given a relation R in set A = { 1, 2, 3}.Is relation R = {(1,1),(2,2), (2,3)}. a) Reflexive b) Symmetric c) Transitive d) All the three
Q3	If $\begin{vmatrix} 2x & -1 \\ 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}$ then x is a) 3 b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) $-\frac{1}{4}$
Q4	A function **f** is said to be continuous for $x \in$ R , if a) it is continuous at x = 0 b) differentiable at x = 0 c) continuous at two points d) differentiable for $x \in$ R
Q5	The general point on the line $\vec{r}$ = $(2\hat{\imath} + \hat{\jmath} - 4\hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} - \hat{k})$ is a) (2, 1, -4) b) (3, 4, -1) c) (-1, 1, 3) d) (2+3λ, 1+2λ, -4-λ)
Q6	Integrating factor for the solution of differential equation $(x - y^3)dx + y\ dx$= 0 is a) $\frac{1}{y}$ b) log y c) y d) y^2
Q7	The corner points of the bounded feasible region determined by a system of linear constraints are (0,3), (1,1) and (3,0) . Let Z = px + qy, where p, q>0. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is

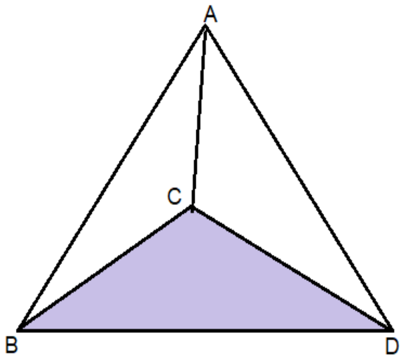
	a) p = 2q b) $p = \frac{q}{2}$ c) p = 3q d) p = q
Q8	The area of a triangle with vertices A, B, C is given by a) $\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$ b) $\frac{1}{2} \lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$ c) $\frac{1}{4} \lvert \overrightarrow{AC} \times \overrightarrow{AB} \rvert$ d) $\frac{1}{8} \lvert \overrightarrow{AC} \times \overrightarrow{AB} \rvert$
Q9	If a is such that $\int_0^a \frac{1}{1+4x^2}\, dx = \frac{\pi}{8}$, then the value of a is a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) 2 d) 1
Q10	If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$ then x+y is a) 7 b) -1 c) 8 d) 6
Q11	The feasible region corresponding to the linear constraints of a linear programming problem is given below.  Which of the following is not a constraint to the given linear programming problem? a) $x + y \geq 2$ b) $x + 2y \leq 10$ c) $x - y \geq 1$ d) $x - y \leq 2$
Q12	If $\lvert\vec{a}\rvert = \sqrt{3}$, $\lvert\vec{b}\rvert = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then $\vec{a} \cdot \vec{b}$ is a) $\sqrt{3}$ b) 2 c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{3}}$
Q13	Given a matrix A of order 3 × 3. If |A| = 3, then find |A.AdjA|. a) 3 b) 27 c) 9 d) 81
Q14	A four digit number is formed by using the digits 1, 2, 3, 5 with no repetition. The probability that number is divisible by 5 is a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{1}{6}$
Q15	Order of differential equation corresponding to family of curves y = $Ae^{2x} + Be^{-2x}$ is a) 2 b) 1 c) 3 d) 4
Q16	If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendiculars unit vectors, then the value of $\lvert 2\hat{a} + \hat{b} + \hat{c} \rvert$ is a) $\sqrt{5}$ b) $\sqrt{3}$ c) $\sqrt{2}$ d) $\sqrt{6}$
Q17	The set of all points where the function f(x) = x + |x| is differentiable, is

	a) (0,∞) b) (-∞, 0) c) (-∞,0) U (∞,0) d) (-∞,∞)
Q18	Direction ratios of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ are a) 2, 6, 3 b) -2, 6, 3 c) 2, -6, 3 d) none of these
	ASSERTION-REASON BASED QUESTIONS **In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).Choose the correct answer out of the following choices.** **(a) Both (A) and (R) are true and (R) is the correct explanation of (A).** **(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).** **(c) (A) is true but (R) is false.** **(d) (A) is false but (R) is true.**
Q19	**ASSERTION (A) :** The function y = $[x(x-2)]^2$ is increasing in (0,1) U (2, ∞) **REASON (R) :** $\frac{dy}{dx}$ = 0, when x = 0, 1, 2
Q20	**ASSERTION (A) :** The relation **f : { 1, 2, 3, 4}** → **{x, y, z, p}** defined by **f = {(1,x), (2,y), (3,z)}** is a bijective function. **REASON (R) :** The function **f : { 1, 2, 3}** → **{x, y, z, p}** defined by **f = {(1,x), (2,y), (3,z)}** is one-one.
	SECTION - B **[This section comprises of very short answer type questions (VSA) of 2 marks each]**
Q21	Find the value of $\sin^{-1}[\sin(\frac{13\pi}{7})]$ **(OR)** Find the value of $tan^2(\frac{1}{2}\sin^{-1}\frac{2}{3})$.
Q22	A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street lightt is 4 m above the ground .At what rate is the tip of his shadow moving? At what rate is this shadow lengthening?
Q23	Show that the function f(x) = log$\|cosx\|$ is strictly decreasing in (0, $\frac{\pi}{2}$). **(OR)**

	Find the intervals in which the function f given by f(x) = $2x^3$ - $9x^2$ + $12x$ + 15 is strictly increasing or strictly decreasing.
Q24	Evaluate $\int \frac{e^{m\tan^{-1}x}}{1+x^2}\,dx$.
Q25	Check whether the function f : $\mathbb{R} \rightarrow \mathbb{R}$ defined by f(x) = $x^3 + x$, has any critical point/s or not ? If yes, then find the point/s.
	SECTION - C **[This section comprises of short answer type questions (SA) of 3 marks each]**
Q26	Find $\int \frac{e^x}{(e^x-1)^2\,(e^x+2)}\,dx$.
Q27	The random variable X has a probability distribution P(X)of the following form, where 'K' is some real number: $P(X) = \begin{cases} k, if\ x = 0 \\ 2k, if\ x = 1 \\ 3k, if\ x = 2 \\ 0, otherwise \end{cases}$ (i)Determine the value of k (ii)Find P(X < 2) (iii)Find P(X>2)
Q28	Evaluate $\int_{-1}^{2} \lvert x^3 - x \rvert$dx. (OR) Evaluate $\int \sin^{-1}\sqrt{\frac{x}{2+x}}$ dx.
Q29	Solve the differential equation: ydx +(x-y^2)dy. (OR) Solve the differential equation: xdy - ydx = $\sqrt{x^2+y^2}$dx.
Q30	Solve the following linear programming problem (LPP) graphically. Maximize Z = x + y Subject to the constraints $\frac{x}{25} + \frac{y}{40} \leq 1$; $2x + 5y \leq 100$, $x \geq 0$, $y \geq 0$ (OR) Solve the following linear programming graphically, Maximize Z = 3x + 4y +370 Subject to the constraints, $y \geq 0$

	$x + y \leq 60$ $x \leq 40$ $y \leq 40$ $x + y \geq 10$
Q31	If y = Ae^{mx} + Be^{nx} , prove that $\frac{d^2y}{dx^2}$ - (m + n)$\frac{dy}{dx}$ + mny =0
	SECTION - D **[This section comprises of Long answer type questions (LA) of 5 marks each]**
Q32	Make a rough sketch of the region$\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$ and find the area of the region using integration.
Q33	Let ℕ be the set of all natural numbers and R be a relation on ℕ× ℕ defined by (a, b)R(c, d)⇔ ad =bc for all (a, b), (c, d) ∈ ℕ×ℕ .Show that R is an equivalence relation on ℕ×ℕ. Also, find the equivalence class of (2, 6), i.e.,[(2,6)] (OR) Show that the function $f : \mathbb{R} \rightarrow \{x \in \mathbb{R} : -1 < x < 1\}$ defined by f(x) = $\frac{x}{1+\|x\|}$, x∈ ℝ is One-one and onto function.
Q34	Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations x- y + z = 4; x - 2y -2z = 9 ; 2x + y +3z =1
Q35	Find the shortest distance between the following two lines : $\vec{r}$ = (1+λ)$\hat{\imath}$ + (2 - λ)$\hat{\jmath}$ + (λ+1)$\hat{k}$; $\vec{r}$ = (2$\hat{\imath}$-$\hat{\jmath}$-$\hat{k}$) +μ(2$\hat{\imath}$ +$\hat{\jmath}$ +2$\hat{k}$) (OR) Find the equation of a line passing through the point P(2,-1,3) and perpendicular to the lines: $\vec{r}$ = ($\hat{\imath}$+$\hat{\jmath}$-$\hat{k}$)+λ(2$\hat{\imath}$-2$\hat{\jmath}$+$\hat{k}$) and $\vec{r}$ = (2$\hat{\imath}$-$\hat{\jmath}$-3$\hat{k}$)+μ($\hat{\imath}$+2$\hat{\jmath}$+2$\hat{k}$)
	SECTION – E **[This section comprises of 3 case – study /passage based questions of 4 marks each with sub parts. The first two case study questions have three sub**

	parts(i),(ii) of marks 1, 1, 2 respectively. The third case study questions has two sub parts of 2 marks each.]
Q36	Case – Study: Read the following passage and answer the questions given below.  These days competitive examinations are online, a student has to go to a particular place to give the examination at the given time. For this , company has to make perfect arrangements and students are expected to be well prepared . But as a human nature sometimes a student guesses or copies or knows the answer to a multiple choice question with four choices each. i) If the probability that a student makes a guess is 1/3 and that he copies the answer is 1/6 . what is the probability that he knows the answer? ii) If answer is correct what is the probability that he guesses it? III) What is the conditional probability that this answer is correct and he knew it? OR iii) what is the probability that he copied it given that his answer is correct?
Q37	Case – Study 2: Read the following passage and answer the questions given below. A village panchayat wants to dug out a square base tank. For preparing fertilizers and wants capacity to be 250 cubic meters. On calculations it was found tht cost of the land is Rs. 50 per square meter and cost of digging increases with dept and cost of the whole tank is Rs. 400 (Depth) . Tank is shown as  

<table>
<tr><td></td><td>i) If the side of the square base is' x' m and the height of the tank is 'h 'm. then establish relations between' x 'and 'h '
ii) Find the cost C for digging the tank in terms of 'x' and' h'
iii) Find the cost C in terms of ' h 'only
OR
iii) Find the value of 'h' for which cost C is minimum.</td></tr>
<tr><td>Q38</td><td>Case study- 3
Read the following passage and answer the questions given below


A building is to be constructed in the form of triangular pyramid, ABCD as shown in the figure. Let its angular points are A(0,1,2), B(3,0,1), C((4,3,6) and D(2,3,2) and G be the point of intersection of the medians of triangle ABCD
i)Find the coordinates of point G
ii)Find the length of vector</td></tr>
</table>

<u>MARKING SCHEME</u>

SAMPLE QUESTION PAPER -03

MATHEMATICS (CODE – 041)

SESSION 2023-2024

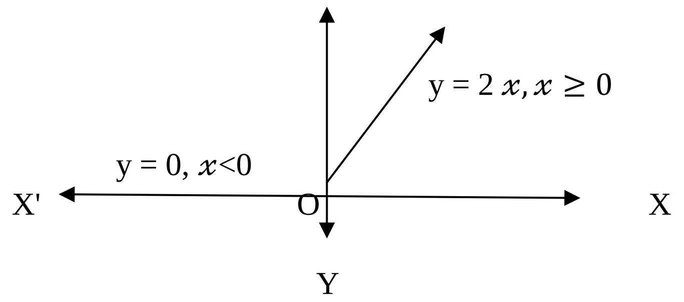
Time: 3 hours **Maximum marks: 80**

<u>General Instructions</u>:

7. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there areinternal choices in some questions.
8. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
9. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
10. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
11. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
12. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** of 4 marks each withsub-parts.

Q. No.	**SECTION A** **(Multiple Choice Questions)** **Each question caries 1 mark**	**Mark**
Q1	(c), In a skew-symmetric matrix, the (i, j)th element is negative of the (j,i)th element. Hence, the (i, i)th element = 0.	1
Q2	(d), Relation R in set A is not reflexive as for a ∈ A, (a,a) ∉ R, e.g. (3,3) ∉ R but 3 ∈ A. Relation R in set A is not symmetric as for a_1, a_2 ∈ A, (a_1, a_2) ∈ R, (a_2, a_1) ∉ R but (3, 2) ∉ R Relation R in set A is not transitive as (2, 3) ∈ R, (2,2) ∈ R but (3, 2) ∉ R	1
Q3	(d), as $\begin{vmatrix} 2x & -1 \\ 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}$ $\Rightarrow 4x + 4 = 3 - 0$ $\Rightarrow x = -\frac{1}{4}$	1
Q4	(d), as differentiable functions is continuous also.	1
Q5	(d), as given line is $\vec{r} = (2\hat{\imath} + \hat{\jmath} - 4\hat{k}) + \lambda\,(3\hat{\imath} + 2\hat{\jmath} - \hat{k})$ ∴ position vector of a point through which line passes. $\therefore \vec{r} = (2 + 3\lambda)\hat{\imath} + (1 + 2\lambda)\hat{\jmath} + (-4 - \lambda)\hat{k}$ ∴ General point is $(2 + 3\,\lambda,\ 1 + 2\,\lambda,\ -4 - \lambda)$	1

Q6	(c), Equation is $(x - y^3)\, dy$ + y dx = 0 $\Longrightarrow$ $\frac{dy}{dx}$ = - $\frac{x-y^3}{y}$ $\Longrightarrow$ $\frac{dx}{dy}$ + $\frac{1}{y}$. x =y^2 Integrating factor = $e^{\int \frac{1}{y} dy}$ = $e^{\log y}$ = y	1
Q7	(b), Z = px + qy --------(i) At (3,0) , Z = 3p ------(ii) and at (1,1), z = p + q --------(iii) From (ii) & (iii), 3p = p + q $\Longrightarrow$ 2p = q	1
Q8	(b), The area of the parallelograms with adjacent sides AB and AC = $\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$. Hence, the area of the triangle with vertices A, B, C = $\frac{1}{2} \lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$.	1
Q9	(b), as $\int_0^a \frac{1}{1+(2x)^2}\, dx = [\frac{1}{2} \tan^{-1} 2x]_0^a$ $= \frac{\pi}{8}$ $\frac{1}{2} \tan^{-1}(2a) = \frac{\pi}{8}$ 2a = tan$\frac{\pi}{4}$ a = $\frac{1}{2}$	1
Q10	(d) $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$ x+3y=4 and y = -1 therefore x = 7 x+y= 7+(-1) = 6	1
Q11	(c) , We observe, (0,0) does not satisfy the inequality $x - y \geq 1$ So, the half plane represented by the above inequality will not contain origin Therefore, it will not contain the shaded feasible region.	1
Q12	(a) as $\vec{a}.\vec{b} = \lvert\vec{a}\rvert \, \lvert\vec{b}\rvert$ cos 60° = √3 × 2 × $\frac{1}{2}$ = √3	1
Q13	(b), as |A.Adj A| = $\lvert A \rvert^3$ = $\lvert 3 \rvert^3$ = 27	1
Q14	(b), As total possibilities for four digit number is n(S) = 4! Favourable possibilities for number to be divisible by 5, n(A) = 3! × 1 = 3! ∴ Required probability = $\frac{n(A)}{n(S)} = \frac{3!}{4!} = \frac{1}{4}$	1
Q15	(a) Two arbitrary constants present therefore order is 2	1
Q16	(d), as $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors. ∴ $\lvert\widehat{a}\rvert = \lvert\widehat{b}\rvert = \lvert\widehat{c}\rvert = 1$ Consider, $\lvert 2\hat{a} + \hat{b} + \hat{c}\rvert^2 = (2\hat{a} + \hat{b} + \hat{c})^2$ $= 4\hat{a} + \hat{b}^2 + \widehat{c^2} + 4\hat{a}.\hat{b} + 4\hat{a}.\hat{c} + 2\hat{b}.\hat{c}$ $= 4\lvert\hat{a}\rvert^2 + \lvert\hat{b}\rvert^2 + \lvert\hat{c}\rvert^2 + 0 + 0 + 0$	1

	$= 4 + 1 + 1 = 6$ $\Rightarrow \ |2\hat{a} + \hat{b} + \hat{c}|^2 = \sqrt{6}$	
Q17	(c) , **Method 1:** $f(x) = x + |x| = \begin{cases} 2x, x \geq 0 \\ 0, x < 0 \end{cases}$  There is a sharp corner at $x = 0$, so f(x) is not differentiable at x = 0. **Method 2:** $Lf'(0) = 0$& $Rf'(0) = 2$; so, the function is not differentiable at x = 0 For $x \geq 0, f(x) = 2x$ (linear function) & when $x < 0$, $f(x) = 0$ (constant function) Hence $f(x)$ is differentiable when $x \in (-\infty, 0) \cup (0, \infty)$.	1
Q18	(b) line $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$, dr's are $< -2, 6, 3 >$	1
	<u>ASSERTION-REASON BASED QUESTIONS</u> **In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).Choose the correct answer out of the following choices.** **(e) Both (A) and (R) are true and (R) is the correct explanation of (A).** **(f) Both (A) and (R) are true but (R) is not the correct explanation of (A).** **(g) (A) is true but (R) is false.** **(h) (A) is false but (R) is true.**	
Q19	(b), Both A and R are true but R is not the correct explanation of A.	1
Q20	(d), **Assertion is false.** As element 4 has no image under f, so relation f is not a function. **Reason is true.** The given function **f :{1, 2, 3}** $\rightarrow$ $\boldsymbol{\{x, y, z, p\}}$ is one – one, for each **a** $\in$ **{1, 2, 3},** there is different image in $\{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{p}\}$ under f.	1
	SECTION - B	

<table>
<tr><td></td><td>[This section comprises of very short answer type questions (VSA) of 2 marks each]</td><td></td></tr>
<tr><td>Q21</td><td>

$\sin^{-1}[\sin(\frac{13\pi}{7})] = \sin^{-1}[\sin(2\pi - \frac{\pi}{7})]$

$= \sin^{-1}[\sin(-\frac{\pi}{7})]$

$= -\frac{\pi}{7}$

(OR)

Let $\theta = \sin^{-1}\frac{2}{3} \Rightarrow \sin\theta = \frac{2}{3}, \cos\theta = \frac{\sqrt{5}}{3}$

$tan^2(\frac{1}{2}\sin^{-1}\frac{2}{3}) = tan^2\frac{\theta}{2} = \frac{sin^2\frac{\theta}{2}}{cos^2\frac{\theta}{2}} = \frac{2sin^2\frac{\theta}{2}}{2cos^2\frac{\theta}{2}}$

$= \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\frac{\sqrt{5}}{3}}{1+\frac{\sqrt{5}}{3}}$

$= \frac{3-\sqrt{5}}{3+\sqrt{5}} = \frac{(3-\sqrt{5})^2}{3^2-(\sqrt{5})^2}$ [on Rationalisation]

$= \frac{(3-\sqrt{5})^2}{4} = \frac{9+5-6\sqrt{5}}{4}$

$= \frac{7-3\sqrt{5}}{2}$

</td><td>1

1

1

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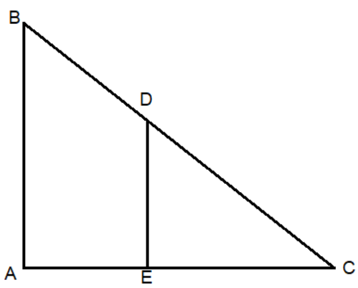
<tr><td>Q22</td><td>

Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6m be at a distance of x from AB and the length of his shadow EC by y m.



Using similarity of triangles , we have

$\frac{4}{1.6} = \frac{x+y}{y}$

3y = 2x

Differentiating both sides w.r.t 't ', we get

$3\frac{dy}{dt} = \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{2}{3}(0.3)$

$\frac{dy}{dt} = (0.2)$

At any time t seconds, the tip of his shadow is at a distance of (x+y) m from AB

The rate at which his shadow is lengthening = 0.2m/s

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<tr><td>Q23</td><td>

f '(x) = $\frac{\mathbf{1}}{\mathbf{cosx}}$ **.** (-sin x) = - tan x.,tan x > 0 for (0, $\frac{\pi}{2}$).

f '(x) < 0. Hence function is strictly decreasing.

(OR)

Consider f(x) = $2x^3$- $9x^2$ + 12 x + 15

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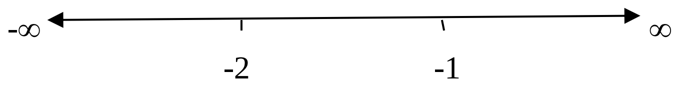
1</td></tr>
</table>

f '(x) = $6x^2$ - 18 x + 12 = 6(x^2- 3 x + 2)

= $6(x-1)\ (x-2)$

For critical points f '(x) = 0 $\Rightarrow x = 1, 2$ 1



	x < 1	1< x < 2	x > 2
6	+	+	+
$x-1$	-	-	-
$x-2$	-	-	+
f '(x)	+	-	+
	↑	↓	↑

1

Strictly increasing for (-∞, 1) U (2, ∞), strictly decreasing in (1, 2)

Q24 Consider $\int \frac{e^{\tan^{-1} x}}{1+x^2}\, dx = \frac{1}{m} \int e^t\ dt$

$= \frac{1}{m}\, e^t$ + C = $\frac{1}{m}\, e^{m \tan^{-1} x}$ + c

Let m $\tan^{-1} x$ = t

$\Rightarrow \quad \frac{m}{1+x^2}\ dx = dx$

Q25 **f (x) = $x^3 + x$, for all $x \in$ ℝ.**

$\frac{d}{dx}$(f(x)) = f '(x) = 3x^2 + 1; for all $x \in$ ℝ, $x^2 \geq 0 \Rightarrow$ f '(x) > 0 **1**

Hence, no critical point exits. **1**

SECTION - C

[This section comprises of short answer type questions (SA) of 3 marks each]

Q26	$\int \frac{e^{m\tan^{-1}x}}{1+x^2}\,dx$ let u = $\tan^{-1}x$ du = $\frac{1}{1+x^2}dx$ $\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx = \int e^{mu}\,du$ $= \frac{e^{mu}}{m} + c$ $= \frac{e^{m\tan^{-1}x}}{m} + c$	1 1 1
Q27	We have i) $\sum P(X_i) = 1$ k+2k+3k=1 6k=1 k=$\frac{1}{6}$ ii)P(X<2)=P(X)=0+P(X)=1 k+2k=3k =3. $\frac{1}{6}$ =$\frac{1}{2}$ iii) P(X>2) = 0	1 1 1
Q28	Consider $\int_{-1}^{2}\|x^3 - x\|dx$, Now x^3-x =0 ⟹ x(x^2 −1) =0 ⟹x(x+1)(x−1) =0 ⟹x = 0,-1,1 For -1 < x< 0 , x^3- x is positive For 0 < x <1 , x^3- x is negative For 1 <x < 2 , x^3- x is positive ∴ $\int_{-1}^{2}\|x^3 - x\|\,dx = \int_{-1}^{0}(x^3 - x)dx - \int_{0}^{1}(x^3 - x)dx + \int_{1}^{2}(x^3 - x)dx$ $= \left\|\frac{x^4}{4} - \frac{x^2}{2}\right\|_{-1}^{0} - \left\|\frac{x^4}{4} - \frac{x^2}{2}\right\|_{0}^{1} + \left\|\frac{x^4}{4} - \frac{x^2}{2}\right\|_{1}^{2}$ =(0-0) −($\frac{1}{4}$ - $\frac{1}{2}$) − ($\frac{1}{4}$ - $\frac{1}{2}$) + (0-0) + ($\frac{16}{4}$ - $\frac{4}{2}$) − ($\frac{1}{4}$ - $\frac{1}{2}$) $= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} \Longrightarrow 2 + \frac{3}{4} \Longrightarrow \frac{11}{4}$ (OR) Consider $\int Sin^{-1}\sqrt{\frac{x}{2+x}}$ dx. Let x = $2tan^2\theta$ ⟹dx = 4tan θ $Sec^2\,\theta$ d θ $= \int Sin^{-1}\sqrt{\frac{2tan^2\theta}{2+2tan^2\theta}} \times$ 4tan θ $Sec^2\,\theta$ d θ = 4$\int Sin^{-1}\,(Sin\theta)$.(tan θ $Sec^2\,\theta$)d θ.	1 1 1 1

	$= 4\int \theta\ (\tan\theta\, Sec^2\,\theta)\mathrm{d}\,\theta.$ $=4\left[\theta\ tan^2\theta - \int 1.\frac{tan^2\theta}{2}\ d\theta\right]$ $\Big[\because$ $\int (\tan\theta\, Sec^2\,\theta)\mathrm{d}\,\theta\ = \frac{tan^2\theta}{2}\ \Big]$ $= 2\ [\theta\ tan^2\theta - \int (sec^2\theta - 1)d\theta]$ $=\ 2\ [\theta\ tan^2\theta - tan\theta + \theta] + \mathrm{C}$ $= 2\ \left[tan^{-1}\sqrt{\frac{x}{2}}.\ \frac{x}{2} - \sqrt{\frac{x}{2}} - tan^{-1}\sqrt{\frac{x}{2}}\right] + \mathrm{C}$ $=(x - 2\,)tan^{-1}\sqrt{\frac{x}{2}}\ - 2\sqrt{\frac{x}{2}} + \mathrm{C}$	1 1
Q29	$ydx + (x - y^2)dy = 0$ Reducing the given differential equation to the form $\frac{dx}{dy} + Px = Q$ We get, $\frac{dx}{dy} + \frac{x}{y} = y$ I.F $= e^{\int pdy} = e^{\int \frac{x}{y}dy}$ $= e^{logy} = y$ The general solution is given by $x.I.F = \int Q.IF\ dy$ $\Rightarrow xy = \int y^2\ dy$ $\Rightarrow xy = \frac{y^3}{3} + c,$ Which is the required general solution **(OR)** $x\,dy - y\,dx = \sqrt{x^2 + y^2}dx.$ It is a Homogeneous Equation as $\frac{dy}{dx} = \frac{\sqrt{(x^2+y^2)} + y}{x}$ $= \sqrt{1 + \left(\frac{y}{x}\right)^2} + \frac{y}{x} = f\left(\frac{y}{x}\right)$ Put $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$ Separating variables , and by integrating we get	½ ½ 1 1 ½ ½

$$\log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log K,\ k>0$$ 1

$$\log\left|y+\sqrt{x^2+y^2}\right| = \log x^2\,k$$

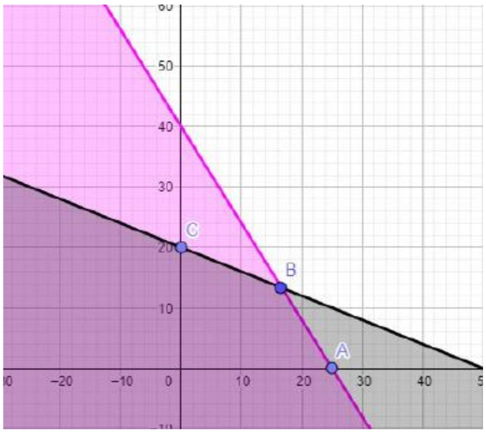
$$\Rightarrow\ y+\sqrt{x^2+y^2} = \pm\ Kx^2$$

$$\Rightarrow\ y+\sqrt{x^2+y^2} = Cx^2$$

Which is the required general solution 1

Q30

2(graph)



Points	Z= x+ y	Values
A(25,0)	25+0	25
B($\frac{50}{3}$,$\frac{40}{3}$)	$\frac{50}{3}+\frac{40}{3}$	30
C(0.20)	0+20	20

Plotting the in equations as graph we notice shaded portion is feasible solution .Possible points for maximum Z are A(25,0),B($\frac{50}{3}$, $\frac{40}{3}$) and C(0,20).

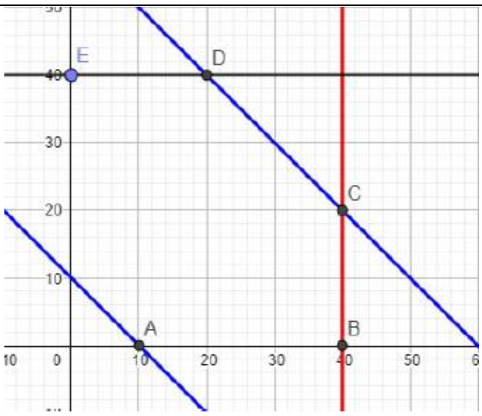
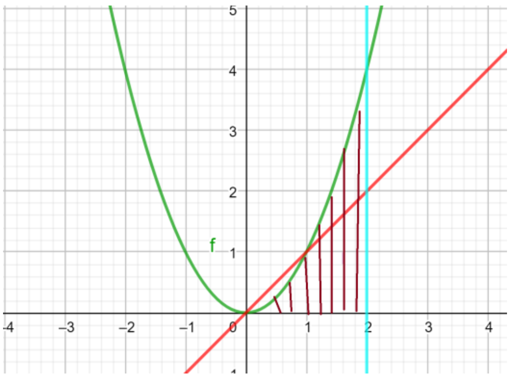
Z is maximum at B($\frac{50}{3}$,$\frac{40}{3}$) , i.e. x = $\frac{50}{3}$, y = $\frac{40}{3}$

1

(OR)

2(graph)

Points	Z = 3x + 4y +370	Values
A(10,0)	30+0+370	400
B(40,0)	120+0+370	490
C(40,20)	120+80+370	570
D(20,40)	60+160+370	590
E(0,40)	0+160+370	530
F(0,10)	0+40+370	410

	 On plotting the in equations we notice shaded portion is feasible solution. Possible points for minimum Z are A(10,0),B(40,0),C(40,20),D(20,40),E(0,40),F(0,10) Z is minimum for A(10,0),i.e, x = 10 , y = 0	1
Q31	We have, y $= Ae^{mx} + Be^{nx}$ …(i) $\Rightarrow \quad \frac{dy}{dx} = Ame^{mx} + Bne^{nx}$ …(ii) and $\frac{d^2y}{dx^2} = Am^2e^{mx} + Bn^2e^{nx}$ …(iii) Consider $\frac{d^2y}{dx^2}$ - (m + n) $\frac{dy}{dx}$ + mny $= Am^2e^{mx} + Bn^2e^{nx}$ - (m + n)($Ame^{mx} + Bne^{nx}$) + mn($Ae^{mx} + Be^{nx}$) [from (i), (ii) and (iii)] $= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + Amne^{mx}$ $+ Bmne^{nx}$	1 1 1
	SECTION - D **[This section comprises of Long answer type questions (LA) of 5 marks each]**	
Q32	 The points of intersection of the parabola y=x^2 and the line y=s are (0,0) and (1,1) Required area $=\int_0^1 ydx + \int_1^2 ydx$ parabola line	1 1 1 1+1

	$=\int_0^1 x^2dx + \int_1^2 xdx$ = 11/6 (after integration and applying the limits)	
Q33	Let (a, b) be an arbitrary element of $\mathbb{N}\times\mathbb{N}$. Then (a, b) $\in \mathbb{N}\times\mathbb{N}$ and a ,b$\in \mathbb{N}$	
	We have ab = ba ;(as a ,b$\in \mathbb{N}$ and multiplication is commutative on $\mathbb{N}$)	
	⇒(a,b)R(a,b),according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$	1
	Thus (a, b)R(a, b),∀(a,b) $\in \mathbb{N}\times\mathbb{N}$	
	So,R is reflexive relation on $\mathbb{N}\times\mathbb{N}$.	
	Let (a, b),(c, d) be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that (a,b)R(c,d)	
	Then (a,b)R(c,d) ⇒ ad=bc⇒bc =ad;(changing LHS and RHS)	
	⇒cb=da; (As a, b, c, d $\in \mathbb{N}$ and multiplication is commutative on $\mathbb{N}$)	
	⇒(c, d) R (a, b); according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$	1.5
	Thus (a,b)R(c,d) ⇒ (c, d) R (a, b) So R is symmetric relation on $\mathbb{N}\times\mathbb{N}$	
	Let (a, b),(c, d), (e, f) be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that (a,b)R(c,d) and (c, d) R (e, f)	
	$\left.\begin{array}{l}\text{Then (a,b)R(c,d)} \Rightarrow ad = bc \\ \text{(c, d) R (e, f)} \Rightarrow cf = de\end{array}\right\\} \Rightarrow (ad)\ (cf) = (bc)\ (de) \Rightarrow af = be$	
	⇒ (a,b) R (e,f) ; according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$	
	Thus (a,b)R(c,d) and (c, d) R (e, f) ⇒ (a,b) R (e,f)	
	So R is transitive relation on $\mathbb{N}\times\mathbb{N}$	1.5
	as the relation R is reflexive, symmetric and transitive so, it is equivalence relation on $\mathbb{N}\times\mathbb{N}$	
	[(2, 6)] = { (x,y) $\in \mathbb{N}\times\mathbb{N}$: (x,y) R (2,6)}	1
	= { (x,y) $\in \mathbb{N}\times\mathbb{N}$: 3x = y}	
	= {(x, 3x) : x $\in \mathbb{N}$ } = { (1, 3), (2, 6), (3, 9),…………}	
	(OR)	
	we have $f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \geq 0 \end{cases}$	
	Now we consider the following cases	
	Case 1: when x≥ 0 we have f(x)=$\frac{x}{1+x}$	
	Injectivity; Let x,y ∈R^+U{0}	
	such that f(x)=f(y)	
	$\frac{x}{1+x} = \frac{y}{1+y}$	1
	x + xy = y + xy	
	x = y	
	so, f is injective function.	
	Surjective : When x≥0, f(x)=$\frac{x}{1+x}$ ≥0	
	and f(x)=$1-\frac{1}{1+x}$<1, as x≥ 0	1
	let y∈[0,1), thus for each y∈[0,1) there exists x=$\frac{y}{1-y}$ ≥ 0 such that f(x)=$\frac{\frac{y}{1-y}}{1-\frac{y}{1-y}}$= y	
	so , f is onto function on $[0,\infty)\ to [0,1)$	

	Case 2: when x<0 we have f(x)= $\frac{x}{1-x}$ Injectivity; Let x,y $\in R^-$ such that f(x)=f(y) $\frac{x}{1-x} = \frac{y}{1-y}$ x - xy = y - xy x = y so, f is injective function Surjective : When x<0 f(x)=$\frac{x}{1-x}$ <0 and f(x)=$1 - \frac{x}{1-x}$ $= 1 + \frac{1}{1-x} > -1$<1, -1< f(x) < 0 let y$\in (-1,0)$, be an arbitrary real number and there exists x=$\frac{y}{1+y} < 0$ such that f(x)=$\frac{\frac{y}{1+y}}{1-\frac{y}{1+y}}$= y so , for y$\in (-1,0)$there exists x=$\frac{y}{1+y}$<0 such that f(x) = y Hence f is onto function on (-∞, 0) to $(-1,0)$ Case 3: (Injectivity) Let x>0 & y<0 such that f(x)=f(y) $\frac{x}{1+x} = \frac{y}{1-y}$ x-xy=y+xy x-y=2xy LHS>0 but RHS <0 which is in admissible hence f(x) $\neq f(y)$when x$\neq y$ so f is one-one and onto function	1 1 1
Q34	Consider A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ And B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ AB = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ = $\begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$ AB = $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ = 8I …(i)	2

	Consider equations $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1$ Corresponding matrix equation is $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $\Rightarrow$ BX = C is matrix equation. Its solution is X = B^{-1}C From (i), we have AB = 8I $\Rightarrow$ ($\frac{1}{8}$ A) B = I $\Rightarrow$ $B^{-1} = \frac{1}{8}$ A $\therefore$ From (ii), X = $\frac{1}{8}$ AC = $\frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\therefore$ $x = 3, y = -2, z = -1$ is required solution.	1 1 1
Q35	Consider the line $\vec{r}$ = (1+λ)$\hat{\imath}$ + (2 - λ)$\hat{\jmath}$ + (λ+1)$\hat{k}$ i.e. $\vec{r}$ = ($\hat{\imath}$ + 2 $\hat{\jmath}$ + $\hat{k}$) + λ($\hat{\imath}$ - $\hat{\jmath}$ + $\hat{k}$) Here $\vec{a_1}$ = $\hat{\imath}$ + 2 $\hat{\jmath}$ + $\hat{k}$, $\vec{b_1}$ = $\hat{\imath}$ - $\hat{\jmath}$ + $\hat{k}$……..(i) Consider ,the line $\vec{r}$ = (2$\hat{\imath}$-$\hat{\jmath}$-$\hat{k}$) +μ(2$\hat{\imath}$ +$\hat{\jmath}$ +2$\hat{k}$) Here $\vec{a_2}$ = $\widehat{2\imath}$ - $\hat{\jmath}$ -$\hat{k}$, $\vec{b_2}$ = 2$\hat{\imath}$ + $\hat{\jmath}$ +2$\hat{k}$……….(ii) Shortest distance between the lines $= \left\| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{\|\vec{b_1} \times \vec{b_2}\|} \right\|$ Now $\vec{a_2}$ - $\vec{a_1}$ =$\widehat{2\imath}$ - $\hat{\jmath}$ - $\hat{k}$-$\hat{\imath}$ -2$\hat{\jmath}$ - $\hat{k}$ =$\hat{\imath}$ -3$\hat{\jmath}$ -2$\hat{k}$ [from (i) and (ii)] $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$ = $\hat{\imath}$(-2-1) - $\hat{\jmath}$(2 - 2) + $\hat{k}$(1+2) = -3$\hat{\imath}$ + 3$\hat{k}$ $\|\vec{b_1} \times \vec{b_2}\| = \sqrt{9+9} = 3\sqrt{2}$	½ ½ ½ ½ ½ ½

	$\therefore$Shortest distance $= \left\lvert \frac{(\hat{\imath} - 3\hat{\jmath} - 2\hat{k})(-3\hat{\imath} + 3\hat{k})}{3\sqrt{2}} \right\rvert$ $= \left\lvert \frac{-3-6}{3\sqrt{2}} \right\rvert = \frac{3}{\sqrt{2}}$ $= \frac{3\sqrt{2}}{2}$ units (OR) Let line through point (2, -1,3) is $\vec{r} = (2\hat{\imath}-\hat{\jmath}-3\hat{k}) + \lambda'(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$……….(i) If line (i) is perpendicular to lines $\vec{r} = (\hat{\imath}+\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}-2\hat{\jmath}+\hat{k})$ and $\vec{r} = (2\hat{\imath}-\hat{\jmath}-3\hat{k})+\mu(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ Then $(a\hat{\imath} + b\hat{\jmath} + c\hat{k}).(2\hat{\imath}-2\hat{\jmath}+\hat{k}) = 0 \Rightarrow 2a -2b +c = 0$ And $(a\hat{\imath} + b\hat{\jmath} + c\hat{k}).(\hat{\imath}+2\hat{\jmath}+2\hat{k}) =0 \Rightarrow a +2b + 2c = 0$ $\Rightarrow \frac{a}{-4-2} = \frac{-b}{4-1}\ \frac{c}{4+2}$ i.e $\frac{a}{-6} = \frac{b}{-3} = \frac{c}{6}$ $\Rightarrow$a; b:c is -6 :-3 ; 6 or 2 : 1: -2 From (i),line is $\vec{r} = (2\hat{\imath}-\hat{\jmath}-3\hat{k}) + \lambda'(2\hat{\imath} + \hat{\jmath} -2\hat{k})$	1 1 1+1 2 1
	SECTION – E **[This section comprises of 3 case – study /passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts(i),(ii) of marks 1, 1, 2 respectively. The third case study questions has two sub parts of 2 marks each.]**	
Q36	A: Guesses B: Copies, C= Knows, E= Correct i) P(A)+P(B)+P(C)=1 $\frac{1}{3} + \frac{1}{6} + P(C) = 1$ $P(C) = \frac{1}{2}$ ii) There are four choices for guesses $P(A) = \frac{1}{4}$ iii) P(B/C) = 1 OR iii) using Baye's theorem $P(B/E)=\frac{\frac{1}{6}X\frac{1}{8}}{\frac{29}{48}}$	1 1 1 1

	$=\frac{1}{29}$	
Q37	i) Volume = x.x.h = 250 $x^2h = 250$ ii) Cost (C) = 50 X $x^2 + 400(h)^2$ $=50x^2+400(h)^2$ iii) $C = 50\frac{250}{h}+400h^2$ $=\frac{12500}{h}+400h^2$ OR iii) h = 2.5M	1 1 1 1 2
Q38	i) Clearly , G be the centroid of the triangle ABCD, there fore coordinates of G are $\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right)$ = (3,2,3) ii) Since A = (0,1,2) and G = (3,2,3) $\overrightarrow{AG} = (3-0)\hat{\imath} + (2-1)\hat{\jmath}+(3-2)\hat{k}$ $= 3\hat{\imath}+\hat{\jmath}+\hat{k}$ $\lvert\overrightarrow{AG}\rvert^2 = 3^2+1^2+1^2$ $=11$ $\lvert\overrightarrow{AG}\rvert = \sqrt{11}$	2 2

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