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ZONAL INSTITUTE OF EDUCATION AND TRAINING, MYSORE

ASSERTION & REASON AND CASE BASED QUESTIONS FOR CLASS XII MATHEMATICS

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DIRECTOR'S MESSAGE

The material on '***Assertion & Reason and Case based Questions for class XII Mathematics 2023-24***' was prepared during the three-day online workshop by experienced teachers from the Feeder Regions namely Bangalore, Chennai, Ernakulum and Hyderabad who have invested their knowledge and expertise to complement the classroom learning experience of the students. The material will serve as an invaluable aid for self-study since it is presented in a manner that is easy to comprehend.

It is hoped that the material will be very useful for the students to perform better in the forthcoming examinations.

Mr. K P Sudhakaran, Principal, KV No.1 CPCRI Kasaragod, Ernakulam Region, as Associate Course Director, Mr. D. Sreenivasalu, Training Associate (Maths) as Course Co-ordinator, Mrs. Beena Prince, PGT(Maths), KV Port Trust and Mrs. Sreedevi PG, PGT(Maths) KV NTPC Kayamkulam, Ernakulam Region as Resource Persons and all the participant-Teachers of the workshop deserve commendations for their sincere efforts and involvement in the preparation of the material.

It is hoped that the material on '***Assertion & Reason and Case based Questions for class XII Mathematics 2023-24***' prepared by experienced teachers will be found useful by teachers and students during their preparations for class XII.

Wishing you all the very best in your academic journey!

MENAXI JAIN
Director

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SYLLABUS

MATHEMATICS (XII)
(Code No. 041)
Session – 2023-24

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Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3×3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. **Continuity and Differentiability**

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, $like\ x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{a^2 - x^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$, $\int \sqrt{x^2 \pm a^2}\, dx$, $\int \sqrt{a^2 - x^2}\ dx,$

$\int \sqrt{ax^2 + bx + c}\, dx$ $\int \frac{px+q}{ax^2+bx+c} dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}$ *+py=q, where p and q are functions of x or constants.*

$\frac{dx}{dy}$ *+ px=q, where p and q are functions of y or constants.*

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

CHAPTER: RELATIONS AND FUNCTIONS

ASSERTION AND REASONING QUESTIONS

	In the following question a statement of Assertion (A) is followed by a statement of reason (R). Pick the correct option: A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true but R is not the correct explanation of A. C. A is true but R is false. D. A is false but R is true
1	**Assertion (A):** If n (A) =p and n (B) = q then the number of relations from A to B is 2^{pq} **Reason (R)** : A relation from A to B is a subset of A x B
2	**Assertion (A):** The relation R in the set A = {1, 2, 3, 4, 5, 6} defined as R={ (x, y) :y is divisible by x} is not an equivalence relation. **Reason (R)** **:**The relation R will be an equivalence relation, if it is reflexive, symmetric and transitive.
3	**Assertion (A):** If R is the relation defined in set {1, 2, 3, 4, 5, 6} as R = {(a, b) :b = a + 1} , then R is reflexive **Reason (R)** **:** The relation R in the set {1 , 2, 3} given by R = {(1, 2), (2, 1)} is symmetric.
4	**Assertion (A)** : A relation R ={ (1,1),(1,2),(2,2),(2,3)(3,3)}defined on the set A={1,2,3} is reflexive. **Reason (R)** **:** A relation R on the set A is reflexive if (a,a) $\in R$,for all a $\in$ A
5	**Assertion (A):** If R is the relation in the set A= {1, 2, 3 , 4, 5 } given by R={(a, b): |a - b| is even} R is an equivalence relation. **Reason (R)** **:** All elements of {1, 3, 5} are related to all elements of {2, 4}
6	**Assertion (A):** The function f :R*→R* defined by f(x) = 1/x is one-one and onto, where R* is the set of all non-zero real numbers. **Reason (R)** **:** The function g: N → R* defined by f(x) = 1/x one-one and onto.

7	**Assertion (A)**: A relation R ={ (1,1),(1,2),(2,2),(2,3)(3,3)}defined on the set A={1,2,3} is symmetric **Reason (R)** : A relation R on the set A is symmetric if (a,b) $\in R \Longrightarrow (b\ ,\ a) \in R$
8	**Assertion (A):** Let f : R →R defined by f(x) = $x^2 + 1$. Then pre-images of 17 are ±4. **Reason (R)** **:** A function f : A→ B is called a one-one function, if distinct elements of A have distinct images in B.
9	**Assertion (A):** The function f : R→R given by f(x) = x^3 is injective. **Reason (R)** **:** The function f : X →Y is injective, if f(x) = f(y) ⇒ x = y for all x,y∈X
10	**Assertion (A):** The function f: R⟶R, f(x)=|x| is not one-one **Reason (R)** **:** The function f(x)=|x| is not onto
11	**Assertion (A):** Let A = {2, 4, 6} and B = {3, 5, 7, 9} and defined a function f = {(2, 3), (4, 5), (6, 7)} from A to B. Then, f is not onto. **Reason (R)** **:** A function f : A → B said to be onto, if every element of B is the image of some elements of A under f.
12	**Assertion (A):** Let a relation R defined from set B to B such that B = {1, 2, 3, 4}.and R= {(1, 1),(2,2),(3,3),(1,3),(3,1)}, then R is transitive. **Reason (R)** **:** A relation R in set A is called transitive, if (a, b) ∈ R and (b, c) ∈ R ⇒ (a,c) ∈ R, ∀ a,b,c∈ A
13.	**Assertion (A):** A function cap f : X → Y is said to be one-one and onto (or bijective) **Reason (R)** **:** if f is both one-one and onto.
14	**Assertion (A):** The relation R on the set N×N , defined by (a, b) R (c, d) ⇔ a+d = b+c for all (a, b), (c, d) ∈ N×N is an equivalence relation **Reason (R)** **:** Any relation R is an equivalence relation, if it is reflexive, symmetric and transitive

15	**Assertion (A)** :The function f : N → N given by f(x) = 2x is not onto **Reason(R)** :The function f is onto, f(x) = f(y) ⇒ 2x = 2y ⇒ x = y.
16	**Assertion (A)** : Let L be the set of all lines in a plane and R be the relation in L defined as R={(L_1, L_2) : L_1, is perpendicular to L_2,). R is not an equivalence relation. **Reason (R)** : R is symmetric but neither reflexive nor transitive.
17	**Assertion (A):**{(T_1, T_2,) : T_1, is congruent to T_2}. Then R is an equivalence relation. **Reason (R)** : Any relation R is an equivalence relation, if it is reflexive, symmetric and transitive .
18	**Assertion (A)** : The relation R in the set {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but Neither symmetric nor transitive. **Reason (R)** : R is not symmetric as (1, 2) ∈ R but (2,1) ∉ R. Similarly, R is not transitive as (1,2) ∈ R and (2,3) ∈ R but (1, 3) ∉ R.
19	**Assertion (A)** : Show that the relation R in the set A of all the books in a library of a college, given by R = { (x, y) : x and y have the same number of pages } is not an equivalence relation. **Reason (R)** : Since R is reflexive, symmetric and transitive.
20	**Assertion (A)** : The relation R in **R** defined as R = {(a, b) : a ≤ b)} is not an equivalence relation. **Reason (R)** : Since R is not reflexive but it is symmetric and transitive.
21	**Assertion (A)** : The relation R in **R** defined as R = {(a, b) : a ≤ b^2} is not anequivalence relation. **Reason (R)** : Since R is neither reflexive nor symmetric, nor transitive.

22	**Assertion (A)** : Set A has 3 elements and set B has 5 elements then number of injective function from set A to set B will be 60 **Reason (R)** : If set A has m elements and set B has n elements where $n \geq m$, the number of injective function is ${}^{n}p_{m}$
23	**Assertion (A)** : A function f(x) = cosx, $x \in [0, 3\pi/2]$ is bijective **Reason (R)** : For one - one function each elements in domain has unique image in codomain
24	**Assertion (A):** The function R→R defined by f(x) = IxI is neither one one nor onto **Reason (R)** : The signum function R →R given by f(x) $= \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ bijective function
25	**Assertion (A):** The possible numberof reflexive relations of set A whose n(A) =4 is 2^{12} **Reason (R)** **:** Number of reflexive relation on a set contain n elements is 2^{n^2-n}
EXCERCISE	
1	**Assertion (A) :** The relation R in the set Z of integers given by R={(a, b) : 2 divides a-b } is reflexive and symmetric. **Reason (R) :** R is reflexive, as 2 divides (a - a) for all $a \in Z$.
2	**Assertion (A) :** Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both and b are either odd or even). R is equivalence relation. **Reason (R) :** Since R is reflexive, symmetric but R is not transitive.
3	**Assertion (A)** : Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3) ,(3,3), (3,2)}. R is not an equivalence relation. **Reason (R)** : R is not Reflexive relation but it is symmetric and transitive

4	**Assertion (A)** : If n (A) = p and n (B) = q. The number of relation from set A to B is pq. **Reason (R)** : The number of subset of A × B is 2^{pq}
5.	**Assertion (A):** Domain and Range of a relation $R = \{(x, y): x - 2y = 0\}$ defined on $A=\{1, 2, 3, 4\}$ are respectively {1, 2,3, 4} and {2, 4, 6, 8} **Reason(R):** Domain and Range of a relation R are respectively the sets $\{a: a \in A \ and \ (a, b) \in R.\} \ and \ \{b: b \in A \ and \ (a, b) \in R\}$
6	**Assertion (A**) :Let R be any relation in the set A of human beings in a town at a particular time If R={ (x, y) : x is wife of y}, then R is reflexive. **Reason (R):** If R= {(x, y) :x is father of y}, then R is neither reflexive nor symmetric nor transitive.
7	**Assertion (A)**: If X = { 0, 1, 2 } and the function f: X → Y defined by $f(x) = x^2-2$ is surjection then Y = { -2, -1, 0 } **Reason(R):** Iff: X → Y is surjectiveif f(X) = Y

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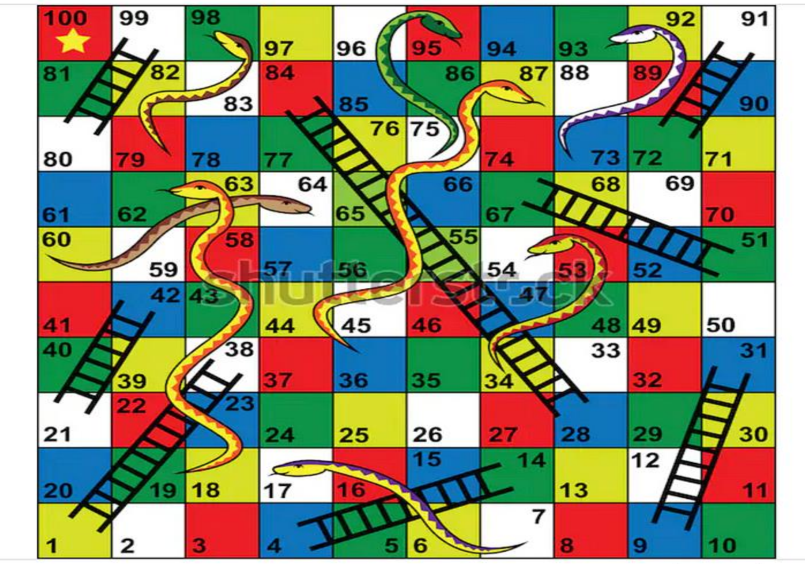
ANSWER KEY

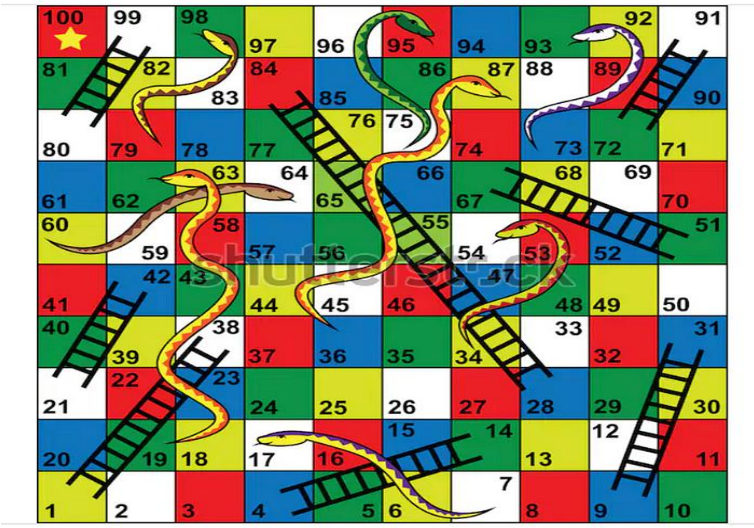
1.	**Answer: A** **Solution:** A is true - No of elements of AXB = pxq, So the number of relations from A to B is 2^{pq} R is true – every relation from A to B is a sub set of AXB
2.	**Answer : A** **Solution:** A is true-R is reflexive and transitive but not symmetric ie $(2,4)\epsilon R \nRightarrow (4,2)\epsilon R$ R-true- Definition of an equivalence relation
3.	**Answer :D** **Solution** : A is false – $(x,x)\notin R$ R is true - $(1, 2)\in R \Rightarrow (2, 1) \in R$
4.	**Answer: A** **Solution:** A is true - $(a,a) \in R$,for all $a \in A$ R is true – Correct explanation for reflexive relation
5.	**Answer: C** **Solution:** A is true- Since it an equivalence relation R is false – the modulus of difference between the two elements from each of these two subsets will not be even
6.	**Answer:C** **Soluion:** A is true – Every elements has distinct images and range is R* R is false – g is one one but range is a subset of R*
7.	**Answer: D** **Solution:** A is false - $(2,3)\epsilon R \nRightarrow (3,2)\epsilon R$ R is true - A relation R on the set A is symmetric if $(a,b) \in R \Longrightarrow (b\,,\,a) \in R$
8.	**Answer: B** **Solution:** A is true - $x^2 + 1=17$, $x^2 =16$, $x=\pm4$ R is true - but reason is not the correct explanation of A
9.	**Answer: A** **Solution** : A is true – $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ R is true – and is the correct explanation of A
10.	**Answer: B** **Solution:** A is true – f(1) = f(-1)=1,so not one one R is true – range of f = $[0,\propto)\subset R$ not a correct explanation of A
11.	**Answer: A** **Solution:** A is true – Range of f $\subset B$ R is true - Range of f $= B$ and reason is correct
12.	**Answer: A** **Solution:** A is true - $(1,3)\in R,(3,1)\in R \Rightarrow (1,1) \in R$ R is true – correct definition of transitive relation

13.	**Answer: A** **Solution:** A is true R is true with correct reason
14.	**Answer: A** **Solution**: A is true- R is if it is reflexive, symmetric and transitive R is true and reason is correct
15.	**Answer: C** **Solution**: A is true – range is a subset of N R is false - not the definition of onto
16.	**Answer: A** **Solution**: A is true - (L1, L1) $\notin$ R – not reflexive and (L1, L2)$\in$ R, (L1, L3) $\notin$ R – (L2, L3)$\in$ R$\Rightarrow$ not transitive but symmetric R is true – reason is correct
17.	**Answer: A** **Solution**: A is true R is true and reason is correct
18.	**Answer: A** **Solution**: A is true R is true and reason is correct
19.	**Answer: D** **Solution :** A is false – R is an equivalence relation R is true
20.	**Answer: C** **Solution:** A is true- not symmetric, ie (2,4)$\in$ R but (4,2) $\notin$ R R is false – R is reflexive and transitive but it is not symmetric.
21.	**Answer: A** **Solution:** A is true-not reflexive ,ie(1/2,1/2) $\notin$ R -not symmetric (1,2)$\in$ R but (2,1) $\notin$ R- not transitive (3,2) $\in$ R, (2,1.5)$\in$ R but (3,1.5)$\notin$ R R is true – reason is correct
22.	**Answer: A** **Solution:** A is true $^5p_3 = 60$ R is true
23.	**Answer: D** **Solution:** A is false- $\cos\frac{\pi}{2}=0=\cos\frac{3\pi}{2}$ R is true
24.	**Answer:C** **Solution:** A is true- f(1) = f(-1)=1,so not one one and range of f = $[0,\propto) \subset R$ R is False– f(1) = f(2)=1,so not one one and range of f $=\{0,1,-1\} \subset R$
25.	**Answer:A** **Solution**: A is true R is true and R is the correct reason

EXERCISE	
1.	Answer:B
2.	Answer:C
3.	Answer: C
4.	Answer: D
5.	Answer: D
6.	Answer: D
7.	Answer:A

CASE STUDY BASED QUESTIONS

1	Vivek and Anuj are playing snake and ladder game at home during vacation. While rolling the dice ?Vivek‘s cousin Anand observed and noted that possible out comes of the throw every time belongs to set { 1,2,3,4,5,6}. Let X be the set of players and Y be the set of all possible out comes. X= { V, A} Y= { 1,2,3,4,5,6}  (i) Let R: Y to Y be defined by R= { (a,b) / b is divisible by a }. Then R is A. Reflexive B. symmetric C. reflexive &transistive D. Equivalence (ii) How many functions are possible from X to Y (iii) How many relations are possible from X to Y
2	In a school library around 8000 books are there . It is divided and kept in different places in the big library room based on the language-

	English, Malayalam, Hindi, Sanskrit. It is again divide based on poems, Novels, Literature, short story, Dictionary etc.  Let B = set of all books in the library. A relation R is defined on B , given by R= { (x,y); x and y have same number of pages} (i) Is R a reflexive relation? (ii) Is R a symmetric relation? (iii) Is R a transitive relation? (iv) Is it an equivalence relation?
3	Arya and Medha are playing snake and ladder game at home during vacation. While rolling the dice Arya 's cousin Anand observed and noted that possible out comes of the throw every time belongs to set { 1,2,3,4,5,6}. Let X be the set of players and Y be the set of all possible out comes. X= { V, A} Y= { 1,2,3,4,5,6}  Let R: Y to Y be defined by R= { (a,b) : $\lvert a - b \rvert$ is even }. (i) Is R a reflexive relation? (ii) Is R a symmetric relation? (iii) Is R a transitive relation? (iv) is it an equivalence relation?

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A scout master wants to make different groups of students sothat they can be given different tasks. Students started making groups with their friends, then the scout master interfere and told them to make groups as per a rule “a student will make group with roll number in such a way that the difference of roll number is divisible by 3.

(i) Write a relation **R** in set-builder form for the rule told by the scout master.
(ii) Which roll number of students will be in the group of students with roll number 2, if there are 30 students in the class?
(iii) Which roll number of students will be in the group of students with roll number 3, if there are 30 students in the class?

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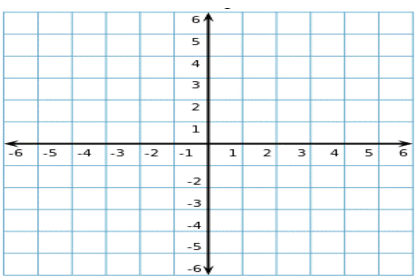
Mohan and Ritu were playing a game .The rule for the game is that when Mohan plots a point say P on a Cartesian plane, then Ritu has to find points on the plane in such a way the distance of the points from the origin O is same as that of the distance between the origin and the point P(ie; OP). Mohan’s sister Avantika put the rule of the game in set form as R= { (P, Q) : OP = OQ}



(i)Is R a reflexive relation?
(ii) Is R a symmetric relation?
(iii) Is R a transitive relation?
(iv) is it an equivalence relation?
(v) what figure Ritu get if She joins all such points

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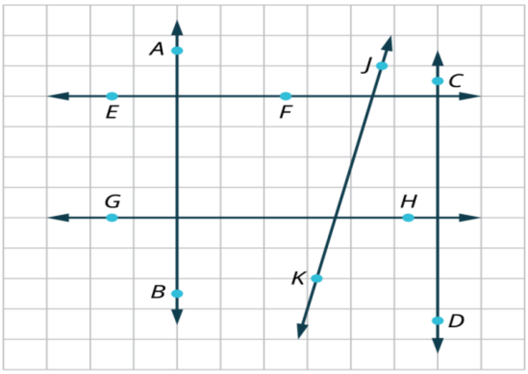
6	Abhimanu is having a collection of all lines in XY plane and his friend Arjun made a relation on this collection of lines as $R_1 = \{ (L_{1,} L_2) / L$ 1 is parallel to L2}. Arjun also made another relation on the same collection as $R_{2=}$ { (L1, L2) / L1 is perpendicular to L2}.  They ask their class mate to check the following: (i) Is the relation R1 form an equivalence relation (ii) Find the set of lines related to y= 2x + 4 with respect to R1 (iii) Is the relation R2 form an equivalence relation?
7	In class 6, the teacher conducted a survey on the fruits and vegetables the students like most.The class leader is asked to record the data . from the the data it was found that the topmost 3 fruits liked by most of them are Apple, Orange, Mango whereas the 3 vegetables most of them like are Tomatoes, Carrot, Cucumber.The teacher put it in 2 sets F={Apple, Orange, Mango } and V = { Tomatoes, Carrot, Cucumber}  (i) How many relations are possible from F to V?

	(ii) How many functions are possible from F to V? (iii) How many one to one functions are possible from F to V? (iv) How many bijections are possible from F to V?
8	48 children joined a vacation camp organized by Sports authority of a locality and they were allotted registration number .They want to make 4 teams with 12 members each out of the 48 students for football match . The coach asked the children to form teams in such way that " two students are in the same team if the difference of their registration number is divisible by 4".  Now answer the question based on the above information: (i) Is it possible to form the teams by the rule given by the Coach? (ii) Which registration numbers are the members of the team with number 1? (iii) Which registration numbers are the members of the team with number 4?
9	During a SwachBharth Abhiyan organizing committee wanted to collect and segregate Metal, paper, glass, batteries , organic and plastic waste , In the set of all participants a relation R defined as R= { (x,y)/ both the participants collect the same type of waste} 

	Based on the given information answer the following: (i) Check whether R is an Equivalence relation in the set of all participants (ii) In how many groups the participants are divided on the basis of their waste collection, assume that there are participants to collect all type of the waste.
10	In a master chef competition final round 3 chef were selected and Judges assigned three dishes D= { D1, D2, D3} to the participants P= {P1,P2, P3} and asked them to prepare dishes as per the folowing rules: Rule A :everybody has to prepare exactly one dish Rule B:no 2 paricipants is allowed to prepare same dish Rule C : all the dish must be prepared in the competition  Answer the following in the context of functions (i) In how many ways all participants can choose a Dish as per rule A? (ii) In how many ways everybody can choose a dish to prepare as per the rule B? (iii) How many ways all participants can prepare exactly one dish as per rule C?

ANSWER KEY

1	SOLUTIONS: (i) C (ii) 6^2 (iii) 2^{12}
2	SOLUTIONS/ANS: (i)Yes (ii) yes (iii) yes (iv) yes
3	SOLUTIONS/ANS: (i) Yes (ii) yes (iii) yes (iv) yes
4	SOLUTIONS/ANS: (i) R = { (a, b)/ | a - b| is divisible by 3 } (ii) { 2, 5, 8,11,14, …..} (iii) {3,6,9,12,15…….}
5	SOLUTIONS/ANS: (i) Yes (ii) yes (iii) yes (iv) yes (v) circle
6	SOLUTIONS/ANS: (i) R1 is reflexive, symmetric, transistive so, equivalence (ii) y = 2x + c ; c is any arbitary constant (iii) No(R is not reflexive, transistive but symmetric)
7	SOLUTIONS/ANS: (i) 2^9 (ii) 3^3 (iii) 3!/ (3-3) ! = 3!= 6 (iv) 3!
8	SOLUTIONS/ANS: (i) Yes (ii) { 1, 5, 9, 13, 17, 21…….} (iii) { 4, 8, 12, 16, …..}
9	SOLUTIONS/ANS: (i) Yes , it is an equivalence relation (ii) 6 groups
10	SOLUTIONS/ANS: (i) No of function 3 x 3 x 3= 27 (ii) No of one to one functions from D to P= 3! (iii) No of onto functions from D to P = 3!

CHAPTER : INVERSE TRIGONOMETRIC FUNCTIONS

ASSERTION AND REASONING QUESTIONS

Q.No	Questions
1	**Assertion(A):** The range of the function $f(x)=2\sin^{-1}x+3\pi/2$, where $x\epsilon[-1,1]$, is $[\pi/2, 5\pi/2]$. **Reason(R):** The range of the principal value branch of $\sin^{-1}x$ is $[0,\pi]$
2	**Assertion(A):** Maximum value of $(\cos^{-1}x)^2$ is π^2 **Reason(R):** Range of principal value branch of $\cos^{-1}x$ is $[-\pi/2, \pi/2]$.
3	**Assertion(A):** Range of $\sin^{-1}x+2\cos^{-1}x$ is $[0,\pi]$ **Reason(R):** The range of the principal value branch of $\sin^{-1}x$ is $[-\pi/2,\pi/2]$
4	**Assertion(A):** All trigonometric functions have their inverses over their respective domains **Reason(R):** The inverse of $\tan^{-1}x$ exists for some $x\ \epsilon R$
5	**Assertion(A):** The value of $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})=-\pi/2$ **Reason(R):** The principal value branch of $\sin^{-1}x$ is $[\pi/2,\pi/2]$ and the principal value branch of $\cos^{-1}x$ is $[0,\pi]$
6	**Assertion(A)**: The principal value of $\sin^{-1}(1/\sqrt{2})=\pi/4$ **Reason(R):** The principal value branch of $\sin^{-1}x$ is $[\pi/2,\pi/2]$ and $\sin(\pi/4)=1/\sqrt{2}$
7	**Assertion(A):** The value of $\sin^{-1}(\sin 3\pi/5)=3\pi/5$ **Reason(R):** $\text{Sin}^{-1}(\text{Sin } x)=x$, if $x\epsilon[-\pi/2,\pi/2]$

8	**Assertion(A):** The value of $\tan^{-1}[2sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$=π/3 **Reason(R):** The range of principal value branch of $\cot^{-1}$ is (0, π)
9	**Assertion(A):** $\sin^{-1}x = (\sin x)^{-1}$ **Reason(R):** Any value in the range of principal value branch is principal value of that inverse trigonometric function.
10	**Assertion(A):** The domain of the function $\sec^{-1}2x$ is $(-\infty, -1/2] \cup [1/2, \infty)$ **Reason(R):** $\text{Sec}^{-1}(-2) = -\pi/4$
11	**Assertion(A):** If $\tan^{-1}(\sin(-\pi/2)) = -\pi/4$ **Reason(R):** $\cos^{-1}(1/2) = \pi/3$
12	**Assertion(A):** $\cos^{-1}(\cos 13\pi/6) = \pi/6$ **Reason(R):** $\cos(13\pi/6) = \cos \pi/6$
13	**Assertion(A):** $\tan(1) > \tan^{-1}(1)$ **Reason(R):** tanx is an increasing function in (-π/2, π/2) of the Assertion(A)
14	**Assertion(A):** The principal value of $cos^{-1}(\frac{-1}{2})$ is $\frac{2\pi}{3}$ **Reason(R):** The principal value of cos^{-1} x lies between $(0,\pi)$
15	**Assertion(A):** the principal value of $cos^{-1}(cos\frac{7\pi}{6})$ is $\frac{5\pi}{6}$ **Reason(R):** $cos^{-1}(\cos x) = x \quad \forall \ x \in [o\ ,\pi]$
16	**Assertion(A):** The value of $\sin[\frac{\pi}{3} - sin^{-1}(\frac{-1}{2})]$ is 1 **Reason(R):** The principal value of $sin^{-1}(\frac{-\sqrt{3}}{2}) = \frac{-\pi}{3}$
17	**Assertion(A):** The domain of the function $sin^{-1}(2x-1)$ is [0,1]

	Reason(R): The domain of the function sin^{-1} x is[- 1 ,1]
18	**Assertion(A):** If $0 \leq x \leq \pi$,thenthe value of sin^{-1}(cos x) is ($\frac{\pi}{2}$ –x) **Reason(R):** sin^{-1}(sin x) = x, ∀ x ∈ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
19	**Assertion(A):** The principal value of tan^{-1} (tan $\frac{3\pi}{5}$) is - $\frac{2\pi}{5}$ **Reason(R):** tan^{-1} (tan x) = x ∀ x ∈ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
20	**Assertion(A):** Inverse of sine function exist in[0, π] **Reason(R):** sin^{-1} function becomes bijective if we restrict the domain to [-1 ,1]
21	**Assertion(A):** The principal value of tan^{-1} (-1) is $-\frac{\pi}{4}$ **Reason(R):** The range of principal value branch of $tan^{-1} is$ ($\frac{-\pi}{2}$,$\frac{\pi}{2}$) and tan^{-1} (-x) = -tan^{-1} x
22	**Assertion(A):** Principal value of sin^{-1} (sin $\frac{17\pi}{18}$) is $\frac{\pi}{18}$ **Reason(R):** Domain of principal value branch of sin^{-1} is [-1, 1]
23	**Assertion(A):** The value of $tan^{-1}\sqrt{3}$ - $cot^{-1}(-\sqrt{3})$ is $\frac{-\pi}{2}$ **Reason(R):** The principal branch of tan^{-1} is [$\frac{-\pi}{2}, \frac{\pi}{2}$] and cot^{-1} (-x) =π - cot^{-1} x
24	**Assertion** : $\sin^{-1}$(sin$\frac{2\pi}{3}$) is$\frac{2\pi}{3}$ **Reason :** $\sin^{-1}$ (sinθ) = θ, θ ∈ [$\frac{-\pi}{2}, \frac{\pi}{2}$]
25	**Assertion(A):** principal value of$\cos^{-1}$ cos (-680) = $\frac{2\pi}{9}$ **Reason(R)** : $\cos^{-1}$(-x) =π-$\cos^{-1}$x

ANSWER KEY

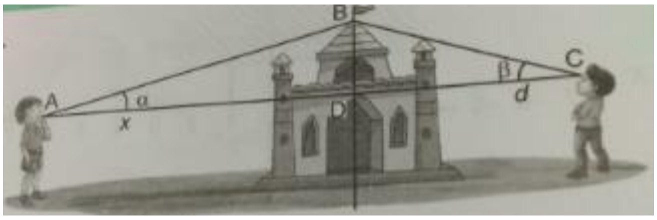
Q. No	ASSERTION REASON SOLUTIONS
1	<u>Solution</u>: $-\pi/2 \leq \sin^{-1} x \leq \pi/2$ $-\pi \leq 2\sin^{-1} x \leq \pi$ $\pi/2 \leq 2\sin^{-1} x+3\pi/2 \leq 5\pi/2$. Hence assertion is true The range of the principal value branch of $\sin^{-1} x$ is $[-\pi/2,\pi/2]$.Hence Reason R is False. Assertion(A) is true Reason(R) is False. Hence correct option is c.
2	Maximum value of $(\cos^{-1}x)$ is π Maximum value of $(\cos^{-1}x)^2$ is π^2Hence assertion is true Range of principal value branch of $\cos^{-1}x$ is $[0, \pi]$ Hence Reason R is False. Assertion(A) is true Reason(R) is False. Hence correct option is c.
3	Assertion(A) is false, since Range of $\sin^{-1}x+2\cos^{-1}x$ is $[-\pi/2, 5\pi/2]$. Reason(R) is true. Hence correct option is d.
4	Assertion(A) is false , since All trigonometric functions have their inverses over their principal branches Reason(R) is true. Hence correct option is d.
5	$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \pi/3-(\pi-\pi/6)$ $= -\pi/2$ Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A) So correct option is a
6	Both Assertion (A)and Reason(R)are true and Reason(R)is the correct explanation of the Assertion(A)So correct option is a
7	Assertion (A) is false and Reason (R) is true.So correct option is d $\sin^{-1}(\sin 3\pi/5) = \sin^{-1}[\sin (\pi-3\pi/5)] = \sin^{-1}[\sin (2\pi/5) = 2\pi/5$
8	Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of the Assertion(A).SoCorrect option:b
9	Assertion (A) is false and Reason (R) is true.So correct option is d
10	$\sec^{-1}(-2) = 2\pi/3$ Hence Reason R is False.

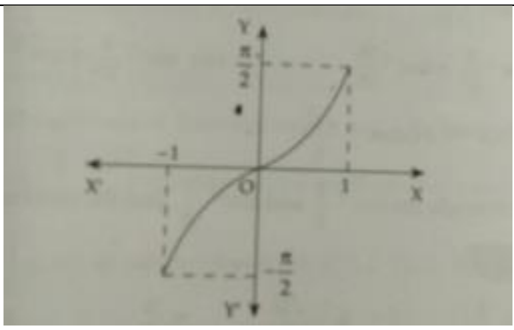
	The domain of the function $sec^{-1}x$ is $(-\infty,-1] \cup [1, \infty)$. So the domain of the function $sec^{-1}2x$ is $(-\infty,-1/2] \cup [1/2,\infty)$. So Assertion(A) is true. Hence correct option is c.
11	$\tan^{-1}(\sin(-\pi/2)) = \tan^{-1}(-\sin(\pi/2)) = \tan^{-1}(-1) = -\pi/4$ and $\cos^{-1}(1/2) = \pi/3$ So both Assertion and Reason are correct.But Reason(R) is not the correct explanation of the Assertion(A).So correct option is b
12	$\cos(13\pi/6) = \cos(2\pi+\pi/6)= \cos \pi/6$ $\cos^{-1}(\cos 13\pi/6) = \cos^{-1}(\cos \pi/6) = \pi/6$ So both Assertion and Reason are correct andReason(R) is the correct explanation of the Assertion(A). So option a is correct
13	tanx is an increasing function in $(-\pi/2, \pi/2)$ $1 > \pi/4$ $\tan 1 > \tan \pi/4$ $\tan 1 > 1$ since $1 > \pi/4$ $\tan 1 > 1 > \pi/4$ $\tan 1 > 1 > \tan^{-1}(1)$ So $\tan(1) > \tan^{-1}(1)$ So both Assertion and Reason are correct Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is a.
14	$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - COS^{-1}(1/2) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is a.
15	$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = =\cos^{-1}\left(\cos(2\pi - \frac{5\pi}{6}\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6} \in (0,\pi)$ Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is a.
16	in $[\frac{\pi}{3} - \sin^{-1}(\sin(\frac{-\pi}{6})]$ = $\sin(\frac{\pi}{3} - (\frac{-\pi}{6})) = \sin\frac{\pi}{2} = 1$ Also $\sin^{-1}(\frac{-\sqrt{3}}{2}) = \sin^{-1}(\sin(\frac{-\pi}{3})] = \frac{-\pi}{3}$ So both Assertion and Reason are correct.But Reason(R) is not the correct explanation of the Assertion(A).So correct option is b
17	<u>Solution:</u> $-1 \leq 2x-1 \leq 1$, $-1 + 1 \leq 2x \leq 1+1$ $o \leq x \leq 1$ Hence Reason(R) is the correct explanation

18	<u>Solution:</u> $0 \le x \le \pi$ implies $o \ge -x \ge -\pi$ $-\pi \le -x \le o$ $\frac{\pi}{2} - \pi \le \frac{\pi}{2} - x \le \frac{\pi}{2} - o$ $\frac{-\pi}{2} \le \frac{\pi}{2} - x \le \frac{\pi}{2}$ $\sin^{-1}(\cos x) = \sin^{-1}(\sin(\frac{\pi}{2} - x)) = (\frac{\pi}{2} - x)$ Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is
19	$\tan^{-1}(\tan\frac{3\pi}{5})$ $= \tan^{-1}(\tan(\pi - 2\pi/5)$ $= \tan^{-1}(-\tan 2\pi/5) =$ $-\tan^{-1}\tan(2\pi/5) =$ $-\frac{2\pi}{5}$ Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is a.
20	Inverse of sine function exists in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Assertion (A) is false and Reason (R) is true.So correct option is d
21	$\tan^{-1}(-x) = -\tan^{-1} x$ $\tan^{-1}(-1) = -\tan^{-1} 1$ $= -\frac{\pi}{4}$ Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is a.
22	$\sin^{-1}\left(\sin\frac{17\pi}{18}\right) = \sin^{-1}(\sin \pi - \frac{\pi}{18})$ $= \sin^{-1}(\sin\frac{\pi}{18})$ $= \frac{\pi}{18}$, since $\sin^{-1}(\sin x) = x$ for every $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ So both Assertion and Reason are correct.But Reason(R) is not the correct explanation of the Assertion(A).So correct option is
23	$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$ $\cot^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right) = \frac{\pi}{3} - \frac{5\pi}{6} = \frac{-\pi}{2}$ Hence Reason(R) is the correct explanation of the Assertion(A) So correct option is a.
24	$\sin^{-1}(\sin\frac{2\pi}{3}) = \sin^{-1}(\sin(\pi - \frac{\pi}{3})$ $\sin^{-1}(\sin\frac{\pi}{3})$ $= \pi/3$

	Assertion (A) is false and Reason (R) is true. So correct option is d
25	$\cos^{-1}\cos(-680^0)$ $= \cos^{-1}\cos(680^0)$ $=\cos^{-1}\cos(720^0-40^0)$ $=\cos^{-1}\cos(40^0)$ $=40^0$ $= \frac{2\pi}{9}$ So both Assertion and Reason are correct.But Reason(R) is not the correct explanation of the Assertion(A).So correct option is b

CASE BASED QUESTIONS

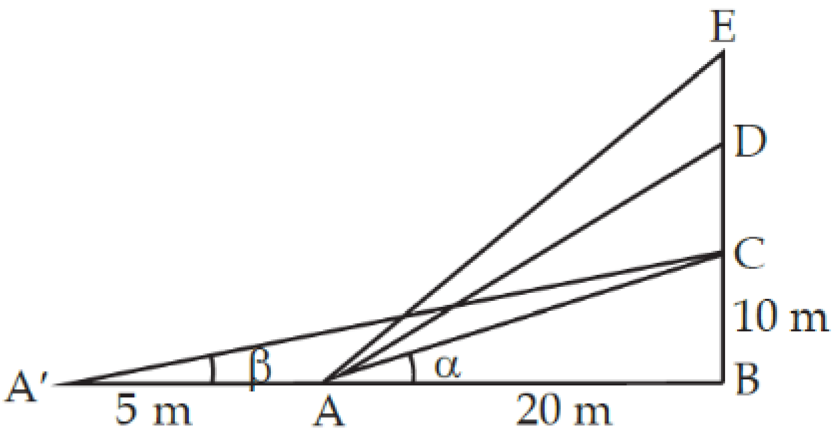
1	Two men on either side oaf a temple 30 m high from the level of eye observe its top at the angles of elevation α, β respectively . the distance between two men is $40\sqrt{3}$meters and the first person and the temple is $30\sqrt{3}$ based on the information answer the following  1)find angle CAB 2)Find the principal value of $\sin^{-1}\sin(\alpha + \frac{\pi}{3})$ 3) Find the principal value of $\cos^{-1}\cos(\alpha + \frac{\pi}{3})$
2	Ram and Krishna are students of class XII. one teacher told them about inverse trigonometric functions and she sketched the graph of$\sin^{-1} x$ as follows Based on the above answer the following



1) Fin the domain of $\sin^{-1}x$
2) Find the range of principal value branch of$\sin^{-1}$ x
3) Find the domain of $\sin^{-1}(1-x)$

3 The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. 'A' is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby, Ram Robert and Rahim suggested to the film to place the hoarding board at three different locations namely C, D and E. 'C' is at the height of 10 metres from the ground level. For the viewer 'A', the angle of elevation of 'D' is double the angle of elevation of 'C'. The angle of elevation of 'E' is triple the angle of elevation of 'C' for the same viewer.

Look at the figure given and based on the above information answer the following:



(i) Measure of ∠CAB =
(a) $\tan^{-1}(2)$
(b) $\tan^{-1}(1/2)$
(c) $\tan^{-1}(1)$

	(d) $\tan^{-1}(3)$ (ii) Measure of ∠DAB = (a) $\tan^{-1}(3/4)$ (b) $\tan^{-1}(3)$ (c) $\tan^{-1}(4/3)$ (d) $\tan^{-1}(4)$ (iii) Measure of ∠EAB (a) $\tan^{-1}(11)$ (b) $\tan^{-1}(3)$ (c) $\tan^{-1}(2/11)$ (d) $\tan^{-1}(11/2)$ (iv) A' is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between ∠CAB and ∠CA'B is (a) $\tan^{-1}(1/12)$ (b) $\tan^{-1}(1/8)$ (c) $\tan^{-1}(2/5)$ (d) $\tan^{-1}(11/21)$

ANSWERS(CASE BASED QUESTIONS)

1	1) $\tan\alpha = \frac{BD}{AD} = \frac{1}{\sqrt{3}}$ CAB $=\frac{\pi}{6}$ 2) $\sin^{-1}\ \sin(\alpha+\frac{\pi}{3}) = \sin^{-1}\ \sin(\frac{5\pi}{6}) = \frac{\pi}{6}$ 3) $\cos^{-1}\ \cos(\alpha+\frac{\pi}{3}) = \cos^{-1}\ \cos(\frac{2\pi}{3}) = (\frac{2\pi}{3})$
2	Ans [-1,1] $[-\frac{\pi}{2}\frac{\pi}{2}]$ [0,2]
3	Answer: (i) b (ii) c (iii) d (iv) a

CHAPTER: MATRICES

ASSERTION AND REASONING QUESTIONS

	In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true and (R) is not the correct explanation of (A). (c) (A) is true and (R) is false (d) (A) is false but (R) is true.
1.	**Assertion(A) :** If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then x=2, y =2, z = -5, and w = 4. **Reason(R) :** Two matrices are equal if their orders are same and their corresponding elements are equal.
2.	Let A and B be two symmetric matrices of order 3. **Assertion(A) :** A(BA) and (AB)A are symmetric matrices. **Reason(R) :** AB is symmetric matrix, if matrix multiplication of A with B is commutative.
3.	**Assertion(A) :** If A = $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then AB and BA both are defined. **Reason(R) :** For two matrices A and B , the product AB is defined , if the number of columns in A is equal to the umber of rows in B.
4.	**Assertion(A) :** Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a diagonal matrix. **Reason(R) :** Identity matrix of order 3 is a diagonal matrix.

5.	**Assertion(A) :** For any square matrix B with real number entries , $B + B^T$ is skew symmetric matrix and $B - B^T$ is symmetric matrix . **Reason(R) :** A square matrix B can be expressed as the sum of symmetric and skew symmetric matrix.
6.	**Assertion(A) :** If A is a skew- symmetric matrix, then A^2 is a symmetric matrix. **Reason(R) :** If A is a skew- symmetric matrix, then $A^T = -A$
7.	Let $A(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ **Assertion(A) :** $\left(A(\frac{\pi}{3})\right)^3 = -I$, I is identity matrix. **Reason(R) :** $A(\theta_1).A(\theta_2) = A(\theta_1 + \theta_2)$
8.	Let A,B and C be three square matrices of the same order. **Assertion(A) :** If AB = O, then A = O or B = O. **Reason(R) :** A+B = A+C → B = C.
9.	**Assertion(A) :** If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$, is a skew- symmetric matrix then x = 2 **Reason(R) :** For a skew- symmetric matrix, $a_{ij} = 0$ for I = j .
10.	**Assertion(A) :** $\begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$ is a scalar matrix. **Reason(R) :** All the elements of the principal diagonal are equal, it is called a scalar matrix.

11.	**Assertion(A) :** A = $\begin{bmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{bmatrix}$ is a symmetric matrix. Then x = 6 **Reason(R) :** If A is a symmetric matrix, then A^T= A.
12.	**Assertion(A) :** A =$\begin{bmatrix} 2 & -2 & 0 \\ 6 & 4 & -2 \end{bmatrix}$ and B =$\begin{bmatrix} 2 \\ 6 \\ 10 \end{bmatrix}$ then $(AB)^T = [-8 \quad 16]$ **Reason(R) :** AB = $[-8 \quad 21]$
13.	**Assertion(A) :** If the matrix A is both symmetric and skew- symmetric matrix, then A is a zero matrix. **Reason(R) :** A^T= A and A^T= -A
14.	**Assertion(A) :** If A is a matrix of order m$\times$ n, and BA^T, A^TB both are defined , then B is m $\times$ n. **Reason(R) :** The product AB is defined , then number of columns of A is same as number of rows of B.
15.	Let A and B are symmetric matrices of same order **Assertion(A) :** AB –BA is skew- symmetric **Reason(R) :** AB +BA is symmetric
16.	**Assertion(A) :** If $\begin{bmatrix} 0 & -5 & 7 \\ 5 & 2k-2 & 4 \\ -7 & -4 & 0 \end{bmatrix}$is a skew- symmetric matrix then k =1 **Reason(R) :** In askew symmetric matrix all the principal diagonal elements are zero.
17.	Let A and B are square matrices of order 2 x 2 and 2 x 3 respectively , then **Assertion(A) :** AB exists but BA does not exists

	Reason(R) : If X and Y are matrices then XY exists only if number of columns of X and number of rows of Y are the same
18.	If $\begin{bmatrix} x+y & -2 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}$ then **Assertion(A) :** x= -1 and y =3 **Reason(R) :** Any two matrices can be compared
19.	If A is a square matrix ,then **Assertion(A) :** $A + A^1$ is symmetric **Reason(R) :** A square matrix X is said to be symmetric if $X^1 = X$
20.	**Assertion(A) :** The matrix A = $\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$ is a skew -symmetric **Reason(R) :** A square matrix is skew-symmetric if its principal diagonal elements are zeros
21.	**Assertion(A) :** The square matrix $\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}$,can be expressed as the sum of a symmetric matrix $\begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$and a skew symmetric matrix$\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$ **Reason(R):** For every square matrix A,$\frac{A+A^1}{2}$is symmetric and $\frac{A-A^1}{2}$ is skew - symmetric
22.	If a matrix has 12 elements then **Assertion(A) :** The number of possible dimensions of the matrix is 6 **Reason(R):** If a matrix has a total of m elements then the possible factorizations of m will be the possible dimensions of the matrix .
23.	Let A be a square matrix of order 2 x 2 ,then

	Assertion(A) : The number of possible matrices A with its each entry either 0 or 1 is 16. **Reason(R):** If an operation can be performed in m ways following which another operation can be performed in n ways then the two operations can be performed successively in m x n ways
24.	Let A = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ **Assertion(A) :** A^{-1} does not exists **Reason(R):** For a square matrix A , A^{-1} exist if $|A| \neq 0$
25.	**Assertion(A) :** A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}$ is invertible **Reason(R):** A is a square matrix and $|A| = -8$

ANSWERS

1.	w = 4, z +5 = 0, z = -5, xy =4 and x +y =4 , so x =2 , y =2 . First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
2.	$\{A(BA)\}^T = (BA)^T \cdot A^T = (A^T B^{T)} A^T$= (AB)A= A(BA) $[(AB)A]^T = A^T (AB)^T$= A($B^T A^T$)= A (BA) = (AB)A . First statement is correct. $(AB)^T = B^T A^T$ = BA = AB , Second statement is correct and it is not the correct explanation of A Option (b)
3.	A = 2×3, B = 3×2 , AB is 2×2, B = 3×2 , A = 2×3, BA = 3×3 First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
4.	First statement is correct. Second statement is correct and it is not the correct explanation of A Option (b)

5.	First statement is not correct. Second statement is correct Option (d)
6.	First statement is correct. Second statement is correct . A = -A^T, Squaring, $A^2 = (-A^T)^2 = (A^T)^2 = (A^2)^T \Rightarrow A^2$ is symmetric matrix. So Second statement is correct and it is the correct explanation of A Option (a)
7.	Second statement is correct . A($\theta 1$)$.A(\theta 2) =$ $\begin{bmatrix} cos\theta 1 & sin\theta 1 \\ -sin\theta 1 & cos\theta 1 \end{bmatrix}X\begin{bmatrix} cos\theta 2 & sin\theta 2 \\ -sin\theta 2 & cos\theta 2 \end{bmatrix}$ = $\begin{bmatrix} \cos(\theta 1+\theta 2) & \sin(\theta 1+\theta 2) \\ -\sin(\theta 1+\theta 2) & \cos(\theta 1+\theta 2) \end{bmatrix}$ = A($\theta 1 + \theta 2$), Second statement is correct . A(2θ) $= A(\theta).A(\theta)$ = $(A(\theta))^2$. So $\left[A(\frac{\pi}{3})\right]^3$ = A$\left[3 \times \frac{\pi}{3}\right]$= A($\pi$)= $\begin{bmatrix} cos\pi & sin\pi \\ -sin\pi & cos\pi \end{bmatrix}$=$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$= -I, So Second statement is correct and it is the correct explanation of A Option (a)
8.	If A =$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq O,$ B=$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \neq O,$ But AB = O First statement is not correct. A+B = A+C, -A+(A+B) = -A+ (A+C)= (-A+A) +B = (-A+A) +C = O+B=O+C$\Rightarrow B = C$. Second statement is correct . Option (d)
9.	For a skew- symmetric matrix, $a_{ij} =-a_{ji}$, for all i and j, so x = 2. First statement is correct. Second statement is correct and it is not the correct explanation of A Option (b)
10.	First statement is correct. Second statement is not correct Option (c)

11.	First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
12.	AB= $\begin{bmatrix} -8 \\ 16 \end{bmatrix}$, $(AB)^T$ = $[-8 \quad 16]$ First statement is correct. Second statement is not correct Option (c)
13.	A^T= A andA^T= -A ⇒ A = -A , 2A = O , A=O , $A\ is\ a\ zero\ matrix.$ First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
14.	A is m $\times$ $n.A^T$is n $\times$ $m.$ $BA^T$is defined , then B must have n columns. A^TB is defined , so then B must have m rows. First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
15.	$(AB - BA)^T$ = -(AB-BA), : So AB –BA is skew- symmetric Also, $(AB + BA)^t$ = AB +BA , So AB +BA is symmetric. First statement is correct. Second statement is correct and it is not the correct explanation of A Option (b)
16.	Because in a skew symmetric matrix a_{ii} =o ,for all I, Therefore 2k-2=0 ⇒ k=1 Option (a)
17.	First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
18.	Because matrices of the same order only can be compared Option (c)

19.	First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
20.	Clearly $A^1 = -A$, therefore A is skew symmetric ,A is true and R is false Option (c)
21.	Because , A $= \frac{A+A^1}{2} + \frac{A-A^1}{2}$ Option (a)
22.	First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
23.	First statement is correct. Second statement is correct and it is the correct explanation of A Option (a)
24.	Here IAI = $-5 \neq 0$, Therefore A^{-1}exists.So, A is false but R is true Option (d)
25.	IAI = -8 .Therefore R is true. Also IAI = $-8 \neq 0$. Therefore A is invertible so A is true. But IAI = -8 is not the reason for A to be invertible .The reason is that IAI = $-8 \neq 0$. Option (b)

CASE BASED QUESTIONS

1.	Three students Ram, Mohan and Ankit go to a shop to buy stationary. Ram purchases 2 dozen note books, 1 dozen pens and 4 pencils. Mohan purchases 1 dozen note book , 6 pens and 8 pencils. Ankit purchases 6 note books , 4 pens and 6 pencils. A note book costs ₹ 15, a pen costs ₹4.50 and a pencil costs ₹ 1.50. Let A and B be the matrices representing the number of items purchased by the three students and the prices of items respectively. Based on the above information answer the following questions.

	(i) What is the order of the matrix B representing the prices of the items (ii) What is the order of the matrix A representing items purchased by the three students (iii) What is the order of the matrix AB. **OR** Find the total amount of bill by all the three students.
2.	Three schools DPS, CVC and NVS decided to organize a fair for collecting money for helping the food victims. They sold handmade fans , mats and plates from recycled material at a cost of ₹ 25, ₹ 100, and ₹50 each respectively. The number of articles sold are given below: <table><tr><td>School/Article</td><td>DPS</td><td>CVC</td><td>NVS</td></tr><tr><td>Handmade fans</td><td>40</td><td>25</td><td>35</td></tr><tr><td>Mats</td><td>50</td><td>40</td><td>50</td></tr><tr><td>Plates</td><td>20</td><td>30</td><td>40</td></tr></table>Based on the above information answer the following questions. (i) What is the total money collected by all the three schools. (ii) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all the three schools.
3.	On her birthday ,Seema decided to donate some money to children of an orphanage home. If there were 8 children less , everyone would have got ₹10 more. However if there were 16 children more , everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be ₹ y. (i) Write the matrix equation to represent the information given above. (ii) Find the number of children who were given some money by Seema. (iii) How much amount is given to each child by Seema. **OR** How much amount Seema spends in distributing the money to all the children of Orphanage.

4.	Gautam buys 5 pens , 3 bags and 1 Instrument box and pays a sum of ₹160 . From the same shop Vikram buys 2 pens , 1 bag and 3 Instrument boxes and pays a sum of ₹190. Ankur buys 1 pen , 2 bags and 4 Instrument boxes and pays a sum of ₹ 250 Based on the above information answer the following questions. (i) Write the matrix equation to represent the information given above. (ii) Find P = A^2-5A
5.	Two farmers Ramakrishnan and Charan Singh cultivate only three varieties of rice namely Basmati, Permal, and Naura. The sale in Rs of these varieties of rice by both the farmers in the month of September and October are given in the following matrices A and B. A= $\begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix}$ B = $\begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$ (i) How the total sales in of September and October for each farmer in each variety can be represented in form of matrix. (ii) How the decrease in sales from September to October can be represented in form of matrix. (iii) If Ramakrishnanreceives 2% profit on gross sales, compute his profit for each variety sold in October. **OR** If Charansingh receives 2% profit on gross sales, compute his profit for each variety sold in September
6.	To promote the making of toilets for women, an organisation tried to generate awareness through (i)house calls (ii) emails and (iii) announcements . The cost for each mode per attempt is given below (i) ₹ 50 (ii) ₹ 20 (iii) ₹40 The number of attempts made in the villages X,Y and Z are given below (i) (ii) (iii) X 400 300 100 Y 300 250 75 Z 500 400 150

	Also the chance of making of toilets corresponding to one attempts of given modes is (i) 2% (ii) 4% (iii) 20% Based on the above information answer the following questions : (i) What is the cost incurred by the organization on village X (ii) What is the cost incurred by the village Y (iii) What is the cost incurred by the village Z **OR** What is the total number of toilets that can be expected after the promotion in village X
7.	A trust fund has ₹ 35000 that must be invested in two different types of bonds say X and Y .The first bond pays 10% interest per annum ,which will be given to an old age home and second one pays 8% interest per annum which will be given to women welfare association .Let A be a 1 x 2 and B be a 2 x 1 matrix representing the investment and interest rate on each bond respectively. Based on the above information answer the following questions (i) What is the total amount of interest received on both bonds if ₹15000 is invested in bond X . (ii) What is the amount of investment in bond Y if the amount of interest given to old age home is ₹500.
8.	A manufacturer produces three stationary products Pencil, Eraser, and Sharpener which he sells in two markets. Annual sales are indicated below: Products in numbers given below in the table

Market	Pencil	Eraser	Sharpener
A	10,000	2,000	18,000
B	6,000	20,000	8,000

	If the unit sale price of Pencil, Eraser, and Sharpener are₹2.50, ₹1.50 and₹1.00 respectively . (i) Write the matrix equation to represent the information given above. (ii) Find the total revenue of market A and B
9.	Amit, Biraj and Chirag were given the task of creating square matrix of order 2. Below are the matrices created by them namely A , B and C respectively. A = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, C = $\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ (i) Find A +B+C (ii) Evaluate $(A^T)^T$ (iii) Find AC – BC **OR** Find the matrix (a+b) B , a = 4, b = -2
10.	A manufacturer produces three stationary products Pencil, Eraser, and Sharpener which he sells in two markets. Annual sales are indicated below: Products in numbers given below in the table <table><tr><td>Market</td><td>Pencil</td><td>Eraser</td><td>Sharpener</td></tr><tr><td>A</td><td>10,000</td><td>2,000</td><td>18,000</td></tr><tr><td>B</td><td>6,000</td><td>20,000</td><td>8,000</td></tr></table> If the unit cost of the above three commodities are ₹2.00, ₹1.00 and ₹0.50 respectively and If the unit sale price of Pencil, Eraser, and Sharpener are₹2.50, ₹1.50 and₹1.00 respectively . (i) Find the cost incurred in Market A and B. (ii) Find the profit in Market A and B.

<u>**ANSWERS**</u>

1.	B = $\begin{bmatrix}15\\4.5\\1.3\end{bmatrix}$, A = $\begin{bmatrix}24 & 12 & 4\\12 & 6 & 8\\6 & 4 & 6\end{bmatrix}$ (i) 3x1 (ii) 3x3 (iii) 3x1 or ₹ 756.
2.	(i) M = $[25 \quad 100 \quad 50]$, A = $\begin{bmatrix}40 & 25 & 35\\50 & 40 & 50\\20 & 30 & 40\end{bmatrix}$, MA= $[7000 \quad 6125 \quad 7875]$ Total money collected by all the three schools.= ₹ 21000 (ii)B = $\begin{bmatrix}20 & 30 & 40\\50 & 40 & 50\\40 & 25 & 35\end{bmatrix}$, MB = $[7500 \quad 6000 \quad 7750]$ Total money collected by all the three schools.=₹ 21250.
3.	(x-8) (y+10) = xy,→ 5x-4y = 40 (x+16)(y-10) = xy), → 5x-8y = -80 (i)$\begin{bmatrix}5 & -4\\5 & -8\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$ = $\begin{bmatrix}40\\-80\end{bmatrix}$ (ii) 32 (iii) Rs. 30 Or ₹ 960
4.	(i) $\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}160\\190\\250\end{bmatrix}$ (ii) $A^2 - 5A = \begin{bmatrix}32 & 20 & 18\\15 & 13 & 17\\13 & 13 & 23\end{bmatrix} - \begin{bmatrix}25 & 15 & 5\\10 & 5 & 15\\5 & 10 & 20\end{bmatrix}$ = $\begin{bmatrix}7 & 5 & 13\\5 & 8 & 2\\8 & 3 & 3\end{bmatrix}$
5.	(i) A +B = $\begin{bmatrix}15000 & 30000 & 36000\\70000 & 40000 & 20000\end{bmatrix}$ (ii) A-B =$\begin{bmatrix}5000 & 10000 & 24000\\30000 & 20000 & 0\end{bmatrix}$ (iii) In October Profit of ₹100, ₹200, ₹120 received by Ramakrishnan in the sale of each variety of rice.
6.	Let ₹A , ₹ B and ₹ C be the cost incurred by the organisations for villages X , Y ,Z respectively. Then A,B,C will be given by the matrix equation $\begin{bmatrix}400 & 300 & 100\\300 & 250 & 75\\500 & 400 & 150\end{bmatrix}\begin{bmatrix}50\\20\\40\end{bmatrix} = \begin{bmatrix}A\\B\\C\end{bmatrix}$

	Hence $\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$ (i) 30000 (ii) 23000 (iii) 39000 **OR** The total number of toilets that can be expected in each village is given by the following matrix equation $$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$ Therefore X = 40 , Y= 31 Z =56 Therefore X = 40
7.	If ₹ 15000 is invested in bond X then amount invested in bond Y = 35000-15000 = 20000 Let A = investment = [15000 20000] and B = $\begin{bmatrix} 10\% \\ 8\% \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$ The amount of interest received on both bonds is given by AB = [15000 20000]$\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$ = [3100] Therefor(i) Total amount = ₹3100 Let ₹x is invested in bond X then we have x x 10/100 = 500 Therefore X = ₹5000 Therefore amount invested in bond Y = 35000-5000 =30000 Ans: (ii) ₹30000
8.	(i)$\begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$ (Ii)Revenue of Market A = $\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$= ₹46000

	Revenue of Market B = $[6000 \quad 20000 \quad 8000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$= ₹53000 Total revenue of market A and B = ₹ 99000
9.	(i) A+B+C = $\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix} + \begin{bmatrix}4 & 0\\1 & 5\end{bmatrix} + \begin{bmatrix}2 & 0\\1 & -2\end{bmatrix} = \begin{bmatrix}7 & 2\\1 & 6\end{bmatrix}$ (ii) $A^T = \begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$, $(A^T)^T = \begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$ = A (iii) AC – BC $= \begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & -2\end{bmatrix} - \begin{bmatrix}4 & 0\\1 & 5\end{bmatrix}\begin{bmatrix}2 & 0\\1 & -2\end{bmatrix}$ $= \begin{bmatrix}4 & -4\\1 & -6\end{bmatrix} - \begin{bmatrix}8 & 0\\7 & -10\end{bmatrix} = \begin{bmatrix}-4 & -4\\-6 & 4\end{bmatrix}$ **OR** (a+b) B = (4-2)$\begin{bmatrix}4 & 0\\1 & 5\end{bmatrix} = 2\begin{bmatrix}4 & 0\\1 & 5\end{bmatrix} = \begin{bmatrix}8 & 0\\2 & 10\end{bmatrix}$
10.	(i) Cost incurred in Market A = $[10000 \quad 2000 \quad 18000]\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$ = ₹31000 Cost incurred in Market B =$[6000 \quad 20000 \quad 8000]\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$ =₹36000 (ii)Revenue of Market A = $[10000 \quad 2000 \quad 18000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$= ₹46000 Revenue of Market B = $[6000 \quad 20000 \quad 8000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$= ₹53000 Profit in market A =₹46000 - ₹31000= ₹15000 Profit in market B =₹53000-₹36000=₹17000

CHAPTER : DETERMINANTS

ASSERTION-REASON QUESTIONS

	In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true but (R) is not the correct explanation of (A). (c) (A) is true but (R) is false. (d) (A) is false but (R) is true
1	**Assertion (A):**If A= $\begin{bmatrix} \alpha & 3 \\ 3 & \alpha \end{bmatrix}$ and $\|A\|^3$= -125, then α = ±2 **Reason (R) :**Determinant of a square matrix A= $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given as $a_{22} - a_{12}a_{21}$
2	**Assertion (A):** If A is a square matrix of order 3x3, then |3A|= 27|A| **Reason (R)** : If A is a square matrix of order n, then |kA| = k^n|A|
3	**Assertion (A):** The minor of the element of second row and third column in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$ is 13 **Reason (R) :** The positive value of x, which makes the given pair of determinants $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$ equal is 4.
4	**Assertion (A):** If A= $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ then A^{-1} = $\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ **Reason (R):** Determinant is a number associated to a matrix.
5	**Assertion (A):** A square matrix A has inverse, if and only if A is singular. **Reason (R):** Let A be a square matrix of order 2x2, then the value of |kA| is equal to k^2|A|.
6	**Assertion (A):** If A is an invertible matrix of order 3x3, then $\|A^{-1}\|$ =|A|. **Reason (R) :** If A is a 3x3 invertible matrix such that $\|A^{-1}\|$= $\|A\|^k$, then the value of k is -1

7	**Assertion (A):** If A = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$, then |A| = 105 **Reason (R)** : The determinant of a diagonal matrix is equal to the product of the diagonal elements
8	**Assertion (A):** Determinant of a skew symmetric matrix of order 3 is zero. **Reason (R)** : For any matrix A, |A|= |A^T | and |-A| = -|A|
9	**Assertion (A):**If A and B are square matrices of the same order 3 such that 2AB=I and |B| = $\frac{1}{24}$, then |A| =3 **Reason (R) :**If A and B are square matrices of the same order n and k is a scalar, then |kA| =$(k)^n$|A| and |AB|= |A| |B|
10	**Assertion (A):** The matrix A= $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ -1 & 2 & x \end{bmatrix}$ is singular for x=5 **Reason (R):** A square matrix A is singular if |A|=0
11	**Assertion (A**):If A is a square matrix of order3 such that |A|=5,then |adj A|= 25 **Reason (R):** If A is a non-singular matrix of order n, then |adj A|= $\|A\|^{n-1}$
12	**Assertion (A**):If A is a square matrix of order 3 such that |A|=4, then |adj(adjA)|=4^4 **Reason (R):** If A is a non-singular square matrix of order n, then |adj(adjA)|= $\|A\|^{(n-1)^2}$
13	**Assertion**: If A is invertible matrix, then A^T is invertible **Reason**: Inverse of invertible symmetric matrix is invertible symmetric matrix
14	**Assertion**: If A and B are square matrices of order 2 such that 3AB = I and det(A) = 9, then det(B) = $\frac{1}{81}$

	Reason: If A and B are square matrices of order n and k is a scalar, then $\det(kA) = k^n\det(A)$ and $\det(AB) = \det(A).\det(B)$
15	**Assertion:** If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ $then\ x = \pm 6$ **Reason:** If A is a symmetric matrix, then $A^T = A$
16	**Assertion:** If A is a square matrix of order 3, det $A^T = -5$, then det A = -5 **Reason**: det A = det A^T
17	**Assertion:** If A is a square matrix such that A(adjA) = 4I, then det A = 2 **Reason**: A(adjA) =$\lvert A \rvert$I
18	**Assertion:** If A is a 3x3 non-singular square matrix, then $\lvert A^{-1}adjA\rvert = \lvert A\rvert$ **Reason**: If A and b are invertible matrices such that B is inverse of A, then AB = BA = I
19	Let A and B are two square matrices of order 2 **Assertion**: A (adjA) = (detA) I **Reason**: adj(AB) = (adjA)(adjB)
20	**Assertion:** Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are distinct prime numbers less than 20, then maximum value of det A is 317 **Reason**: Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19
21	**Assertion:** The system of linear equations 5x + ky = 5 and 3x + 3y = 5 has a unique solution for $k \neq 5$ **Reason**: The system of linear equations 5x + ky = 5 , 3x + 3y = 5 is inconsistent for k = 5
22	**Assertion:** If A is a non singular matrix of order 2x2 such that $A^2 - 5A + 7I = O$, then $A^{-1} = 1/7(5I - A)$ **Reason:** For any two matrices A and B, $(AB)^{-1} = A^{-1}B^{-1}$

23	**Assertion:** If A is square matrix of order 2 such that (detA)A^{-1} = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then adjA = $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ **Reason:** $A^{-1} = \frac{1}{\lvert A \rvert}\, adjA$
24	**Assertion:** Range of the function f(x) = $\begin{vmatrix} cosx & -2sinx \\ 3sinx & cosx \end{vmatrix}$ is [1, 7] **Reason**: -1≤ sinx ≤ 1
25	**Assertion:** If A = $\begin{bmatrix} 2 & 1+2i \\ 1-2i & 2 \end{bmatrix}$then det A is real a number **Reason:** If A = $(a_{ij})_{2x2}$, where a_{ij} being complex numbers, then det A is always real.

SOLUTIONS

1	**Answer: a** $\lvert A \rvert = \begin{vmatrix} \alpha & 3 \\ 3 & \alpha \end{vmatrix} = \alpha^2 - 9$ $\lvert A \rvert^3 = -125$ $(\alpha^2 - 9)^3 = -125 = (-5)^3$ $(\alpha^2 - 9) = -5$ $\alpha^2 = 4$ $\alpha = \pm 2$ Therefore assertion (A) is true Reason (R) is clearly true which is the definition of determinant and is the Correct explanation of (A). Therefore ,Both (A) and (R) are true and (R) is the correct explanation of (A). Hence Option (a) is the corerct answer.
2	**Answer: a** $\lvert 3A \rvert = 3^3 \lvert A \rvert$ $= 27 \lvert A \rvert$ Therefore assertion (A) is true Reason (R) is clearly true which is a standard result of determinant and is the

	correct explanation of (A). Therefore, Both (A) and (R) are true and (R) is the correct explanation of (A). Hence Option (a) is the correct answer.
3	**Answer: b** Minor of a_{23} = $\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix}$ = 13 Therefore Assertion (A) is true For reason , $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$ = $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$ $2x^2$-15 =32-15 x^2 = 16 $x = \pm 4$ x= 4 Therefore Reason (R) is true, but not the corret explanation. So, Both (A) and (R) are true but (R) is not the correct explanation of (A). Hence Option (b) is the correct answer.
4	**Answer: c** $A^{-1} = \frac{1}{\|A\|}$ adj A $= \frac{1}{1}\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ $= \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ Therefore assertion (A) is true. Also as determinant is associated to a square matrix, Reason is wrong. Therefore, (A) is true but (R) is false Hence Option (c) is the correct answer
5	**Answer: d** A square matrix A has inverse, if and only if A is non- singular ie. $\|A\| \neq 0$ So Assertion is false. For reason , $\|kA\| = k^n\|A\|$ $= k^2\|A\|$ So reason is true.

	Therefore (A) is false but (R) is true. Hence Option (d) is the correct answer.
6	**Answer: d** $\|A^{-1}\| = \frac{1}{\|A\|}$, So assertion is false For reason $\|A^{-1}\| = \|A\|^k$ $\frac{1}{\|A\|} = \|A\|^k$ $\|A\|^{k+1} = 1$ $k + 1 = 0$ K= -1 So reason is true. Therefore (A) is false but (R) is true. Hence Option (d) is the correct answer
7	**Answer: a** |A| = 3x35= 105.So assertion is true. Reason statement is also true. Therefore Both (A) and (R) are true and (R) is the correct explanation of (A). Hence Option (a) is the correct answer
8	**Answer: c** Let A be a skew symmetric matrix of order 3, $A^T = -A$ $\|A^T\| = \|-A\|$ $\|A\| = (-1)^3\|A\|$ $\|A\| = -\|A\|$ $2\|A\| = 0$ $\|A\| = 0$ So assertion is true. For reason, $\|A\| = \|A^T\|$ is true but $\|-A\| = (-1)^n\|A\|$,so reason is false. Therefore (A) is true but (R) is false. Hence Option (c) is the correct answer
9	**Answer: a**

	Reason are standard properties of determinants, so reason is true For Assertion, 2AB=I $\lvert 2AB\rvert=\lvert I\rvert$ $(2)^3\lvert AB\rvert=1$ $8\lvert A\rvert\lvert B\rvert=1$ $8\lvert A\rvert\frac{1}{24}=1$ $\lvert A\rvert=3$ So assertion is true. Therefore Both (A) and (R) are true and (R) is the correct explanation of (A). Hence Option (a) is the correct answer
10	**Answer: d** Reason is the definition of singular matrix and hence true. For assertion $\lvert A\rvert=0$ 1(x-4)-2(2)+0=0 x-8 = 0 => x= 8. Thus matrix A is singular for x=8. Therefore (A) is false but (R) is true. Hence Option (d) is the correct answer
11	**Answer: d** Reason is true which is a standard result. For assertion , by using the reason $\lvert adj\ A\rvert= \lvert A\rvert^{n-1}= 5^{3-1}$ =25.So assertion is true. Therefore Both (A) and (R) are true and (R) is the correct explanation of (A). Hence Option (a) is the correct answer.
12	**Answer: a** For reason, $\lvert adj(adjA)\rvert= \lvert adjA\rvert^{n-1}= \lvert A\rvert^{(n-1)^2}$.So reason is true. For assertion $\lvert adj(adjA)\rvert= \lvert A\rvert^{(n-1)^2}= 4^{(3-1)^2} =4^4$.So assertion is true. Therefore Both (A) and (R) are true and (R) is the correct explanation of (A). Hence Option (a) is the correct answer

13	**Answer: b** since A is invertible $\|A\| \neq 0 \quad => \|A^T\| \neq 0 \; => A^T \; is \; invertible$ So A is true Let P be invertible symmetric matrix, then $\|P\| \neq 0 \; and \; P^T = P$ $(P^{-1})^T = (P^T)^{-1} = P^{-1}$ Hence P^{-1} is also symmetric matrix => R is true But R does not explains A Hence b is correct
14	**Answer: a** The results given in Reason are standard properties of determinants and hence R is true. Using these results we get $\|2AB\| = \|I\|$ $3^2\|A\|\|B\| = 1$ => 9.9.$\|B\|$ = 1 =>$\|B\|$ = 1/81 Hence A is true Option a correct answer
15	**Answer: b** On expanding determinants we get $x^2 - 36 = 36 - 36$ => $x = \pm 6$ So A is true R is property of a symmetric matrix , hence true But R does not explains A Option b is correct answer
16	**Answer: a** Result given in reason is a standard property of determinants hence true Using this formula we can see clearly Assertion is true Hence option a is correct answer
17	**Answer: c** Formula in reason is correct, using this we get detA = 4 Hence A is false and R is true Option c is correct

18	**Answer: b** $$\lvert A^{-1}adjA\rvert = \lvert A^{-1}\rvert\lvert adjA\rvert (since, detAB = detA.detB)$$ $$= \lvert A\rvert^{-1}\lvert A\rvert^{2}(\lvert adjA\rvert = \lvert A\rvert^{n-1}) \;\; and \;\; detA^{-1} = (detA)^{-1}$$ $$= \lvert A\rvert$$ Hence A is true R is definition of inverse of a matrix, so true But R does not explains A Option b is correct answer
19	**Answer: c** For square matrix A A (adjA) = (detA) I Hence A is true But adj(AB) = (adjB)(adjA) (property) Hence R is false Option c is correct answer
20	**Answer: a** $\lvert A\rvert = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ is maximum if ad is max. and bc is min. ad = 17x19 = 323 and bc = 2x3 = 6 Therefore max. value of detA is 323-6 = 317 Option a is correct answer
21	**Answer: b** For unique solution det. of coefficient matrix ≠ 0 $\begin{vmatrix} 5 & k \\ 3 & 3 \end{vmatrix} \neq 0$, we get k ≠ 5 A is true For k = 5 the given lines are parallel hence no solution, therefore inconsistent R is true But R is not correct explanation for A Option b is correct answer
22	**Answer: c** $A^{-1}(A^2 - 5A + 7I) = O$ implies $A - 5I + 7A^{-1} = O$ implies $A^{-1} = 1/7(5I - A)$ A is true Clearly reason is false Option c is correct answer

23	**Answer:d** Formula given in reason is true. Using this result we get adjA = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ A is false and R is true Option d is correct answer
24	**Answer: d** $f(x) = \cos^2 x + 6 \sin^2 x = 1 + 5 \sin^2 x$ Range of f(x) is $[1 + 5(0) , 1 + 5(1)] = [1, 6]$ hence A is false R is true Option d is correct
25	**Answer:c** det A = $2x2 – (1 – 2i)(1 + 2i) = 5$ which is real, so A is true If elements of a matrix are complex numbers then det A may be real or complex. So R is false Option c is correct answer

CASE BASED QUESTIONS

1 Three shopkeepers Ravi ,Raju and Rohit are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepersRavi ,Raju and Rohit are using (20,30,40), (30,40,20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Ravi ,Raju and Rohitspent Rs.250, Rs.270 and Rs.200 on these carry bags respectively

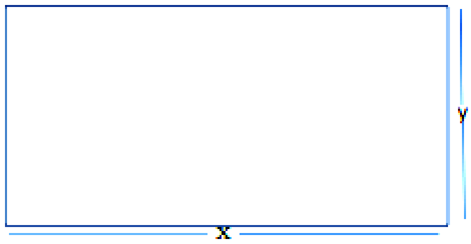


(I) Represent the above situation algebraically & write the system in the form of matrices.

(II) Find the cost of one polythene bag

	(III) Find the cost of one handmade bag and one newspaper envelope bag
2	A trust invested some money in two types of bonds . The first bond pays 10% interest and second bond pays 12% interest. The trust received rupees 2800 as interest.However , if trust hadinterchanged money in bonds, they would have got rupees 100 less as interest.Based on the information answer the following. 1. Represent the above situation algebraically & write the system in the form of matrices. 2. Find the amount invested in first bond. 3. Find the amount invested in second bond.
3	A shopkeeper has 3 varieties of pens A,B,C. Meenu purchased 1 pen of each variety for a total of rupees 21.Jeeva purchased 4 pens of A variety, 3 pens of B variety and 2 pens of C variety for rupees 60.While Shikha purchased 6 pens of A variety,2pens of B variety and 3 pens of C variety for rupees 70.  Based on the above information answer the following questions. 1. Represent the above situation algebraically & write the system in the form of matrices. 2. Find the cost of pen of variety A 3. Find the cost of pen of variety B OR Find the cost of pen of variety C
4	The management committee of a residential colony decided to award some of its members for honesty, some for helping others and some others for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12.Three times the sum of awardees for co-operation and supervision added to two times the number of awardees for honesty is 33.If the sum of the number of awardees for honesty and supervision is twice

	the number of awardees for helping others, Based on the above information answer the following  1. Find the number of awards for honesty 2. Find the number of awards for helping others 3. Find the number of awards for supervising the workers
5	On her birthday Reena decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got rupees 10 more. However if there were 16 children more everyone would have got rupees 10 less. Based on this information answer the following questions. 1. Represent the above situation algebraically & write the system in the form of matrices. 2. Find the number of children 3. Find the amount donated by Reena to each child
6	Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of Rs.160. From the same shop Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of Rs. 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs. 250 Based on the above information answer the following questions i) Express the above given situation into a matrix equation of the form AX = B ii) Find det A iii) a) Find A^{-1} **OR** b) Determine $P = A^2 - 5A$
7	Raman wants to donate a rectangular plot of land for school in her village. When she was asked to give the dimensions of the plot, she told that if its

	length is decreased by 50m and breadth is increased by 50m, then its area does not alter, but if length is deceased by 10m and breadth is decreased by 20m, then area will decrease by 5300m^2  Based on the above information, answer the following questions i) If the length and breadth of the plot are x and y respectively, then find the system of linear equations in x and y ii) Find the length and breadth of the plot iii) Find the area of the rectangular plot
8	The daily income of Sita and Savita are in the ratio 1:2 and their expenditures are in the ratio 2:1. Their savings are Rs.500 and Rs. 2500 respectively Based on the above information answer the following questions i) If their incomes are x and 2x; and their expenditures are 2y and y respectively, then write the linear equations for the above situation ii) Find the income of Sita iii) Find the expenditure of Savita **OR** Find their total income
9	The upward speed v(t) of a rocket at time t is given by V(t) = at^2 + bt + c, 0 $\leq$ t $\leq$ 100, where a. b. c are constants. It has been found that the speed at t = 3sec, t = 6 sec and t = 9sec are respectively 64, 133 and 208 miles per second If $\begin{bmatrix}9&3&1\\36&6&1\\81&9&1\end{bmatrix}^{-1} = \frac{1}{18}\begin{bmatrix}1&-2&1\\-15&24&-9\\54&-54&18\end{bmatrix}$ Answer the following questions

	i) Find the value of b + c ii) Find the speed V(t) in terms of t
10	A diet is to contain 30 units of vitamin A, 40 units of vitamin B and 20 units of vitamin C. Three types of foods F_1, F_2 and F_3 are available. One unit of F1 contains 3 units of vitamin A, 2 units vitamin B and 1 unit of vitamin C. One unit of F2 contains 1 unit of vitamin A, 2 units of vitamin B and 1 unit of vitamin C. One unit of F3 contains 5 units of vitamin A, 3 units of vitamin B and 2 units of vitamin C. Based on the above information answer the following questions i) If the diet contains x units food F1, y units food F2 and z units of food F3, then write the above information in terms of x, y and z ii) Find the values of x, y and z

SOLUTIONS

1	(1) Let the cost of one polythene bag be x, one handmade bag be y and one newspaper envelope bag be z 20x+30y+40z=250 30x+40y+20z=270 40x+20y+30z=200 $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 25 \\ 27 \\ 20 \end{bmatrix}$ AX =B X = A^{-1}B = ($\frac{1}{\|A\|}$ adjA) B $= \frac{1}{-27} \begin{bmatrix} 8 & -1 & -10 \\ -1 & -10 & 8 \\ -10 & 8 & -1 \end{bmatrix} \begin{bmatrix} 25 \\ 27 \\ 20 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ x=1, y=5,z=2 (2) x=1 (3) y=5,z=2
2	(1) Let x and y be the money invested in first and second bond respectively $X\left(\frac{10}{100}\right) + y\left(\frac{12}{100}\right) = 2800$

	X $\left(\frac{12}{100}\right)$ + y $\left(\frac{10}{100}\right)$ = 2700 ⇨ 10x + 12y = 280000 12x + 10y = 270000 $\begin{bmatrix}10 & 12\\12 & 10\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}280000\\270000\end{bmatrix}$ AX = B X = A^{-1}B = $\left(\frac{1}{\|A\|}\right.$ adjA) B = $\frac{1}{-44}\begin{bmatrix}10 & -12\\-12 & 10\end{bmatrix}\begin{bmatrix}280000\\270000\end{bmatrix} = \frac{1}{-44}\begin{bmatrix}-440000\\-660000\end{bmatrix}$ = $\begin{bmatrix}10000\\15000\end{bmatrix}$ (2) x= 10000 (3) y = 15000
3	(1) Let the cost of a pen of variety A be x, B be y and C be z x+y+z = 21 4x+3y+2z =60 6x+2y+3z=70 $\begin{bmatrix}1 & 1 & 1\\4 & 3 & 2\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}21\\60\\70\end{bmatrix}$ AX =B X = A^{-1}B = $\left(\frac{1}{\|A\|}\right.$ adjA) B = $\frac{1}{-5}\begin{bmatrix}5 & -1 & -1\\0 & -3 & 2\\-10 & 4 & -1\end{bmatrix}\begin{bmatrix}21\\60\\70\end{bmatrix} = \begin{bmatrix}5\\8\\8\end{bmatrix}$ (2) x=5 (3)y=8 OR z=8
4	Let the number of awardees for honesty be x, number of awardees forhelping others be y and number of awardees fornumber of awardees forsupervising the workers be z x +y + z =12 3(y + z)+2x =33 => 2x+3y+3z =33

	x+ z=2y => x-2y+z=0 $\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 3\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}12\\ 33\\ 0\end{bmatrix}$ AX =B X = A^{-1}B = ($\frac{1}{\|A\|}$ adjA) B = $\frac{1}{3}\begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ -7 & 3 & 1\end{bmatrix}\begin{bmatrix}21\\ 60\\ 70\end{bmatrix} = \begin{bmatrix}12\\ 33\\ 0\end{bmatrix} = \begin{bmatrix}3\\ 4\\ 5\end{bmatrix}$ 1. x=3, 2. y=4, 3. z=5
5	Let the number of children be x and the amount donated by Reena to each child be y. (x-8) (y+10) =xy => xy+10x-8y-80=xy (x+16) (y-10) = xy => xy-10x+16y-160=xy ⇨ 5x-4y= 40 ………………….(I) ⇨ 5x-8y= -80 …………………(ii) $\begin{bmatrix}5 & -4\\ 5 & -8\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}40\\ -80\end{bmatrix}$ AX = B X = A^{-1}B = ($\frac{1}{\|A\|}$ adjA) B = $\frac{1}{-20}\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}\begin{bmatrix}40\\ -80\end{bmatrix} = \frac{1}{-20}\begin{bmatrix}-640\\ -600\end{bmatrix}$= > 2. x = 32 3. y =30
6	Let the cost of 1 pen, 1 bag and 1 instrument box are Rs.x, y and z respectively From the question 5x + 3y + z = 160, 2x + y + 3z = 190 and x + 2y + 4z = 250

	i) $\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}160\\190\\250\end{bmatrix}$ where A = $\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}$, X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$, B = $\begin{bmatrix}160\\190\\250\end{bmatrix}$ ii) $\|A\| = \begin{vmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{vmatrix}$ = -22 iii) a) adjA = $\begin{bmatrix}-2 & -10 & 8\\-5 & 19 & -13\\3 & -7 & -1\end{bmatrix}$ and $A^{-1} = \frac{1}{\|A\|}\, adjA = \frac{1}{-22}\begin{bmatrix}-2 & -10 & 8\\-5 & 19 & -13\\3 & -7 & -1\end{bmatrix}$ b) P = $A^2 - 5A = \begin{bmatrix}32 & 20 & 18\\15 & 13 & 17\\13 & 13 & 23\end{bmatrix} - 5\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix} = \begin{bmatrix}7 & 5 & 13\\5 & 8 & 2\\8 & 3 & 3\end{bmatrix}$
7	i) area of the plot = xy Given that (x - 50)(y + 50) = xy => x – y = 50 and (x - 10)(y - 20) = xy – 5300 => 2x + y = 550 The matrix equation of the above system is $\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}50\\550\end{bmatrix}$ AX = B implies X = A^{-1}B => X = $\frac{1}{3}\begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}\begin{bmatrix}50\\550\end{bmatrix} = \begin{bmatrix}200\\150\end{bmatrix}$ x = 200 and y = 150 ii) length is x = 200m , breadth y = 150m iii) area xy = 200x150 = 30000m^2
8	i) Income – expenditure = savings => we get x – 2y = 500, 2x – y = 2500 ii) AX = B => $\begin{bmatrix}1 & -2\\2 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}500\\2500\end{bmatrix}$ X = A^{-1}B = $\frac{1}{3}\begin{bmatrix}-1 & 2\\-2 & 1\end{bmatrix}\begin{bmatrix}500\\2500\end{bmatrix} = \begin{bmatrix}1500\\500\end{bmatrix}$ x = 1500 and y = 500 Income of Sita is x = Rs. 1500

	iii)Expenditure of Savita is y = Rs. 500 (OR) their total income is x + 2x = 3x = 3x 1500 = Rs.450
9	Given v(3) =64, v(6) = 133 and v(9) = 208 We get 9a + 3b + c = 64, 36a + 6b + c = 133 and 81a + 9b + c = 208 The matrix equation is $\begin{bmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 64 \\ 133 \\ 208 \end{bmatrix}$ AX = B implies X = A^{-1} B => X = $\frac{1}{18}\begin{bmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{bmatrix}\begin{bmatrix} 64 \\ 133 \\ 208 \end{bmatrix}$ X = $\frac{1}{18}\begin{bmatrix} 6 \\ 360 \\ 18 \end{bmatrix}$ => a = 1/3, b = 20, c = 1 i) b + c = 21 ii) v(t) = (1/3)t^2 + 20t + 1
10	i) As per the given information 3x + y + 5z = 30, 2x + 2y + 3z = 40 and x + y + 2z = 20 ii) On solving these equations by matrix method we get x = 5, y + 15 and z = 0

CHAPTER : CONTINUITY AND DIFFERENTIABILITY

ASSERTION AND REASONING QUESTIONS

	Two statements are given below - one labeled Assertion (A) and the other labeled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R). (a) Both (A) and (R) are true and (R) is the correct explanation for (A). (b) Both (A) and (R) are true but (R) is not the correct explanation for (A). (c) (A) is true but (R) is false. (d) (A) is false but (R) is true.
1	**Assertion (A)**: f(x) = 2x + 5 is continuous on the set of all real numbers. **Reason (R)**: f(x) is a polynomial function
2	**Assertion (A)**: f(x) = | x – 3 | is continuous at x = 3 **Reason (R)**: f(x) = | x – 3 | is differentiable at x = 3
3	**Assertion (A)**: f(x) = [x], where [x] is the greatest integer less than or equal to x is continuous at x = 3. **Reason (R)**: f(x) = [x], where [x] is the greatest integer less than or equal to x is not differentiable at x = 3.
4	Consider the function $f(x) = \begin{cases} \frac{x^2-5x+6}{x-3}, & for\ x \neq 3 \\ k, & for\ x = 3 \end{cases}$ is continuous at x = 3 **Assertion (A)**: The value of k is 4 **Reason (R)**: If f(x) is continuous at a point a then $\lim_{x \to a} f(x) = f(a)$
5	Consider the function $f(x) = \begin{cases} \frac{k\|x-3\|}{x-3}, if\ x < 3 \\ 5, \quad if\ x \geq 3 \end{cases}$ is continuous at x = 3. **Assertion (A)**: The value of k is -5.

	Reason (R): $\frac{\lvert x-3\rvert}{x-3} = \begin{cases} 1\,, if\ x \geq 3 \\ -1, if\ x < 3 \end{cases}$
6	**Assertion (A)**: The number of points of discontinuity of the function f(x) = x – [x] in the interval (-2, 5) are 6. **Reason (R)**: The greatest integer function [x] is continuous at all integral values of x.
7	**Assertion (A)**: $\frac{d}{dx}(sin^2 x) = \sin 2x$ **Reason (R)**: Let *f* be a real valued function which is a composite of two functions *u* and *v*; i.e., *f* = *v* o *u*. Suppose *t* = *u*(*x*) and if both $\frac{dt}{dx}$ and $\frac{dv}{dt}$exist, we have $\frac{df}{dx} = \frac{dv}{dt} . \frac{dt}{dx}$.
8	**Assertion (A)**: $\frac{d}{dx} sin(cosx) = \cos(sinx)$ **Reason (R)**: Let *f* be a real valued function which is a composite of two functions *u* and *v*; i.e., *f* = *v* o *u*. Suppose *t* = *u*(*x*) and if both $\frac{dt}{dx}$and $\frac{dv}{dt}$exist, we have $\frac{df}{dx} = \frac{dv}{dt} . \frac{dt}{dx}$.
9	**Assertion (A)**: $\frac{d}{dx} \sin^{-1}(cosx) = -1$ **Reason (R)**: $\sin^{-1}(\sin x) = x,$ $for\ \lvert x\rvert \leq 1\ and\ cosx = \sin\left(\frac{\pi}{2} - x\right)$
10	**Assertion (A)**: $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)\right] = -1$ **Reason (R)**: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
11	**Assertion (A)**: $\frac{d}{dx} \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \frac{2}{1+x^2}$ **Reason (R)**: $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x$
12	**Assertion (A):** $\frac{d}{dx} \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \frac{-2}{1+x^2}$

	Reason (R): $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
13	If 2x + 3y = sin y **Assertion (A)**: $\frac{dy}{dx} = \frac{-2}{3-\cos y}$ **Reason (R)**: 2x + 3y = sin y is an explicit function.
14	**Assertion (A)**: $\frac{d}{dx}\left(e^{\log x}\right) = 1$ **Reason (R)**: $e^{\log x} = x$
15	**Assertion (A)**: $\frac{d}{dx}\left(e^{\sin x}\right) = e^{\cos x}$ **Reason (R)**: $\frac{d}{dx}(e^x) = e^x$
16	**Assertion (A)**: $\frac{d}{dx}\log(\log x) = \frac{1}{x \log x}$ **Reason (R)**: $\frac{d}{dx}\log x = \frac{1}{x}$
17	**Assertion (A)**: $\frac{d}{dx}\log_{10} x = \frac{1}{x}$ **Reason (R)**: $\frac{d}{dx}\log_e x = \frac{1}{x}$
18	**Assertion (A)**: $\frac{d}{dx}(5^x) = 5^x \log 5$ **Reason (R)**: $\frac{d}{dx}(a^x) = a^x \log a$
19	**Assertion (A)**: $\frac{d}{dx}(a^a) = a^a \log a$ **Reason (R)**: $\frac{d}{dx}(a^x) = a^x \log a$
20	**Assertion (A)**: $\frac{d}{dx}\left(2^{\sin x}\right) = 2^{\sin x} \log 2$ **Reason (R)**: $\frac{d}{dx}(a^x) = a^x \log a$
21	If x = $a \sin t$ and y = $b \cos t$

	Assertion (A): $\frac{dy}{dx} = -\frac{b}{a}\tan t$ **Reason (R)**: if y = f(t) and x = g(t) then $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$
22	**Assertion (A)**: Second derivative of log x is $\frac{-1}{x^2}$ **Reason (R)**: Derivative of e^x is e^x.
23	If $y = log\ x$ **Assertion (A)**: $xy'' + y' = 0$ **Reason (R)**: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$
24	If y = t^3 and x = t^2 **Assertion (A)**: $\frac{d^2y}{dx^2} = \frac{3}{2}$ **Reason (R)**: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$
25	Let $y = \log\left(\frac{1}{1+x}\right)$ **Assertion (A)**: y’(1) = -1/2 **Reason (R)**: xy’ + 1 = e^y.

SOLUTIONS

1	**Answer**: A **Solution**: Every polynomial function is continuous in its domain.
2	**Answer**: C **Solution**: Modulus function is continuous in its domain, hence (A) is correct and every continuous function at a point may not be differentiable at that point.
3	**Answer**: D **Solution**: Greatest integer function is discontinuous at all integral values. Therefore (A) is wrong and every differentiable function at a point is continuous at that point, hence (R) is correct.
4	**Answer**: D

	Solution: if f(x) is continuous at x = a then $\lim_{x \to a} f(x) = f(a)$, therefore (R) is true $\lim_{x \to 3} f(x) = f(3)$, $$\lim_{x \to 3} \frac{x^2 - 5x + 3}{x - 3} = k$$ k = 1 therefore (A) is false.
5	**Answer**: A **Solution**: By the definition of the modulus function $\frac{\lvert x-3 \rvert}{x-3} = \begin{cases} 1, if\ x \geq 3 \\ -1, if\ x < 3 \end{cases}$ is true, hence R is true. $$LHL = \lim_{x \to 3-} f(x) = f(3)$$ $$\lim_{x \to 3-} k \frac{\lvert x - 3 \rvert}{x - 3} = 5$$ k(-1) = 5 k = -5
6	**Answer**: C **Solution**: The greatest integer function [x] is discontinuous at all integral values of x and 6 integer values are there in the interval (-2, 5), hence (A) is true The greatest integer function [x] is discontinuous at all integral values of x, hence R is false.
7	**Answer**: A **Solution**: As per the chain rule $\frac{d}{dx}(sin^2 x) = 2 \sin x \cos x = \sin 2x$, (R) is correct as per the chain rule of the derivatives, hence (R) is true and is the correct explanation for (A).
8	**Answer**: D **Solution**: As per the chain rule, $\frac{d}{dx} sin(cosx) = \cos(cosx) . \frac{d}{dx}(cosx) = -\cos(cosx) . sinx$, hence (A) is false. (R) is correct as per the chain rule of the derivatives, hence (R) is true
9	**Answer**: A **Solution**: $\frac{d}{dx} \sin^{-1}(cosx) = \frac{d}{dx} \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right) = \frac{d}{dx}\left(\frac{\pi}{2} - x\right) = -1$,

	hence (A) is correct and (R) is the correct explanation of (A).
10	**Answer**: B **Solution**:$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)\right]=\frac{d}{dx}\left[\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)\right]$ $=\frac{d}{dx}\tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)=\frac{d}{dx}\left(\frac{\pi}{4}-x\right)=1,$ hence (A) is correct. $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$ is also true, hence (R) is also true but (R) is not correct explanation of (A).
11	**Answer**: A **Solution**: $\frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\frac{d}{dx}(2\tan^{-1}x)=\frac{2}{1+x^2}$, hence (A) is correct $\sin^{-1}\left(\frac{2x}{1+x^2}\right)=2\tan^{-1}x$ is true and correct explanation for (A).
12	**Answer**: A **Solution**: $\frac{d}{dx}\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\frac{d}{dX}\sin^{-1}(\cos 2\theta)$ $=\frac{d}{dx}\sin^{-1}\left(\sin\left(\frac{\pi}{2}-2\theta\right)\right)=\frac{d}{dx}(-2\theta)$ $=\frac{d}{dx}(-2\tan^{-1}x)=\frac{-2}{1+x^2},$ hence (A) is true $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$ is true, hence (R) is also true and correct explanation for (A).
13	**Answer**: C **Solution**: 2x + 3y = siny Differentiate with respect to x then $2+3\frac{dy}{dx}=\cos y.\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{-2}{3-\cos y}$ hence (A) is true But 2x + 3y = siny is an implicit function, hence (R) is false.
14	**Answer**: A **Solution**: $\frac{d}{dx}\left(e^{\log x}\right)=\frac{d}{dx}(x)=1$, hence (A) is true.

	$e^{\log x} = x$ is true, hence (R) is true and (R) is the correct explanation for (A).
15	**Answer**: D **Solution**: $\frac{d}{dx}\left(e^{\sin x}\right) = e^{\sin x}\frac{d}{dx}(\sin x) = e^{\sin x}\cos x$, hence (A) is false. $\frac{d}{dx}(e^x) = e^x$ is true, hence (R) is true, hence (R) is true.
16	**Answer**: A **Solution**:$\frac{d}{dx}\log(\log x) = \frac{1}{\log x}.\frac{d}{dx}\log x = \frac{1}{x\log x}$, hence (A) is true. $\frac{d}{dx}\log x = \frac{1}{x}$ is true, hence (R), hence (R) is true.
17	**Answer**: D **Solution**:$\frac{d}{dx}\log_{10} x = \frac{d}{dx}\left(\frac{\log x}{\log 10}\right) = \frac{1}{\log 10}.\frac{d}{dx}\log x = \frac{1}{x\log 10}$, hence (A) is not true. $\frac{d}{dx}\log_e x = \frac{1}{x}$ is true, hence (R) is true.
18	**Answer**: A **Solution**: (A) is true and (R) is the correct explanation of (A).
19	**Answer**: D **Solution**:$\frac{d}{dx}(a^a) = 0\ as\ a^a\ is\ constanct$, hence (A) is not true. $\frac{d}{dx}(a^x) = a^x\log a$ is true, hence (R) is true.
20	**Answer**: D **Solution**:$\frac{d}{dx}\left(2^{\sin x}\right) = 2^{\sin x}\log 2\ \frac{d}{dx}(\sin x) = 2^{\sin x}\log 2\ .\cos x$, hence (A) is not true. $\frac{d}{dx}(a^x) = a^x\log a$ is true, hence (R) is true.
21	**Answer**: A **Solution**:$\frac{dy}{dt} = -b\sin t\ \ and\ \ \frac{dx}{dt} = a\cos t\ \ then\ \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{b}{a}\tan t$, hence (A) is true. if y = f(t) and x = g(t) then $\frac{dy}{dx} = \frac{dy}{dt}/\frac{dx}{dt}$ is true, hence (R) is true and (R) is the correct explanation of (A).
22	**Answer**: B

	Solution:$\frac{d}{dx}(\log x) = \frac{1}{x}$ $implies$ $\frac{d^2}{dx^2}(\log x) = \frac{-1}{x^2}$, hence (A) is true. $\frac{d}{dx}(e^x) = e^x$ is true, hence (R) is true but (R) is not correct explanation of (A).
23	**Answer**: A **Solution**:$y = \log x$ $implies$ $y' = \frac{1}{x}$ $\Rightarrow xy' = 1$ $\Rightarrow xy'' + y' = 0,$ hence (A) is true. $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ is true, hence (R) is true and (R) is the correct explanation of (A).
24	**Answer**: D **Solution**:$\frac{dy}{dt} = 3t^2$ and $\frac{dx}{dt} = 2t$implies$\frac{dy}{dx} = \frac{3}{2}t$ $implies$ $\frac{d^2y}{dx^2} = \frac{3}{2}.\frac{dt}{dx} = \frac{3}{4t}$, hence (A) is not true. $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ is true, hence (R) is true.
25	**Answer**: A **Solution**: y = - log(1+x) implies y' = -1/1+x implies y'(1) = -1/2, hence (A) is true. y' = -1/1+x ⇒ (1 + x) y' = -1 ⇒ y' + xy' = -1 ⇒xy' = -1 – y' ⇒xy' + 1 = e^y. Hence (R) is true and correct explanation of (A)

CASE STUDY QUESTIONS

	CASE STUDY 1	
	A man travel on a path given by $f(x) = \begin{cases} \lvert x\rvert + 3, & if\ x \leq -3 \\ -2x, & if -3 < x < 3, \\ 6x + 2, & if\ x \geq 3 \end{cases}$ depending on the above information answer the following questions.	
1	Are there any breaks in the path?	2M

2	If so where is the break in the path?	2M

	CASE STUDY 2	
	Let *f* be a real valued function which is a composite of two functions *u* and *v*; i.e., *f* = *v* o *u*. Suppose *t* = *u*(*x*) and if both $\frac{dt}{dx}$and $\frac{dv}{dt}$exist, we have $\frac{df}{dx} = \frac{dv}{dt} . \frac{dt}{dx}$. This is called chain rule in the derivatives. Basing on the above the derivative of the following.	
1	$\sec(\tan\sqrt{x})$	1M
2	$2\sqrt{\cot x^2}$	1M
3	$\cos(x^3)\, sin^2 x^5$	2M

	CASE STUDY 3	
	Ex1: x – y – 6 = 0 Ex2: x + sin xy – y = 0 When a relationship between *x* and *y* is expressed in a way that it is easy to *solve for y* and write *y* = *f* (*x*), we say that *y* is given as an *explicit function* of *x*. In the second case it is implicit that *y* is a function of *x* and we say that the relationship of the second type, above, gives function *implicitly*. With the above information find the derivative of the following.	
1	y + sin y = cos x.	1M
2	$x^2 + xy + y^2 = 5$	1M
3	$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$	2M

	CASE STUDY 4	
	Sometimes the relation between two variables is neither explicit nor implicit, but some link of a third variable with each of the two variables, separately, establishes a relation between the first two variables. In such a situation, we say that the relation between them is expressed via a third variable. The third variable is called the parameter. More precisely, a relation expressed between two variables *x* and *y* in the form *x* = *f* (*t*), *y* = *g*(*t*) is said to be parametric form	

	with *t* as a parameter. In order to find derivative of function in such form, we have by chain rule. $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$. Using the above find dy/dx in the following cases.	
1	x = at^2, y = 2at	1M
2	Find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$.	1M
3	$If\ x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}\ then\ show\ that\ \frac{dy}{dx} = -\frac{y}{x}$	2M

	CASE STUDY 5	
	The derivative of a function is again differentiable then the derivative of first derivative is called second derivative. If y = f(x) is a function then its first derivative is denoted by $f^1(x)$ or dy/dx or y_1 and its second derivative is denoted by $f''(x)\ or\ \frac{d^2y}{dx^2}\ or\ y_2$. Basing the given information answer the following.	
1	Find the second derivative of cos(logx)	1M
2	Find the second derivative of $\tan^{-1}x$.	1M
3	If $y = \sin^{-1} x, then\ show\ that\ (1 - x^2)y_2 - xy_1 = 0$	2M

	CASE STUDY 6	
	Let f(x) be a real valued function, then its LEFT HAND DERIVATIVE (L.H.D): $$L f'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$$ RIGHT HAND DERIVATIVE (R.H.D): $$R f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ Also, a function f(x) is said to be differentiable at x=a if its L.H.D and R.H.D at x=a exist and are equal. For the function $f(x) = \begin{cases} \|x - 3\|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$ Answer the following questions	
1	Find the R.H.D of f(x) at x=1	1 M
2	Show that f(x) is non differentiable at x=3	1 M

3	Find the value of f ʿ (2) OR Find the value of f ʿ (-1)	2 M

	CASE STUDY 7	
	A pottery made a mud vessel , where the shape of the pot is based on f(x) = $\|x-3\|+\|x-2\|$, where f(x) represents the height of the pot	
1	When x$>$ 4 what will be the height in terms of x ?	1 M
2	Find the derivative of f(X) at x=3	1 M
3	What is the function When the x value lies between (2,3). OR If the potter is trying to make a pot using the function f(x)=$[x]$, will he get a pot or not? Why?	2 M

	CASE STUDY 8	
	If f(x) is an even function and g(x) is an odd function , $\|x\|$is a modulus function, $[x]$ is an integer function. By using the definitions of the functions solve the following	
1	If f(x) =cos^{2}x is an even function, then find whether f' (x) is an even function or odd function	1 M
2	If f(x)=log($\frac{u(x)}{v(x)}$) , u(1)=v(1) and u'(1)=v'(1)=2 then find the value of f'(1)	1 M
3	If y= $\|x\|$, find $\frac{dy}{dx}$ for x OR Show that f(x)= x- $[x]$ is discontinuous at x=2	2 M

	CASE STUDY 9	
	Read the following passage and answer the questions given below	

	The relation between the height of the plant (y cm) with respect to its exposure to the sunlight is governed by the following equation y=4x - $\frac{1}{2}x^2$, where x is the number of days exposed to the sunlight.	
1	Find the rate of growth of the plant with respect to the number of days exposed to the sunlight	1 M
2	Does the rate of growth of the plant increase or decrease in the first three days?	1 M
3	What will be the height of the plant after 2 days?	2 M

	CASE STUDY 10	
	Three children X,Y and Z of class XII were discussing the answers after completion of the exam. They got stuck at a question regarding differentiation i.e differentiate y= (x^2 – 5x+8)(x^2 + 7x-6). which each of them solved in three different ways. Check whether all the three got the same answer? verify.	
1	Solve it by product rule	1 M
2	Solve it by expanding the product to obtain a single polynomial	1 M
3	Solve it by logarithmic differentiation	2 M

CASE STUDY SOLUTIONS

	Case Study 1
1	Yes
2	At x = 3, LHL = -- 6 and RHL = 20, hence f is discontinuous at x =3

	Case Study 2
1	$\frac{d}{dx}\left(\sec\left(\tan\sqrt{x}\right)\right) = \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right).\frac{d}{dx}\left(\tan\sqrt{x}\right)$ $= \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right) sec^2\sqrt{x}.\frac{d}{dx}\left(\sqrt{x}\right)$ $= \frac{1}{2\sqrt{x}}\sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right) sec^2\sqrt{x}$
2	$\frac{d}{dx}\left(2\sqrt{\cot x^2}\right) = -2\frac{1}{2\sqrt{\cot x^2}} cosec^2(x^2).\frac{d}{dx}(x^2)$

	$= -\frac{2x}{\sqrt{\cot x^2}} cosec^2(x^2)$
3	$\frac{d}{dx}(\cos(x^3)\, sin^2x^5) = sin^2x^5 \frac{d}{dx}(\cos(x^3)) + \cos(x^3)\frac{d}{dx}(sin^2x^5)$ $= -3x^2 sin^2x^5 . \sin x^3 + 10x^4 \sin x^5 \cos x^5 \cos x^3$

	Case Study 3
1	$\frac{d}{dx}(y + \sin y) = \frac{d}{dx}(\cos x)$ $\rightarrow \frac{dy}{dx} + \cos y \frac{dy}{dx} = -\sin x$ $\rightarrow \frac{dy}{dx} = -\frac{\sin x}{1+\cos y}$
2	$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(5)$ $\rightarrow 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ $\rightarrow \frac{dy}{dx} = -\frac{2x+y}{x+2y}$
3	$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right) \rightarrow y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\tan^{-1}x\right)\right)$ $\rightarrow y = \frac{\pi}{2} - 2\tan^{-1}x \;\rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$

	Case Study 4
1	$x = at^2 \rightarrow \frac{dx}{dt} = 2at$ $y = 2at \rightarrow \frac{dy}{dt} = 2a$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$
2	Let u = $\sin^2$x and v =e^{cosx} $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\sin x \cos x}{-\sin x . e^{\cos x}}$

3	$$x=\sqrt{a^{\sin^{-1}t}} \rightarrow \log x=\frac{1}{2}\sin^{-1}t\log a \rightarrow \frac{1}{x}.\frac{dx}{dt}=\frac{1}{2}\log a.\frac{1}{\sqrt{1-t^2}}$$ Similarly $y=\sqrt{a^{\cos^{-1}t}} \rightarrow \frac{dy}{dt}=\frac{1}{y}.\frac{dy}{dt}=-\frac{1}{2}\log a.\frac{1}{\sqrt{1-t^2}}$ $$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=-\frac{y}{x}$$ OR $$xy=\sqrt{a^{\cos^{-1}t+\sin^{-1}t}} \rightarrow xy=\sqrt{a^{\pi/2}}$$ Diff w.r.t. x $$x\frac{dy}{dx}+y.1=0 \rightarrow \frac{dy}{dx}=-\frac{y}{x}$$

	Case Study 5
1	$$\frac{d}{dx}(\cos(\log x))=-\frac{1}{x}\sin(\log x)$$ $$\frac{d^2}{dx^2}(\cos(\log x))=-\frac{d}{dx}\left(\frac{1}{x}\sin(\log x)\right)$$ $$=-\left\{\frac{1}{x}.\frac{1}{x}\cos(\log x)-\frac{1}{x^2}\sin\log x\right\}$$ $$=\frac{1}{x^2}\sin\log x-\frac{1}{x^2}\cos(\log x)$$
2	$$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$$ $$\frac{d^2}{dx^2}(\tan^{-1}x)=\frac{d}{dx}\left(\frac{1}{1+x^2}\right)=-\frac{2x}{(1+x^2)^2}$$
3	$$y=\sin^{-1}x \rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}} \rightarrow \sqrt{1-x^2}y_1=1 \rightarrow (1-x^2)y_1^2=1$$ $$\rightarrow (1-x^2)2y_1y_2-2xy_1^2=0 \rightarrow (1-x^2)y_2-xy_1=0$$

	Case Study 6
	f(x) = $\begin{cases} 3-x, & 1\le x<3 \\ x-3, & x\ge 3 \\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, & x<1 \end{cases}$

	$f'(x)=\begin{cases} -1, 1 \le x < 3 \\ 1, \quad x \ge 3 \\ \frac{x}{2} - \frac{3}{2}, x < 1 \end{cases}$
1	R.H.D of f(x) = $\frac{d}{dx}$(3-x)=-1
2	At x=3 L.H.D ≠R.H.D
3	f' (2)=-1 OR f^{-1}(-1)=-2

	Case Study 7
	f(x) =$\begin{cases} -2x+5, x < 2 \\ 1, 2 \le x < 3 \\ 2x-5, x \ge 3 \end{cases}$
1	2x-5
2	not differentiable at x=3
3	1 or NO ,because greatest integer function is not differentiable.

	Case Study 8
1	f ' (x)= -2cos x.sin x f '(-x)=-2 cosx (-sinx)= 2cosx.sinx which is an even function.
2	f(x) = log u(x)-log v(x) f '(x)= $\frac{u'(x)}{u(x)} - \frac{v'(x)}{v(x)}$ f ' (1)= $\frac{u'(1)}{u(1)} - \frac{v'(1)}{v(1)}$ = 0
3	y=-x^2 for x<0 , $\frac{dy}{dx}$ =-2x OR L.HL=1 and RHL =0 not continuous at x=2

	Case Study 9
1	$\frac{dy}{dx}$ = 4 – x
2	Let rate of growth be represented by g(x)= $\frac{dy}{dx}$

	g'(x)= $\frac{d}{dx}(\frac{dy}{dx})$=-1<0 , g(x) decreases. So the rate of growth of the plant decreases for the first three days.
3	Height of the plant after 2 days =4x2 - $(\frac{1}{2}2^2) = 6$

	Case Study 10
1	Y=UV, by using product rule, $\frac{dy}{dx}$ =$(x^2$-5x+8)(2x+7)+(x^2 +7x+6)(2x-5) on simplification $\frac{dy}{dx}$ =4x^3 +6x^2-66x+86
2	On expansion, y=x^4 +2x^3-33x^2 +86x-48 then $\frac{dy}{dx}$ is same as 1
3	Apply log, logy= log(x^2 – 5x+8)(x^2 + 7x-6). logy= log (x^2 – 5x+8)+log(x^2 + 7x-6). After differentiation $$\frac{1}{y}\frac{dy}{dx}=\frac{2x-5}{x^2-5x+8}+\frac{2x+7}{x^2+7x+6}$$ $\frac{dy}{dx}$=y($\frac{(2x-5)(\ x^2+\ 7x-6).+(2x+7)((x^2\ \text{–}\ 5x+8\ )}{y}$) After simplification $\frac{dy}{dx}$ is same as 1 and 2.

CHAPTER : APPLICATION OF DERIVATIVES

ASSERTION AND REASONING QUESTIONS

In the following questions , a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices: (a) Both assertion (A) and reason (R) are true and the Reason (R) is the correct explanation of the assertion (A). (b) Both Assertion (A) and Reason (R) are true , but Reason(R) is not the correct explanation of the assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	
1	**Assertion(A) :** The function f(x) = [x (x + 2)^2] is increasing in (0 ,1) U (2 , ∞) **Reason (R):** $\frac{dy}{dx}$ = 0 , when x = 0 , 1 , 2
2	**Assertion(A) :** f(x) = $\frac{1}{x-7}$ is decreasing in x ∈ R – { 7 } **Reason (R) :** f^1 (x) < 0 , for all x ≠ 7
3	**Assertion(A) :** f(x) = e^x is an increasing function for all x ∈ R **Reason (R) :** If f^1 (x) ≤ 0 , then f(x) is an increasing function.
4	**Assertion(A) :** f(x) = log x is defined for all x ∈ (0 , ∞) **Reason (R) :** If f^1 (x) > 0 then f(x) is strictly increasing function.
5	**Assertion (A) :** If f(x) = Log(sin x) , x > 0 is strictly decreasing in ($\frac{\pi}{2}$, π) **Reason (R) :** If f^1 (x) ≥ 0 then f(x) is strictly increasing function.
6	**Assertion (A) :** If f(x) = a(x + sinx) is increasing function if a ϵ (0 , ∞) **Reason (R) :** The given function f(x) is increasing only if a∈ (0 , ∞)

7	**Assertion (A) :** The function y = log (1 + x) - $\frac{2x}{2+x}$, x > -1 is a decreasing function throughout its domain **Reason (R) :** $\frac{dy}{dx}$> 0 if x ∈(-1 , ∞)
8	**Assertion (A) :** f(x) = $\frac{5}{x}$ + 2 is decreasing in R – { 0 } **Reason (R) :** The above mentioned function is increasing in R
9	**Assertion (A) :** The rate of change of a circle with respective r is $2\pi r$ **Reason (R) :** $\frac{dy}{dx}$ represents the rate of change of y with respective x
10	**Assertion (A) :** The rate of change of volume of a sphere with respective its radius is increasing. **Reason (R) :** $\frac{dV}{dr}$> 0
11	**Assertion (A) :** f(x) = x - $\frac{1}{x}$ is strictly increasing in R – { 0 } **Reason (R) :** A function f(x) is called decreasing in I if f^1 (x) < 0 for all x ∈ I
12	**Assertion (A) :** f(x) = (x – 1) e^x + 1 is an increasing function for all x > 0 **Reason (R) :** f^1(x) > 0 for all x ∈ (0 , ∞)
13	**Assertion (A) :** The rate of change of the area of circle with respective radius r when r = 3 cm is 6π cm^2/cm **Reason (R) :** $\frac{dA}{dr}$ at r = 3
14	**Assertion (A) :** f(x) = x^2 e^{-x} is increasing in (0 , 2) **Reason (R) :** f(x) is decreasing in (2 , ∞)
15	**Assertion (A) :** f(x) = x^2 – x + 1 is neither increasing nor decreasing on (-1 , 1) **Reason (R) :** If f^1 (x) > 0 then f(x) is strictly increasing function for all x ∈ I
16	**Assertion (A)** : $f(x) = -\|x+1\| + 3$is defined for all real values of x except x = - 1 **Reason (R) :** Maximum value of $f(x)$ is 3 and Minimum value does not exists

17	**Assertion (A) :** $f(x) = \sin 2x + 3$ is defined for all real values of x. **Reason (R) :** Minimum value of $f(x)$ is 2 and Maximum value is 4.
18	**Assertion (A) :** For $f(x) = x + \frac{1}{x}$ maximum and minimum values both exists. **Reason (R) :** Maximum value of $f(x)$ is less than its minimum value
19	**Assertion (A):** If m and M are respectively minimum and maximum values of $f(x) = (x-1)^2 + 3$ for all $x \in [-3,1]$ then $(m, M) = (f(1), f(-3))$ **Reason (R):** $f(x)$ is strictly increasing on $[-3,1]$
20	**Assertion (A):** f(x) = Sin(Sin x) is defined for all real value of x **Reason (R):** Minimum and maximum values does not exist
21	**Assertion (A):** f(x) = e^x has no minimum and maximum values **Reason (R):** f(x) has no critical points
22	**Assertion (A)** $f(x) = -\lvert x+1\rvert + 3$ is defined for all real values of x except x = - 1 **Reason (R)** : Maximum value of $f(x)$ is 3 and Minimum value does not exist.
23	**Assertion (A) :** $f(x) = \sin 2x + 3$ is defined for all real values of x. **Reason (R)** : Minimum value of $f(x)$ is 2 and Maximum value is 4. A. Both A and R are true and R is the correct explanation of A
24	**Assertion (A) :** For $f(x) = x + \frac{1}{x}$ maximum and minimum values both exists. **Reason (R)** : Maximum value of $f(x)$ is less than its minimum value.
25	**Assertion (A):** If two positive numbers are such that their sum is 16 and sum of the cubes is minimum, then the numbers

	are 8, 8. **Reason (R):** If f be a function definedon an interval I and c ∈ l and let f be twice Differentiable at c, then x = c is a point of local minima if $f(x)$ =0 and$f''(c)$>0 and $f(c)$local minimum value of f.

ANSWERS :

1	Assertion is true since $f^1(x) > 0$, x ∈(0 ,1) U (2 , ∞) Reason is true but not the correct explanation of A Ans : (b)
2	Assertion is true since $f^1(x) = \frac{-1}{(x-7)^2}$ $f^1(x) < 0$, x ∈ R – { 7 } Reason is true and it is the correct explanation of A Ans : (a)
3	$f^1(x) = e^x$ ⟹ $f^1(x) > 0$ So it is an increasing function ⟹ R is not true as it is mentioned that $f^1(x) \leq 0$ Ans : (c)
4	$f^1(x) = \frac{1}{x} > 0$ for all x ∈ (0 , ∞) ⟹ f (x) is an increasing function So A & R both are true and R is the correct explanation of A Ans : (a)
5	$f^1(x) = \frac{Cosx}{Sinx}$ = Cotx > 0 where x ∈($\frac{\pi}{2}$, π) So f is an increasing function ⟹ Assertion is wrong ⟹ Reason is true Ans (d)
6	$f^1(x)$ = a(1 + Cosx) > 0 if a ϵ (0 , ∞) So f is an increasing function ⟹ Assertion is wrong

	$\Longrightarrow$ Reason is true Ans (d)
7	Here $\frac{dy}{dx} = \frac{x^2}{(x+1).(x+2)^2} > 0$ So f is an increasing function $\Longrightarrow$ Assertion is wrong $\Longrightarrow$ Reason is true Ans (d)
8	$f^1(x) = \frac{-5}{x^2} < 0$ f(x) is decreasing in R – { 0 } Assertion (A) is true but Reason (R) is false. Ans : (c)
9	$A = \pi r^2$ $\Longrightarrow \frac{dA}{dr} = 2\pi r$ Both assertion (A) and reason (R) are true and the Reason (R) is the correct explanation of the assertion (A). Ans : (a)
10	Both assertion (A) and reason (R) are true and the Reason (R) is the correct explanation of the assertion (A). Ans : (a)
11	$f^1(x) = 1 + \frac{1}{x^2} > 0$ is increasing function Therefore Both Assertion (A) and Reason (R) are true , but Reason(R) is not the correct explanation of the assertion (A). Ans (b)
12	$f^1(x) = e^x + (x-1)e^x$ $f^1(x) = x.e^x > 0$ is an increasing function Both assertion (A) and reason (R) are true and the Reason (R) is the correct explanation of the assertion (A). Ans : (a)
	$A = \pi r^2$

13	$\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dr}$ at r = 3 is 6π cm^2/cm Ans : (a)
14	$f^1(x) > 0$ for all $x \in (0, 2)$ Assertion (A) is true but Reason (R) is false. Ans : (C)
15	Both Assertion (A) and Reason (R) are true , but Reason(R) is not the correct explanation of the assertion (A). Ans : (b)
16	Ans : (a)
17	Since $-1 \leq \sin 2x \leq 1 \rightarrow 2 \leq \sin 2x + 3 \leq 4$ Ans : (a)
18	$y = f(x) = x + \frac{1}{x}$, $f'(x) = 1 - \frac{1}{x^2}$,$f'(x) = 0 \rightarrow x = \pm 1$ $f''(x) = \frac{2}{x^3}$,$f''(-1) = -2 < 0$, $f(x)$ is maximum at x= -1 And max value is $f(-1) = -2$, $f''(1) = 2 > 0$, $f(x)$ is minimum at x= 1 And minimum value is $f(1) = 2$ Ans : (a)
19	$f'(x) = 2(x-1)0$ for all $x \in [-3,1]$ $f(x)$is decreasing on$[-3,1]$, $f(-3) = 19 = M, f(1) = 3 = m$ Ans : (a)
20	Assertion (A) is true but Reason (R) is false. Ans : (c)

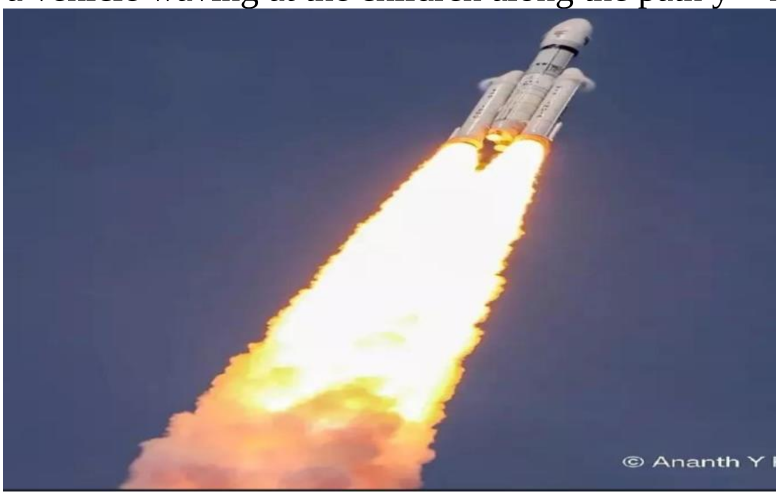
21	Ans : (a)
22	Ans. A
23	Ans. A Explanation $-1 \leq \sin 2x \leq 1 \to 2 \leq \sin 2x + 3 \leq 4$
24	Ans. A. Explanation. $y = f(x) = x + \frac{1}{x}$, $f'(x) = 1 - \frac{1}{x^2}$, $f'(x) = 0 \to x = \pm 1$ $f''(x) = \frac{2}{x^3}$,$f''(-1) = -2 < 0$, $f(x)$ is maximum at x= -1 And max value is $f(-1) = -2$, $f''(1) = 2 > 0$, $f(x)$ is minimum at x= 1 And minimum value is $f(1) = 2$
25	Ans. A. Explanation. Let the numbers be x & y, $x + y = 16$ and $f(x) = x^3 + y^3 = x^3 + (16 - x)^3$ $f'(x) = 3x^2 - 3(16 - x)^2, f'(x) = 0 \to x = 8$ $f''(8) = 96 > 0$ $f(x)$is minimum at$x$=8 when $x = 8, y = 8$ ∴ the numbers are 8,8.

CASE BASED QUESTIONS

1	Mr.Suresh who is a popular businessman consulted a share market expert to know about the investment in a particular company. He predicted that the trend of the company would be governed by the function $f(x) = \frac{x^3}{3} - 4x^2 + 15x + 8$ where x is the years of investment in the company.

	 i. In the first ten years when will he get growth in his investment? ii. There is going to be a lean patch in the investment. When is it going happen? When will the market pick up again?
2	Rain Water Harvesting pits are very essential to conserve water and use it furtheruse.An engineer was asked to design a cuboidal pit with a fixed volu of $256m^3$ and with a square base  i. What will be the value of edge of base so that total surface minimum? ii. Find the height of tank and also the total surface area?
3	The soaring prices of tomatoes in our country in the recent part made the Trade Analysts to come up with an equation $$f(x) = 16x - \frac{1}{2}x^2$$ To find the cost of 1kg of tomatoes after x days.

	 After how many days will the cost of tomatoes be maximum? What will be the maximum cost? ii. When will the cost of tomatoes become Rs.64?
4	The Maths and crafts teachers of a school planned to assign a task to the students. A paper of area ‘k’ sq.uts was given to each one them and were asked to make a cylinder closed at one end and open at the other.  i. Find the value of r for which the cylinder has maximum volume. ii. Find the relation between r and h of the cylinder. Also find the maximum value.
5	A company was given contract to manufacture two varieties of bulbs A & B which will be sold at profits of Rs. 60 & Rs. 80 respectively. There was a condition that sum of squares of the number of bulbs of each type is a constant k.

	  i) What is the ratio of production of 2 bulbs for maximum profit? ii) What is the maximum profit if k = 100?
6	To honor the scientists associated with the success of chandrayaan-3, A school management decided to felicitate them. The students were asked to stand in a path governed by $y = x^2$. The scientists would be asked to move in a vehicle waving at the children along the path $y = x - 2$.  i) A student nearest to their path was given an opportunity to garland the dignitaries. Where on the curve does the student stand? ii) What is the distance of the student from the line?
7	To reduce global warming environmentalists and scientists came up with an innovative idea of developing a spherical bulb that would absorb harmful gases and thereby reduce global warming.But during the process of absorption the bulb would get inflated and it’s radius would beincreasing at 1cm/sec.

	 i) Find the rate at which the volume increases when radius is 6 cm. ii) At an instant when volume was increasing at the rate of $400\pi cm^3/sec$ find the rate at which it's surface area is increasing?
8	An Aero plane is flying with a velocity of 300m/sec at an altitude of 1 km from the Earth's surface.  i) Find an expression for $\frac{dx}{dt}$ in terms of θ, where θ is the angle of elevation of the aeroplane from the bottom of the control tower and x is the horizontal distance between the aeroplane and the control tower. ii) Find the rate at which the angle of elevation of the Aeroplane changes from the control tower at an instant when the horizontal distance of the plane is 500m from it.
9	A cylindrical tank of fixed volume of $144\Pi\ m^3$ is to be constructed with an open top to throw all the garbage in an orphanage. The manager of the orphanage called a contractor for the construction ensure that a tank to dispose off biodegradable waste can be constructed at a minimum cost.

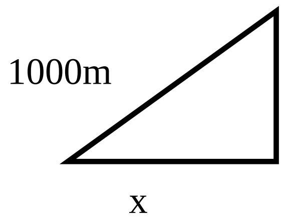
	 i) Find the cost of the least expensive tank that can be constructed if it costs Rs. 80 per sq. m for base and Rs. 120 per sq. m for walls ii) Find the radius and height as well.
10	An insect moves in a straight line to escape from a predator such that the distances travelled by the insect and its predator in time t (in seconds) are given by $s = t^3 – 7t^2 + 15t + 1$ and $s = t^4 + t^3 + 4t + 10$ respectively. i) What were their initial velocities? ii) After how many seconds will the predator be able to catch up the prey if initially it was behind it by 120m? 

ANSWERS

1	$f'(x) = x^2 – 8x + 15$ For critical points, $f'(x) = 0$ $x^2 – 8x + 15 = 0$ $x = 3, x = 5$ In (0, 3), $f'(x) > 0$ and in (3, 5) $f'(x) < 0$ By first derivative test f has maxima at $x = 3$

	ii) when $x > 5$, $f'(x) > 0$ f is Strictly decreasing in (3, 5) ---- lean patch
2	i) Given V = 256 m^3, $l^2 h = 256$ $h = \frac{256}{l^2}$ $S = l^2 + 4lh$ $S = l^2 + 4l \times \frac{256}{l^2} = f(l)$ $f'(l) = 2l - \frac{1024}{l^2}$ for critical points, $f'(l) = 0$ so we get l = 8m by second derivative test f has minima at l = 8m as $f''(l) > 0$ ii) $h = \frac{256}{l^2}$ so h = 4m S = f (8) = 192 m^2
3	$f(x)= 18x- \frac{1}{2}x^2$ $f'(x)=18-x$ for critical points, $f'(x)=0$ ➡ x= 18 $f''(x)= -1<0$ ➡ $f''(18)= -1<0$ by second derivative test f has maxima at x= 18 maximum cost = $18 \times 18 - \frac{1}{2}x^2$ = Rs. 162 ii) $18x-\frac{1}{2}x^2= 64$ $36x- x^2 =128$ $X^2-36x+128= 0$ X= 4.
4	1. $2\pi rh + \pi r^2 = k$ $h= k-\pi r^2/2\pi r$ $v = \pi r^2 h$ $= \pi r^2(k-\pi r^2/ 2\pi r)$ $V= f(r)= ½(kr- \pi r^3)$ $f'(r) = ½(k-3\pi r^2)$

	$f'(r)= 0$ ➡ $r= \sqrt{k/3\pi}$ $f''(r) = - 3\pi r$ $f''(\sqrt{k/3\pi}) = -3\pi\sqrt{k/3\pi} < 0$ by second derivative test, f has local max at $r=\sqrt{k/3\pi}$ ii) $h=(3\pi r^2- \pi r^2)/ 2\pi r$ ➡ $h = 2\pi r^2/2\pi r$ **h= r**
5	1. let A & B variety bulbs be x & y respectively. $X^2+y^2 = k$ $Y=\sqrt{(k-x^2)}$ Profit= 60x + 80y $f(x) = 60x + 80\sqrt{(k-x^2)}$ $f'(x)= 0$ $25x^2 = 9k$ $X = \frac{3}{5}\sqrt{k}$ $f''(x) = -80(2k/ 2(k-x^2)^{3/2}) < 0$ f has max at $x = \frac{3}{5}\sqrt{k}$ $y = \frac{4}{5}\sqrt{k}$ ➡ x: y= 3:4 **ii)** profit= $60 \times \frac{3}{5} \times 10+80 \times \frac{4}{5}\times 10$ = R1000
6	1. let points on $y = x^2$ be $A=(t, t^2)$ $X - Y - 2 = 0$ distance = $\lvert t - t^2 - 2 \rvert/ \sqrt{2}$ $f(t)= \lvert(t - t^2 - 2)\rvert/ \sqrt{2}$ $f'(t)= (2t-1)/ \sqrt{2}$ $f'(t)= 0$ $t= ½$ $f''(t)= 2/\sqrt{2}$ $f''(1/2) = \sqrt{2} > 0$ f has min at t= ½ $\therefore$ A = (1/2 ,1/4) ii) distance = $((½)^2 - ½ + 2)/ \sqrt{2}$ $= \frac{7}{4\sqrt{2}}$ units.

7	$-\frac{dr}{dt}$= 1cm/sec i) $V = 4/3\pi r^3$ $dv/dt = 4\ \pi r^2 dr/dt$ $(dv/dt)_{r=6} = 4\ \pi\ 36 \times 1$ $= 144\ \pi\ cm^3/sec$ ii) Given $dv/dt = 400\ \pi$ $4\ \pi r^2 dr/dt = 400\ \pi$ r= 10 cm $s= 4\ \pi r^2$ $ds/dt = 8\ \pi r\ dr/dt$ $(ds/dt)_{r=10} = 8\ \pi \times 10 \times 1$ $= 80\ \pi\ cm^2/sec$
8	 $\frac{dx}{dt}$ = 300m/sec $\tan\theta = \frac{1000}{x}$ $x= 1000 \cot\theta$ ➡ $\frac{dx}{dt} = -\ 1000\ \text{cosec}^2\theta\ \frac{d\theta}{dt}$ ii) when x = 500 $\tan\theta = 2$ $\cot\theta= ½$,$\text{cosec}\theta= \sqrt{(5/4)}$ $\therefore \frac{dx}{dt}= 1000 \times \frac{5}{4} \times \frac{d\theta}{dt}$ (when x = 500 m) $\frac{300}{-1250} = \frac{d\theta}{dt}$ $\frac{d\theta}{dt} = \frac{-6}{25}\frac{rad}{sec}$
9	$\Pi r^2 h = 144\Pi$ $h = \frac{144}{r^2}$ Area = $\Pi r^2 + 2\Pi rh$ Cost = $80 \times \Pi r^2 + 120 \times 2\Pi rh$ Substituting $h = \frac{144}{r^2}$

	Cost = f (r) $= 80 \text{ X } \Pi r^2 + \frac{34560\Pi}{r}$ $f'(r) = 160\,\Pi r - \frac{34560\Pi}{r^2}$ for critical points $f'(r) = 0$, we get r = 6m $f''(r) = 160\Pi + \frac{69120\Pi}{r^3} > 0$ for r = 6 so by second derivative test, cost is minimum at r = 6m minimum cost = f (6) = Rs. 8640Π
10	i) initial velocities: Prey $\longrightarrow \frac{ds}{dt} = 3t^2 - 14t + 15$ Initial velocity (t = 0) = 15m/sec Predator $\longrightarrow \frac{ds}{dt} = 4t^3 + 3t^2 + 4$ Initial velocity (t = 0) = 4m/sec iii) Let it catch the prey after t seconds, Difference in distances = 120m $(t^4 + t^3 + 4t + 10) - (t^3 - 7t^2 + 15t + 1) = 120$ $(t^4 + 7t^2 - 11t + 9 = 120$ Solving it we get t = 3 seconds

CHAPTER: INTEGRALS

ASSERTION REASONING QUESTIONS

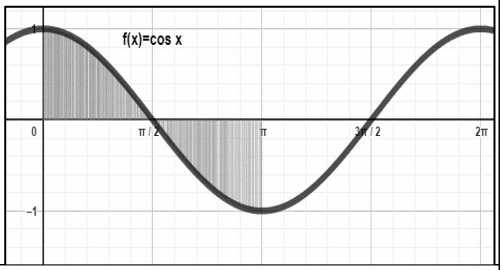
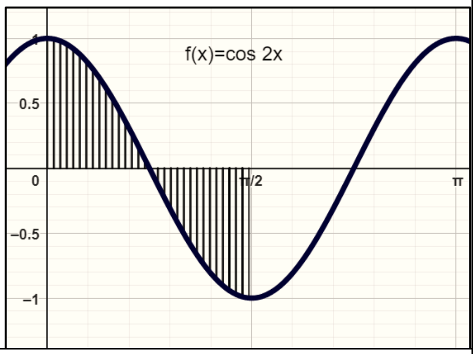
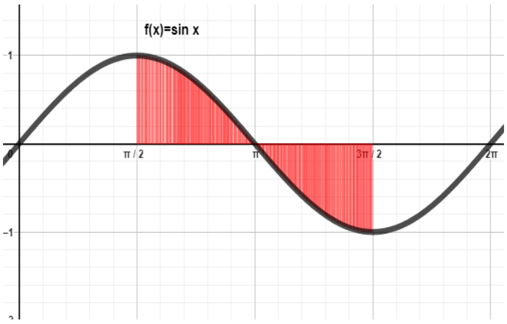
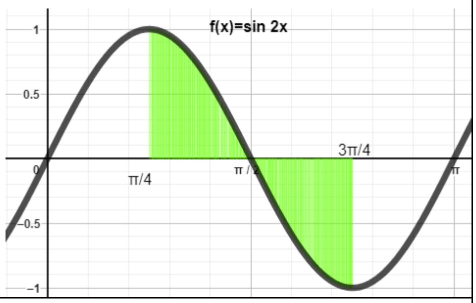
	In the following question a statement of Assertion (A) is followed by a statement of Reason (R). Pick the correct option: (a)Both A and R are true and R is the correct explanation of A. (b)Both A and R are true and R is NOT the correct explanation of A. (c) A is true but R is false. (d)A is false but R is true.
1	**Assertion (A)**: $\int_0^{\pi} \cos x \ dx = 2$ **Reason (R)** : The function f(x)=cos x is decreasing in [0, π]
2	**Assertion (A)**:$\int_0^{\pi/2} \cos 2x \ dx$=1 **Reason (R)**: The function cos 2x is decreasing in [0, $\frac{\pi}{2}$]
3	**Assertion (A)**: $\int_{\pi/2}^{3\pi/2} \sin x \ dx = 2$ **Reason (R)**: The function sin x is decreasing in [$\frac{\pi}{2}$, $\frac{3\pi}{2}$]
4	**Assertion (A)**: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin 2x \ dx$=1 **Reason (R)**: The function sin 2x is decreasing in [$\frac{\pi}{4}$, $\frac{3\pi}{4}$]
5	**Assertion (A)**: $\int_{-\pi/2}^{\pi/2} sin^7 x \ dx = 0$ **Reason(R)**: $\int_{-a}^{a} f(x)dx$ =0, if f is an odd function
6	**Assertion (A)**: $\int_{-1}^{1} \log(\frac{2-3x}{2+3x})dx = 0$ **Reason**: $\int_{-a}^{a} f(x)dx$ =0, if f is an odd function
7	**Assertion (A)**: $\int e^x(sinx - \cos x)\ dx = -e^x cosx + C$ **Reason (R)** : $\int e^x\big(f(x) + f'(x)\big)dx = e^x f(x) + C$
8	**Assertion (A)**: $\int e^x\ (cosx - \sin x)\ dx = -e^x sinx + C$ **Reason(R)** : $\int e^x\big(f(x) + f'(x)\big)dx = e^x f(x) + C$
9	**Assertion (A)**: $\int e^x\ (\frac{1}{x} - \frac{1}{x^2})\ dx = e^x \frac{1}{x} + C$ **Reason(R)** : $\int e^x\big(f(x) + f'(x)\big)dx = e^x f(x) + C$
10	**Assertion (A)**: $\int_0^{10} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} dx = 10$ **Reason R)** : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
11	**Assertion (A)**: $\int \frac{x-3}{(x-1)^3} e^x\ dx = \frac{e^x}{(x-1)2} + c$ **Reason (R)** : $\int e^x$(f(x)+f' (x)) dx = e^xf(x)+c

12	**Assertion (A)**: $\int \frac{e^{tan^{-1}x}}{1+x^2}\, dx = e^{tan^{-1}x} + C$ **Reason (R)** : $\frac{d}{dx}\left(e^{tan^{-1}x}\right) = \frac{e^{tan^{-1}x}}{1+x^2}$
13	**Assertion (A)**: $\int \frac{e^x}{x+1}[1 + (x+1)\log(x+1)]\, dx = e^x \log(x+1) + e^x + c$ **Reason(R)** : We have $\int e^x(f(x)+ f'(x))\, dx = e^x f(x)+c$
14	**Assertion(A)**: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}$ dx is equal to $\frac{\pi}{2}$ **Reason(R)** : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
15	**Assertion (A)**: $\int_0^{\frac{\pi}{2}} sin^2 x dx = \frac{\pi}{4}$ **Reason(R)** : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
16	**Assertion (A)**: $\int_{-1}^{1} (x^3 + \sin x)\, dx = 0$
17	**Assertion (A)**: $\int x^x(1 + \log x)\, dx = x^x + C$ **Reason (R)** : $\frac{d}{dx}(x^x) = x^x(1 + \log x)$
18	**Assertion (A)**: $\int_{-\pi/2}^{\pi/2} cos^7 x\ dx = 0$ **Reason(R)** : $\int_{-a}^{a} f(x)dx$ =0, if f is an odd function
19	**Assertion (A)**: $\int \sqrt{1-x^2}\, dx = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2} sin^{-1}(x) + c$ **Reason(R)** : $\frac{d}{dx}\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2} sin^{-1}(x) + c\right) = \sqrt{1-x^2}$
20	**Assertion (A)**: $\int_{-1/2}^{1/2} \cos x.\ \log\left(\frac{2-3x}{2+3x}\right) dx = 0$ **Reason(R)** : $\int_{-a}^{a} f(x)dx$ =0, if f is an odd function
21	**Assertion (A)**: $\int_{-1}^{1} x\|x\|\, dx = 2/3$ sq. units **Reason(R)** : $\|x\| = \begin{cases} x, if x \geq 0 \\ -x, if\ x < 0 \end{cases}$
22	**Assertion (A)**: $\int_{-1}^{1} \|x\|\, dx$ =1 sq. units **Reason(R)** :the derivative of modulus function f(x)= $\|x\|$does not exist at 0
23	**Assertion(A)**: $\int e^{5logx}\, dx$ is equals to $\frac{x^6}{6} + c$ **Reason(R)** : We have $\int e^{logx}$ = x
24	**Assertion(A)**: $\int x^2 e^{x^3}\, dx = \frac{1}{3} e^{x^3} + c$ **Reason(R)** : $\int e^x dx = e^x + c$
25	**Assertion(A):** $\frac{d}{dx}(f(x)) = \log x$, then $f(x) = \frac{1}{x} + c$

	Reason(R) : $\int f(x)g(x)dx = f(x)\int g(x)dx - \int[f'(x)\int g(x)dx]dx$
26	**Assertion(A):** $\int \frac{x-3}{(x-1)^3} e^x\, dx$ is equal to $\frac{e^x}{(x-1)2} + c$ **Reason(R):** We have $\int e^x$(f(x)+f '(x)) dx = e^xf(x)+c
27	**Assertion (A):** Anti derivative of $\frac{\tan x-1}{\tan x+1}$ with respect to x is $\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\| + c$ **Reason(R) :** $\int \tan x dx = \log\|\sec x\|+c$
28	**Assertion(A):** $\int \frac{e^x(1+x)}{\cos^2(xe^x)}$ dx is equal to $\tan(xe^x) + c$ **Reason(R) :** $\int \sec^2 x dx = \tan x + c$
29	**Assertion(A):** $\int \frac{dx}{16+9x^2}$ is equal to $\frac{1}{3}\tan^{-1}\left(\frac{3x}{4}\right) + c$ **Reason(R) :** $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
30	**Assertion(A):** $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$ **Reason(R):** $\int_{-a}^{a} f(x)dx = 0$ if f is an odd function
31	**Assertion(A):** $\int_{-5}^{5}\|x+2\|\,dx = 29$ **Reason(R):** $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
32	**Assertion(A):** If the value of $\int_0^a 3x^2 dx = 8$ then the value of 'a' is 2 **Reason(R):** $\int x^n dx = \frac{x^{n+1}}{n+1}$+c
32(a)	**Assertion(A):** $\int$ sec^2(7-4x) dx = a tan(7-4x) +c then the value of a is $\frac{-1}{4}$ **Reasoning(R):** $\int f(ax+b)dx = \frac{F(ax+b)}{a}$ +c
33	**Assertion(A):** $\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3}\sin^{-1}(ax)$+c, then a is $\frac{3}{2}$ **Reason(R) :** $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(x/a)$ +c
34	**Assertion(A):** *The value of* $\int_2^3 \frac{x}{x^2+1}$dx is equals to $\frac{1}{2}$ log2 **Reasoning(R):** $\int x^n dx = \frac{x^{n+1}}{n+1}$+c
35	**Assertion(A):** $\int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}$ dx = 2 **Reasoning(R):** $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(x/a)$ +c
36	**Assertion(A):** $\int \frac{(\log x)^2}{x} dx = \frac{(\log x)^3}{3} + c$ **Reason(R):** $\int \frac{1}{x} dx$ =logx+c

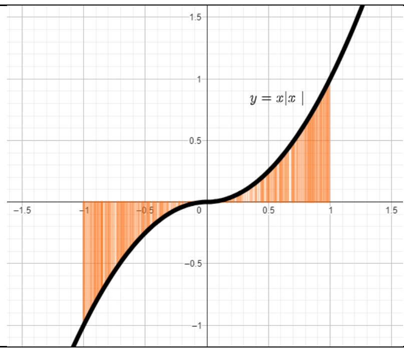
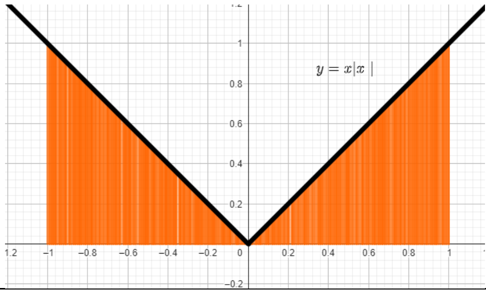
INTEGRALS- ANSWERS

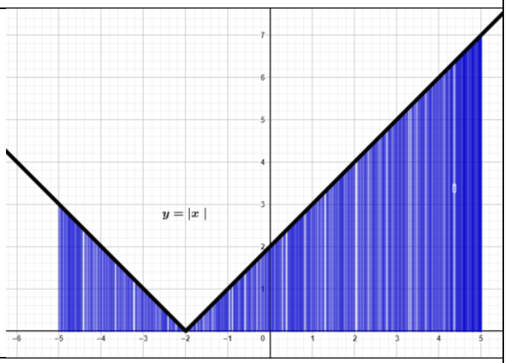
ASSERTION REASONING QUESTIONS

1	Ans:b) **Solution**: $\int_0^{\pi} y\,dx = \int_0^{\pi/2}\cos x\,dx + \left\|\int_{\pi/2}^{\pi}\cos x\,dx\right\|$ $= \sin x]_0^{\pi/2} + \left\|\sin x]_{\pi/2}^{\pi}\right\| \quad =1+1=2$	
2	Ans:b) **Solution**: $\int_0^{\pi/2} y\,dx =$ $\int_0^{\pi/4}\cos 2x\,dx + \left\|\int_{\pi/4}^{\pi/2}\cos 2x\,dx\right\|$ $=\frac{1}{2}\sin 2x]_0^{\pi/4} + \left\|\frac{1}{2}\sin 2x]_{\pi/4}^{\pi/2}\right\|$ $=\frac{1}{2}+\frac{1}{2}=1$ sq. units	
3	Ans:b) **Solution**: $\int_{\pi/2}^{3\pi/2}\sin x\,dx$ $= \int_{\pi/2}^{\pi}\sin x\,dx + \left\|\int_{\pi}^{3\pi/2}\sin x\,dx\right\|$ $=-\cos x]_{\pi/2}^{\pi} + \left\|-\cos x]_{\pi}^{3\pi/2}\right\|$ =1+1=2	
4	Ans:b) $\int_0^{\pi/2} y\,dx = \int_{\frac{\pi}{4}}^{\pi/2}\sin 2x\,dx + \left\|\int_{\pi/2}^{\frac{3\pi}{4}}\sin 2x\,dx\right\|$ $=\frac{-1}{2}\cos 2x]_{\frac{\pi}{4}}^{\pi/2} + \left\|\frac{-1}{2}\cos 2x]_{\pi/2}^{3\pi/4}\right\|$ $=\frac{1}{2}+\frac{1}{2}=1$ sq. units	
5	Ans:a) **Solution**: Let f(x) = $\sin^7 x$ f(-x) = $\sin^7(-x)$ = $-\sin^7 x$ = -f(x) So, f is an odd function By property $\int_{-a}^{a} f(x)dx = 0$ if f(x)is odd function is true	
6	Ans:a) **Solution** f(x) = $\log(\frac{2-3x}{2+3x})$ f(-x) = $\log(\frac{2+3x}{2-3x})$ = $\log(\frac{2-3x}{2+3x})^{-1}$	

	$=-\log(\frac{2-3x}{2+3x})$ $=-f(x)$ So, f is an odd function
7	Ans: a) f(x)= $= -\cos x$ and $f'(x) = \sin x$
8	Ans:d) f(x)= $= \cos x$ and $f'(x) = -\sin x$
9	Ans:a) f(x)= $\frac{1}{x}$ and $f'(x) = -\frac{1}{x^2}$
10	Ans:d) $I = \int_0^{10} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} dx$ $I = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}} dx$(By property) Adding,2I = $\int_0^{10} 1dx$=10 Hence, I =5 sq.units
11	Ans. a) **Solution:** $\int \frac{x-3}{(x-1)^3} e^x\, dx = \int \frac{x-1-2}{(x-1)^3} e^x\, dx$ $= \int e^x[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}]$dx $= \frac{e^x}{(x-1)2} + c(by\ using \int e^x$(f(x)+$f'$ (x)) dx = e^xf(x)+c
12	Ans:a) Integration is the inverse process of differentiation
13	Answer is d **Solution**: $\int \frac{e^x}{x+1}[1 + (x+1)\log(x+1)]\, dx$ =$\int e^x(\frac{1}{x+1}$+log(x+1)) dx = e^xlog(x+1) +c($by\ using \int e^x$(f(x)+f' (x)) dx = e^xf(x)+c Hence Assertion is wrong and Reasoning is correct.
14	Answer is d **Solution**: Let I = $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}$ dx …… (1) By applying the Property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ We get I = $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}$ dx ……… (2) By adding (1) and (2) we get 2I = $\int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}$ I = $\frac{\pi}{4}$, Hence Assertion is wrong and Reasoning is Correct
15	Ans:a) I $= \int_0^{\frac{\pi}{2}} sin^2 x dx$

	$=\int_0^{\frac{\pi}{2}} sin^2(\frac{\pi}{2}-x)dx$ (by property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$) I $= \int_0^{\frac{\pi}{2}} cos^2xdx$ Adding, I+I= $\int_0^{\frac{\pi}{2}}(sin^2x + cos^2x)dx$ $=\int_0^{\frac{\pi}{2}} 1dx= \frac{\pi}{2}$ ∴I = $\frac{\pi}{4}$ Sq. units
16	**Reason (R)** : $\int_{-a}^{a} f(x)dx$ =0, if f is an odd function Ans:a) f(x) = $x^3 + sinx$ f(-x) = $(-x)^3 + \sin(-x)$ $= -x^3 - sinx$ = -f(x) Hence f is an odd function
17	Ans:a) y $= x^x$ log y = x.log x differentiating w.r.t x, $\frac{1}{y}\frac{dy}{dx} = 1 + \log x)$ $\therefore \frac{dy}{dx} = y(1+\log x)$ $= x^x(1+\log x)$
18	Ans:d) Let f(x) = $\cos^7 x$ f(-x) = $\cos^7(-x) = (\cos(-x))^7 = cosx^7$ =f(x) So, f is not an odd function By property $\int_{-a}^{a} f(x)dx = 0$ if f is odd function ……..is true
19	Ans:a) $\frac{d}{dx}(\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}sin^{-1}(x)+c)$ $=\frac{-x^2}{2\sqrt{1-x^2}}+\frac{1}{2}\sqrt{1-x^2}+\frac{1}{2\sqrt{1-x^2}}$ $=\frac{1-x^2}{2\sqrt{1-x^2}}+\frac{1}{2}\sqrt{1-x^2}$ $=\frac{\sqrt{1-x^2}}{2}+\frac{1}{2}\sqrt{1-x^2}=\sqrt{1-x^2}$
20	Ans:a) f(x) = cosx. $\log(\frac{2-3x}{2+3x})$ f(-x) = cos(-x). $\log(\frac{2+3x}{2-3x})$ $= \cos x. \log(\frac{2+3x}{2-3x})^{-1}$ = cosx.$-log(\frac{2+3x}{2-3x})$ = -f(x). hence f is an odd function

21	Ans:b) $\int_{-1}^{1} x\|x\|\, dx = \left\|\int_{-1}^{0} -x^2\, dx\right\| + \int_{0}^{1} x^2 dx = \frac{2}{3}$ 
22	Ans b) $\int_{-1}^{0} -x\, dx + \int_{0}^{1} x\, dx = \frac{1}{2} + \frac{1}{2} = 1$ 
23	Answer is: a) **Solution**: $\int e^{5logx}\, dx = \int e^{logx^5}\, dx = \int x^5$dx $= \frac{x^6}{6} + c$ So, option a) is true
24	Answer is: a) **Solution**: Let x^3 = t,$3x^2$dx = dt, x^2dx =dt $\int x^2 e^{x^3}\, dx = \frac{1}{3}\int e^t dt = e^t$+c $=\frac{1}{3} e^{x^3}$+c So,option a) is true
25	Answer is :d) **Solution**: We know that $\frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{f(x)}) = \mathrm{logx}$, then $\mathrm{f(x)} = \int \mathrm{logxdx}$ Using by parts, taking f(x) = logx, g(x) = 1 = logx$\int dx$-$\int[\frac{1}{x}\int dx]$dx =xlogx-x+c
26	Answer is a **Solution**: $\int \frac{x-3}{(x-1)^3} e^x\, dx = \int \frac{x-1-2}{(x-1)^3} e^x\, dx$ $=\int e^x[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}]$dx (It is of the form $\int e^x$(f(x)+f ‘(x)) dx) $= \frac{e^x}{(x-1)2} + c$
27	Answer: d **Solution**: Anti derivative of $\frac{\mathrm{tanx-1}}{\mathrm{tanx+1}}$ with respect to x is equals to $\int \frac{\mathrm{tanx-1}}{\mathrm{tanx+1}}$dx

	$\int \frac{\tan x-1}{\tan x+1}dx = -\int \tan(\frac{\pi}{4}-x)dx = -\log\left\|\sec\left(\frac{\pi}{4}-x\right)\right\| + c$ Hence Assertion is Wrong and Reasoning is Correct.
28	Answer is a **Solution:** By substitution method Let x e^x = t By differentiating with respect to x we get $e^x(1+x)dx = dt$ Therefore $\int \sec^2 t dt$ = tant+c =tan (x e^x) +c
29	Answer is d **Solution:** $\int \frac{dx}{16+9x^2} = \frac{1}{9}\int \frac{dx}{\frac{16}{9}+x^2}$ Using the formula $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ $= \frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right) + c$ Hence the Assertion is Wrong and Reason is correct
30	Answer is a **Solution**: Let f(x) = $\sin^7 x$ f(-x) = $\sin^7(-x)$ = -$\sin^7 x$ = -f(x) So, f is an odd function By property $\int_{-a}^{a} f(x)dx = 0$ if f is odd function
31	Answer is a **Solution:** $f(x) = \|x+2\| = \begin{cases} x+2\ , & \text{if } x \geq -2 \\ -x-2, & \text{if } x < -2 \end{cases}$ So, by property $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ $\int_{-5}^{5}\|x+2\|\, dx = \int_{-5}^{-2}(x+2)dx + \int_{-2}^{5}(-x-2)dx$ On integrating and substituting limits we get 29 
32	Answer is a **Solution**: $\int_0^a 3x^2 dx = 8$ implies that $[3\frac{x^3}{3}]^a_0$ =8 a^3 = 8 implies that a = 2 Hence the option a is correct.
32(a)	Answer is a **Solution**: Let 7-4x = t, dx = $\frac{-1}{4}$dt $\int \sec^2 t\, dt$ = a tan t +c =-$\frac{1}{4}$ tan(7-4x) +c then a = $\frac{-1}{4}$
33	Answer is a

	Solution: $\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3}\int \frac{1}{\sqrt{\frac{4}{9}-x^2}} dx = \frac{1}{3}\sin^{-1}(\frac{3}{2}x)$ +c Using the formula $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(x/a)$ +c On comparing we get a = $\frac{3}{2}$
34	Answer is b **Solution:** Let $x^2+1 = t, 2xdx = dt$ Therefore $\int_5^{10} \frac{1}{t}dt = \frac{1}{2}\log 2$, using $\int \frac{1}{x}dx = \log x + c$ Hence Reasoning is not the correct explanation of Assertion
35	Answer is b **Solution:** $\int_0^{\frac{\pi}{2}} \sqrt{1+cosx}\, dx = \sqrt{2}\int_0^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) dx = \sqrt{2}[\sin(x/2)]_0^{\frac{\pi}{2}} = 2$ So, Statement A is Correct Also, R is correct but R does not give correct explanation to A.
36	Answer is b **Solution:** In Assertion Let logx =t implies that 1/x dx = dt Therefore, $\int t^2 dt = t^3/3 + c = \frac{(logx)^3}{3} + c$ So, Statement A is Correct Also, R is correct but R does not give correct explanation to A

INTEGRALS- CASE STUDY/ SOURCE-BASED INTEGRATED QUESTIONS

1	The given integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting x = g(t) Consider I = $\int f(x)dx$ Put x = g(t), $\frac{dx}{dt}$ = g'(t) we write dx = g'(t)dt Thus I = $\int f(g(t))g'(t)dt$ This change of variable is very important tools available to us in the name of integration by substitution Based on the above information answer the following questions Evaluate $\int 2x\sin(x^2+2)dx$ (i) To evaluate the above question which quantity we assume as another new variable. (ii) Solve the given question using fundamental integrals.

2	Let f be a continuous and differentiable function defined in a closed interval [a, b] and F be an anti-derivative of f then $\int_a^b f(x)dx = [\ F(x)+c]_a^b = F(b)-F(a)$ called definite integral It is very useful because it gives us a method of calculating the definite integral more easily. There is no need to keep integration constant as it gets cancel by substituting the upper limit and lower limit Based on the above information answer the following: Evaluate $\int_2^3 \frac{xdx}{1+x^2}$ (i) Which technique you will use to evaluate the integral (ii) What is the value of definite integral

3	The rational functions which we shall consider for integration purposes will those denominators can be factorized in to linear and quadratic factors. Assume that we want to evaluate $\int \frac{p(x)}{q(x)}dx$ where $\frac{p(x)}{q(x)}$ is a proper rational function. It is always possible to write the integrand as a sum of simpler rational functions by a method called partial fraction decomposition. After this the integration can be carried out easily using the already known methods. If the rational function is of the form $\frac{px+q}{(x-a)(x-b)}$ then we write the partial fraction is of the form $\frac{A}{x-a} + \frac{B}{x-a}$ Where A and B are to be determined Based on the above information answer the following: Evaluate $\int \frac{dx}{(x+1)(x+2)}$ (i) Whate are the values of A and B when we use partial fractions (ii) After finding the values of A and B how will you evaluate integral and write the final answer

4	The rational functions which we shall consider for integration purposes will those denominators can be factorized in to linear and quadratic factors. Assume that we want to evaluate $\int \frac{p(x)}{q(x)}dx$ where $\frac{p(x)}{q(x)}$ is a proper rational function. It is always possible to write the integrand as a sum of simpler rational functions by a method called partial fraction decomposition. After this the integration can be carried out easily using the already known methods. If the rational function is of the form $\frac{px+q}{(x^2+a)(x+b)}$ then we write the partial fraction is of the form $\frac{Ax+B}{x^2+a} + \frac{C}{x-b}$ Where A, B and C are to be determined Based on the above information answer the following

	Evaluate $\int \frac{(x^2+x+1)dx}{(x^2+1)(x+2)}$ (i) Whate are the values of A, B and C when we use partial fractions (ii) After finding the values of A, B and C how will you evaluate the integral and write the final answer

5	We have I = $\int e^x\left(f(x)+f'(x)\right)dx = \int e^x\left(f(x)\right)dx + \int e^x\left(f'(x)\right)dx$ = $I_1+\int e^x\left(f'(x)\right)dx$ where $I_1+\int e^x\left(f(x)\right)dx$ ……. (1) Taking f(x) as first function and e^x as second function in I_1 and using integrating it by parts, we have I_1 = $f(x)e^x - \int e^x\left(f'(x)\right)dx + c$ Substituting I_1 in (1) we get I = $f(x)e^x - \int e^x\left(f'(x)\right)dx + \int e^x\left(f'(x)\right)dx = f(x)e^x + C$ Thus $\int e^x\left(f(x)+f'(x)\right)dx = f(x)e^x + C$ Based on the above information answer the following (i) Express the integral $\int e^x \sec x(1+\tan x)dx$ in the form $\int e^x\left(f(x)+f'(x)\right)dx$ (ii) What is f(x) and f'(x) (iii) What is the final answer

ANSWERS

INTEGRALS- CASE STUDY/ SOURCE-BASED INTEGRATED QUESTIONS

1	Evaluate $\int 2x\sin(x^2+2)dx$ (i) Let $x^2+2 = t$, (ii) differentiate with respect to t We get 2xdx = dt, Evaluate $sintdt$ =- cost +c = $-\cos(x^2+2)+c$

2	Evaluate $\int_2^3 \frac{xdx}{1+x^2}$ (i) Using substitution method Let $x^2+2 = t$, (ii) differentiate with respect to t We get 2xdx = dt, $\frac{1}{2}\int_2^3 \frac{dt}{t}$ = logt +c = $[\log(1+x^2)]_2^3$ = log10-log5 = log2 (iii) log2

3	Evaluate $\int \frac{dx}{(x+1)(x+2)}$ (i) Since it is a proper fraction, we can write partial fraction $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$, After taking L.C.M and simplifying we get 1 = A(x+2) +B(x+1), On comparing we get A+B =0,2A+B =1 On solving these equations, we get A = 1 and B = -1) (ii) A = 1 and B = -1 Therefore $\int \frac{dx}{(x+1)(x+2)} = \int \frac{1}{x+1} dx - \int \frac{1}{x+2} dx$ =log(x+1) – log (x+2) =log$\frac{x+1}{x+2}$ +c

4	Evaluate $\int \frac{(x^2+x+1)dx}{(x^2+1)(x+2)}$ (i) Since it is a proper fraction, we can write partial fraction $\frac{(x^2+x+1)}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$ After taking L.C.M and simplifying we get $X^2+x+1 = (Ax+B)(x+2) + C(x^2+1)$, On comparing we get A+B =1,2B+C=1, A+2C =1 On solving these equations, we get A = 3/5 and B = 2/5 and C = 1/5 (ii) A = 3/5 and B = 2/5 and C = 1/5 Therefore $\int \frac{(x^2+x+1)dx}{(x^2+1)(x+2)} = \int \frac{\frac{3x}{5}+\frac{2}{5}}{x^2+1} dx + \int \frac{1}{x+2} dx = \frac{3}{5}\log(x+2) - \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1}x + c$

5	Evaluate $\int e^x \sec x(1 + \tan x)dx$ (i) $\int e^x(\sec x + \tan x \sec x)dx$ (ii) F(x) = secx and f'(x) = secxtanx (iii) $e^x \sec x + c$

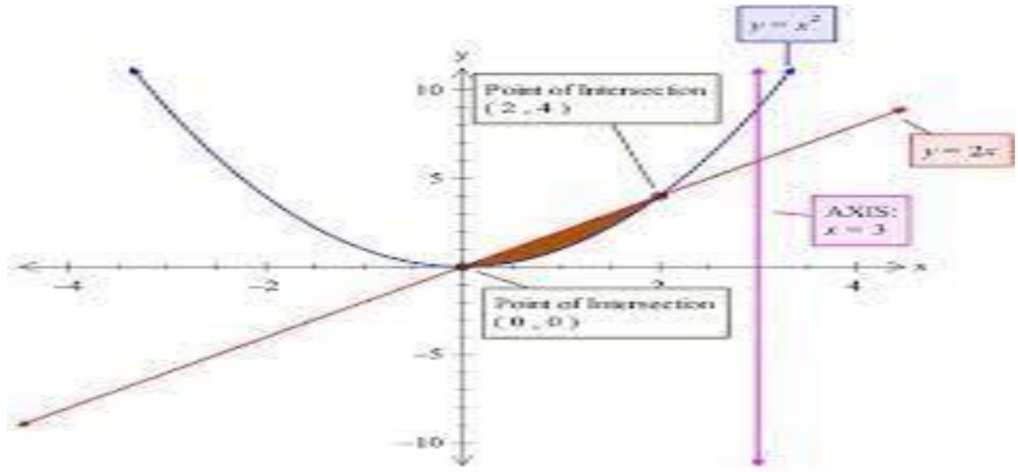
CHAPTER: APPLICATION OF INTEGRALS

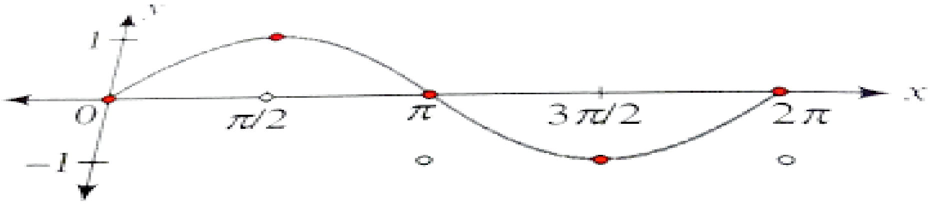
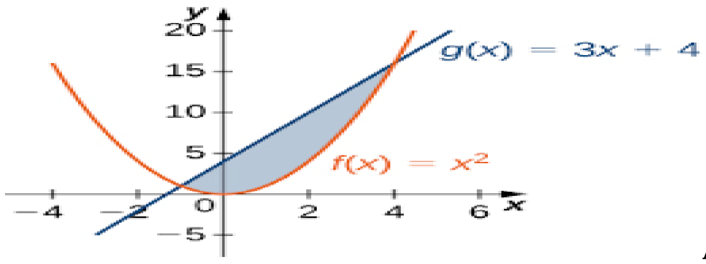
ASSERTION AND REASONING QUESTIONS

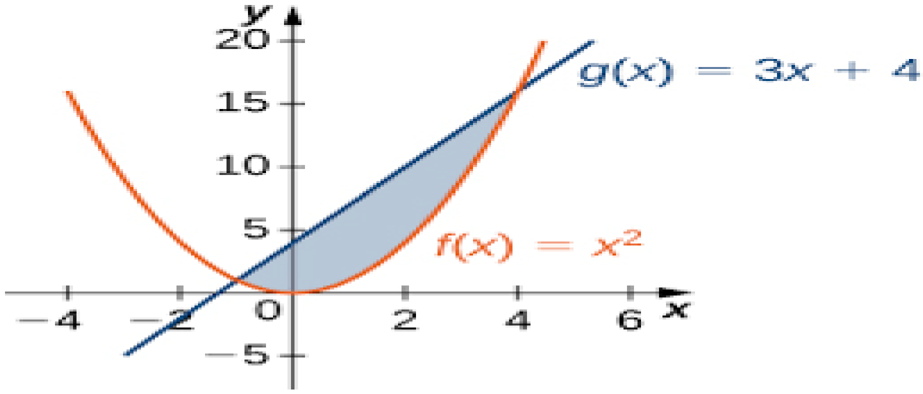
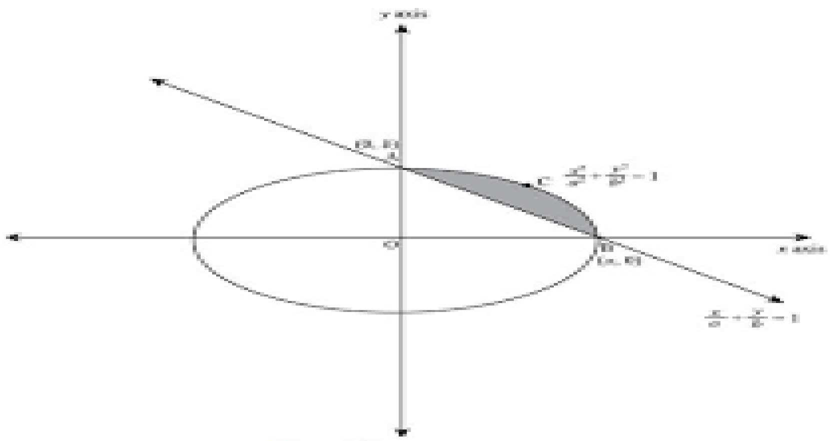
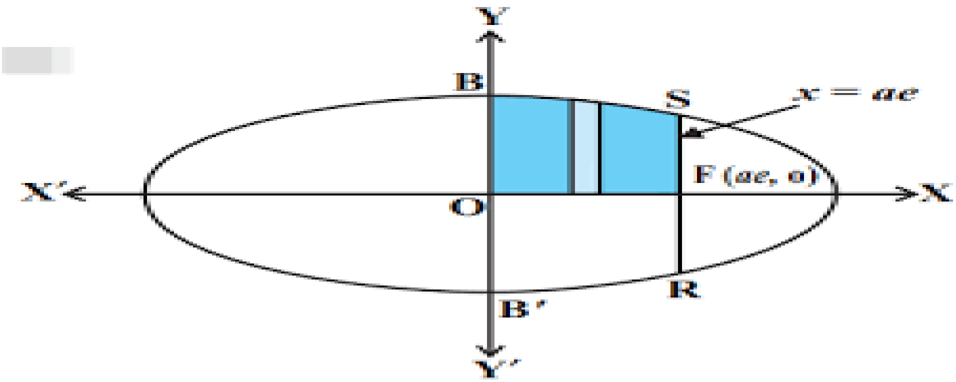
The following questions consist of two statements – Assertion (A) and Reason(R), Answer the questions selecting the appropriate option given below.

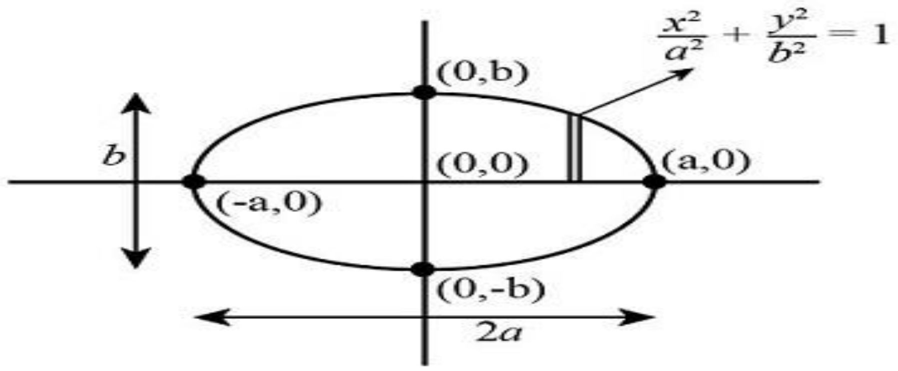
(a) Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true and R is not the correct explanation for A.
(c) A is true and R is false
(d) A is false and R is true

Q.1	**Assertion:** The area bounded by the curve y=Cos x in I quadrant with coordinate axes is 1 sq. unit. **Reason:** $\int_0^{\frac{\pi}{2}}$ Cos xdx =1
Q.2	**Assertion:** Area enclosed by the circle $x^2 + y^2 = 16$ is equal to 16π sq.unit. **Reason:** Area enclosed by circle $x^2 + y^2 = r^2$ is πr^2.
Q.3	**Assertion:** Area bounded by $y = \|x - 1\|$ from x= -2 to x= 0 is 4 sq.unit. **Reason:** $y = \|x -1\|$ is differentiable in R.
Q.4	**Assertion:** The area of the region bounded by y= cos x and the ordinates $x = 0$ *and* $x = \pi$ *is* $2 sq.unit$ **Reason:** cos x is an increasing function in the first quadrant.
Q.5	**Assertion:** Area of the region bounded by the parabola x^2=4y, y=1, y=4 and x= 0 is $\int_1^4 x\ dy$. **Reason:** Area under a curve x=f(y) and right of y-axis lying between the ordinates y=a and y=b is given by $\int_a^b f(y)\ dy$.
Q.6	**Assertion:** Area of the region given by {(x,y): $y^2 \leq 6x$, $2 \leq x \leq 5$, x, $y \geq 0$} $= \int_2^5 \sqrt{6x}$ dy. sq.units. **Reason:** Area under a curve x=f(y) lying to the right of y-axis and between the lines y=a and y=b is given by $\int_a^b f(y)\ dy$.
Q.7	**Assertion:** The area of the region bounded by y= sin x and the ordinates $x = 0$ *and* $x = \pi$ *is* $2 sq.unit$ **Reason:** sin x is an increasing function from $x = 0$ to $x = \pi$.
Q.8	**Assertion:** Area of the region bounded by the parabola x^2=y and y=2x is $\int_0^2 2x\ dx - \int_0^2 x^2\ dx$ **Reason:**

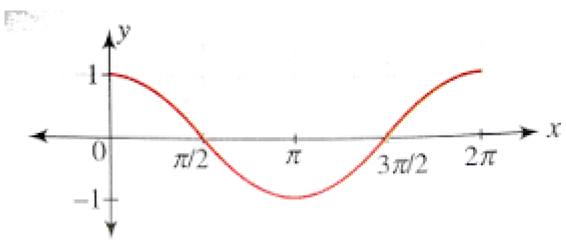
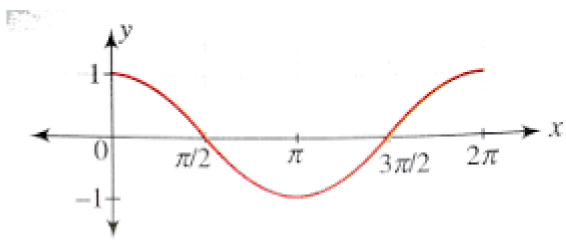
	
Q.9	**Assertion:** Area of the region bounded by the parabola y^2=4x, x=1, x=4 and y=o is $\frac{56}{3}$sq.units. **Reason:** Area under a curve y=f(x) and above x-axis lying between the ordinates x=a and x=b is given by $\int_a^b$ f(x) dx
Q.10	**Assertion:** The area bounded by the curve y=sin x in I quadrant with coordinate axes is 1 sq. unit. **Reason:** $\int_0^{\frac{\pi}{2}}$ sin x dx =1
Q.11	**Assertion:** The area bounded by y^2= 4x and y=x is $\frac{8}{3}$ sq.units. **Reason:** The area bounded by y^2= 4ax and y=mx is ($8a^2/3m^3$) sq. units.
Q.12	**Assertion:** The area enclosed by the circle $x^2 + y^2 = a^2$ is πa^2. **Reason:** The area enclosed by the circle with centre origin and radius a is $4\int_a^b x$ dy.
Q.13	**Assertion:** The area of the region bounded by the curve $y=x^2$ and the line y=4 is $\frac{3}{32}$. **Reason:** Area of the bounded by x=f(y) and the line y=4 is $2\int_0^4 x$ dy.
Q.14	**Assertion:** The area of the region bounded by y = x+1 , x-axis and the lines x =2 and x=3 is 5/ 2 sq.units **Reason:** The intercept made by the line y = x+1 on the coordinate axes are 1 unit left and 1 unit above of origin respectively.
Q.15	**Assertion:** The area bounded by the curve x =y^2 , y-axis and the lines y =3 and y= 4 is 37/ 3 **Reason:** Area bounded by the curve y=f(x) , -axis and the lines y =3 and y= 4 =$\int_3^4$ f(y) dy.

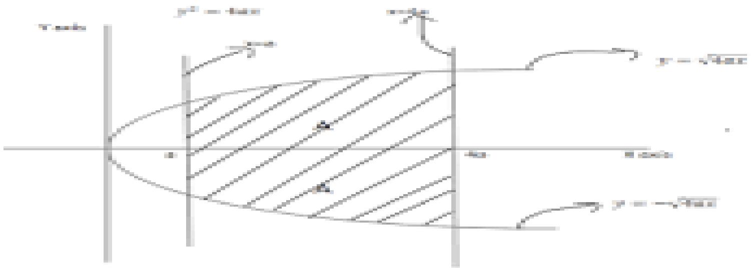
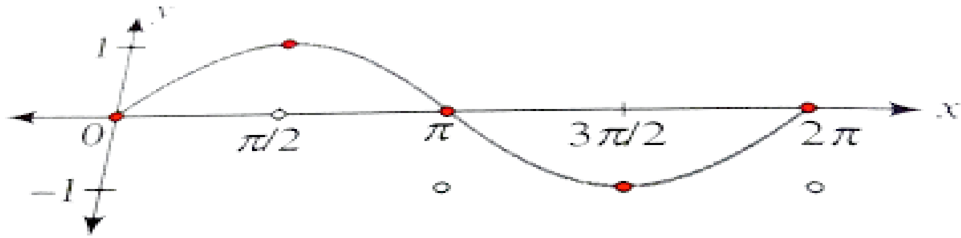
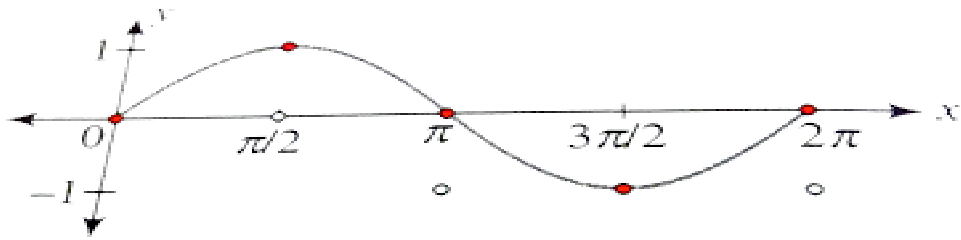
Q.16	**Assertion:** Area bounded by $y = \|x + 2\|$ from x= -2 to x= 0 is 2 sq.unit. **Reason:** $y = \|x + 2\|$ is differentiable in R.
Q.17	**Assertion:** The region bounded by the y =$\sqrt{2^2 - x^2}$is a semicircle above the x-axis. **Reason:** Area of the semicircle y =$\sqrt{2^2 - x^2}$ is half of the area bounded by the equation $x^2 + y^2 = 4$.
Q.18	**Assertion:** Area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant is given by $\int_0^a \sqrt{a^2 - x^2}$ dx. **Reason:** The area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant is $\int_0^a \sqrt{a^2 - y^2}$ dy
Q.19	**Assertion:** The area bounded by the curve y=-Sin x from x=0 to x=2π is 3 sq.unit. **Reason:** The area bounded by the curve y=-Sin x and x axis  In the figure is $4\int_0^{\frac{\pi}{2}} \sin x \, dx$
Q.20	**Assertion:**  **Area is $\int_{-1}^{4}[g(x) - f(x)]$dx** **Reason: If f(x) > g(x) , for all x∈ (a,b) then area bounded by these two curves is $\int_a^b [f(x) - g(x)]$dx**
Q.21	**Assertion:** Area enclosed by $y = x\|x\|$, x-axis and the ordinates $x = -1$ *and* $x = 1$ is given by $\frac{2}{3}$ **Reason:** f(x) = $\|x\|$ = x , x≥0 and –x , x< 0.
Q.22	**Assertion:** Area of bounded region between y=x^2 and y = 3x+4 in first quadrant is $\int_0^4 (3x + 4)$ dx - $\int_0^4 x^2$ dx **Reason:**

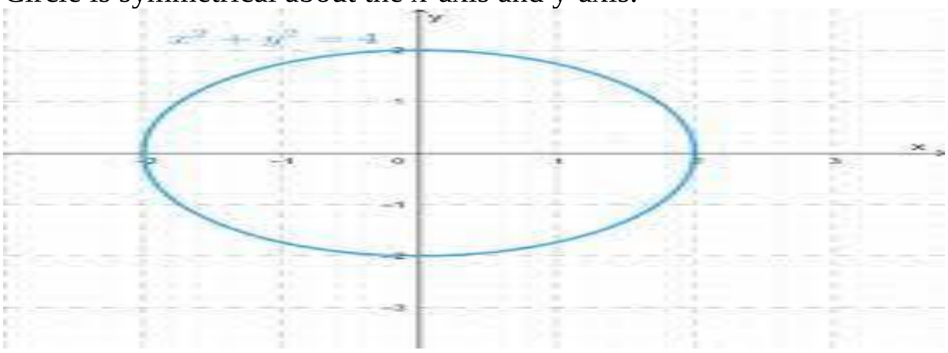
	
Q.23	**Assertion:** Area of minor region bounded by ellipse $b^2x^2 + a^2y^2 = b^2a^2$ and $bx+ay=ab$ is($\pi ab - \frac{1}{2}ab$) in the first quadrant.. **Reason:**  Figure (1)
Q.24	**Assertion:** Area enclosed between ellipse and latus rectum in first quadrant is $\int_0^{ae} x\, dy$ **Reason:** 
Q.25	**Assertion:** Area of ellipse $b^2x^2 + a^2y^2 = b^2a^2$ in first quadrant is $\int_0^{a} y\, dx$ and in second quadrant is $\int_0^{b} x dy$

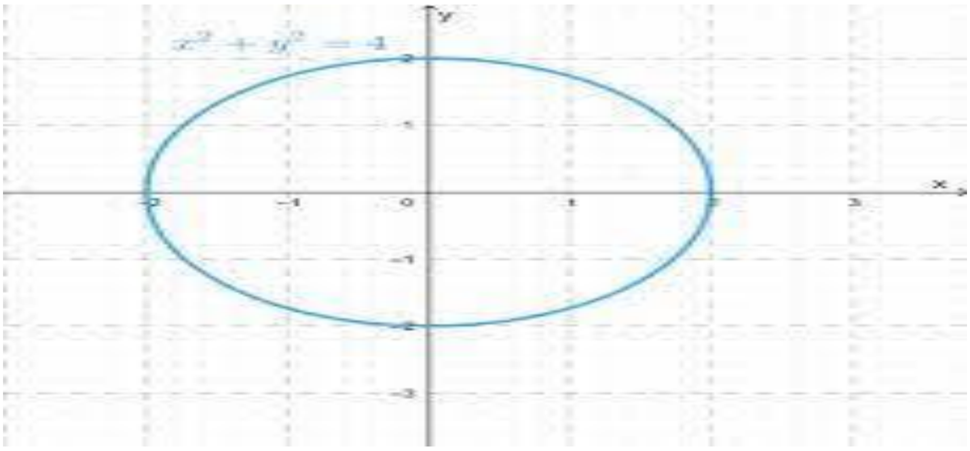
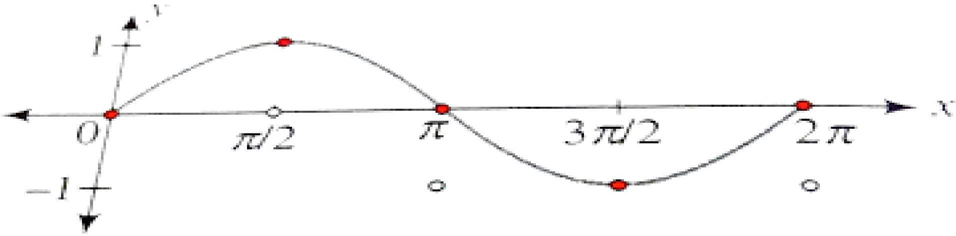
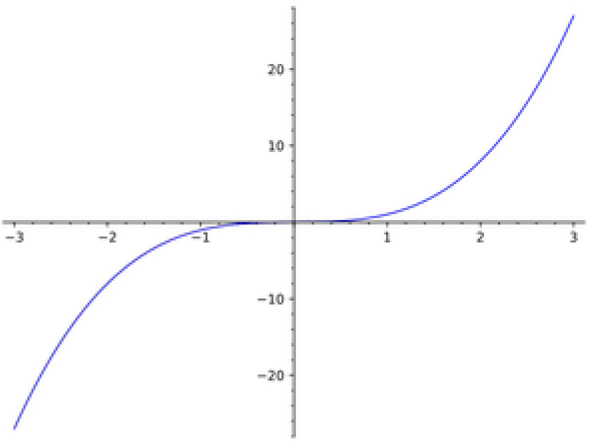
	Reason: 

SOLUTIONS OF
ASSERTION AND REASON QUESTIONS

Q.1	Both A and R are true Area $=\int_0^{\frac{\pi}{2}} \text{Cos } x \ \ dx = (\sin \boldsymbol{\pi}/2 - \sin 0) = 1.$  R is true.	**(a)**
Q.2	Area of circle is πr^2 and r=4 so area is 16π. Both A and R are true and R is the correct explanation for A.	**(a)**
Q.3	Assertion is true Area=$\int_{-2}^{0} -(x-1)\ dx = 4$ sq.unit. $\|x - 1\|=-(x-1)$ when x< 1. But reason is false.$y = \|x - 1\|$ is not differentiable at x=1. A is true and R is false	**(c)**
Q.4	 Area=$\int_0^{\pi/2} \cos x\ dx + \int_{\pi/2}^{\pi} -\cos x\ dx = (1-(-1)) = 2$ Assertion is true. Reason is false cos x is a decreasing function in the first quadrant. A is true and R is false	**(c)**

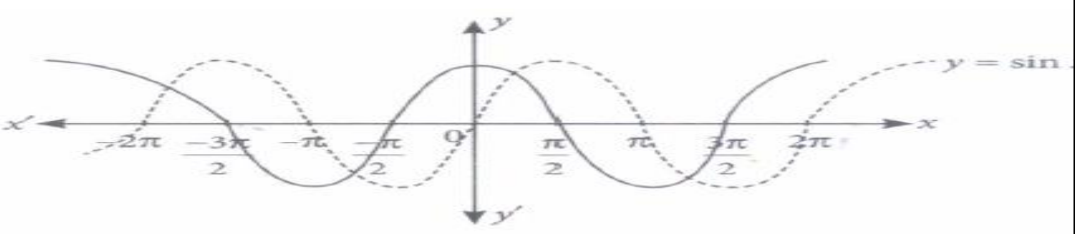
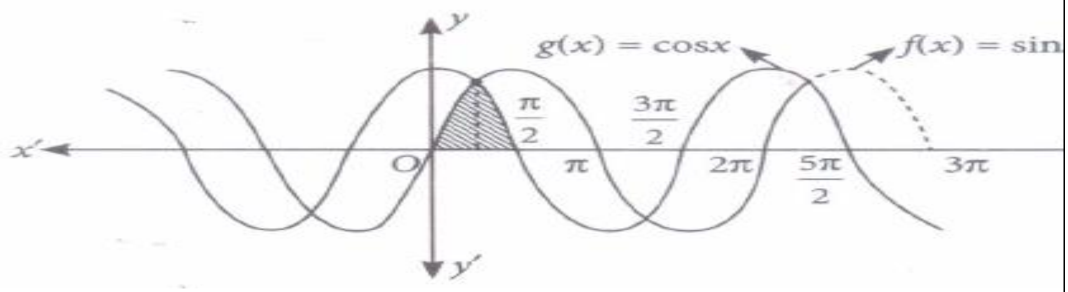
Q.5	A is false and R is true Assertion is false Area=$2\int_1^4 \sqrt{4y}$ dy because parabola $x^2=4y$ is symmetrical about y-axis. Reason is true.	**(d)**
Q.6	 Upper of x-axis. Both A and R are true and R is the correct explanation for A.	**(a)**
Q.7	 Area=$\int_0^{\pi}$ sin x dx=2 sq.unit. Assertion is true. But reason is false because sin x is not increasing from 0 to $\pi/2$ and decreasing from $\pi/2$ to π. A is true and R is false	**(c)**
Q.8	Both A and R are true and R is not the correct explanation for A.	**(a)**
Q.9	Assertion is true Area=$2\int_1^4 \sqrt{4x}$ dx=$\frac{56}{3}$ Reason is true and R is the correct explanation for A. Both A and R are true and R is the correct explanation for A.	**(a)**
Q.10	A is false and R is true Assertion is false because sin x is periodic function but reason is true.  Area = $\int_0^{\frac{\pi}{2}}$ sin x dx=-(cos π/2- cos 0)=1.	**(d)**
Q.11	Assertion and reason both are correct Area bounded by $y^2=4ax$ and y=mx is ($8a^2/3m^3$) sq. units. (General formula)	**(a)**

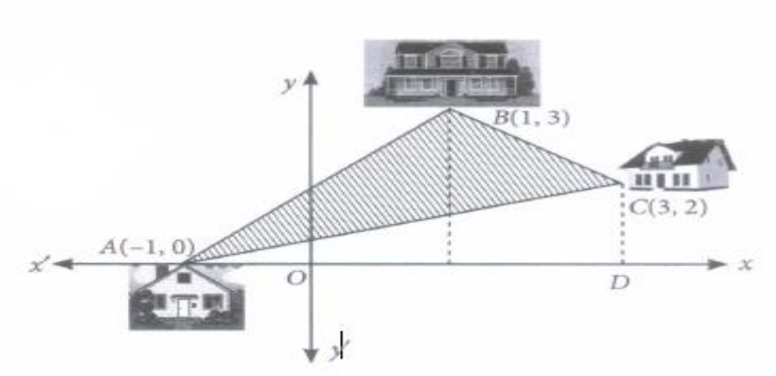
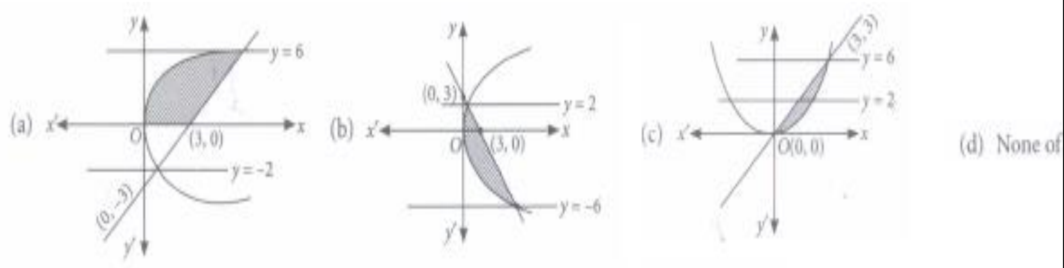
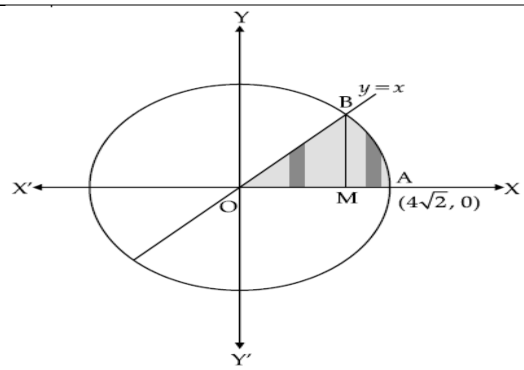
	Reason is a particular case when a=1 and m=1. Both A and R are true and R is the correct explanation for A.	
Q.12	Both A and R are true and R is the correct explanation for A. Circle is symmetrical about the x-axis as well as the y-axis.	**(a)**
Q.13	A is false and R is true Assertion is false Area=2 $\int_0^4 \sqrt{y}$ dy= 16/3 $\neq \frac{3}{32}$ Reason is true. The curve y=x^2 is symmetrical about the y-axis.	**(d)**
Q.14	A is false and R is true Assertion is false Area=$\int_2^3 (x+1)\,dx = 7/2 \neq 5/2$ Reason is true.	**(d)**
Q.15	Both A and R are true and R is the correct explanation for A. Area=$\int_3^4 y2\,dy$=37/ 3	**(a)**
Q.16	Assertion is true Area=$\int_{-2}^{0}(x+2)\,dx = 2$ sq.unit. $\lvert x+2 \rvert$ = (x+2) when x$>$ -2 But reason is false.$y = \lvert x+2 \rvert$ is not differentiable at x= -2. A is true and R is false	**(c)**
Q.17	Both A and R are true and R is not the correct explanation for A. Circle is symmetrical about the x-axis and y-axis. 	**(b)**
Q.18	Both A and R are true and R is not the correct explanation for A. Circle is symmetrical about the x-axis and y-axis.	**(b)**

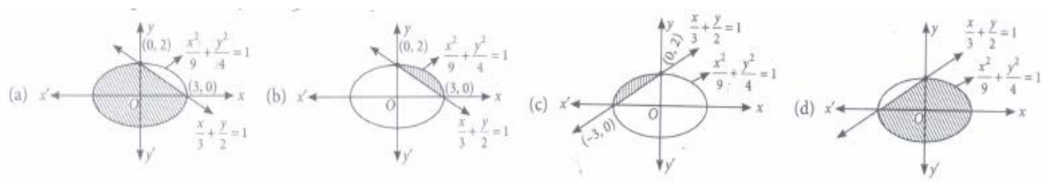
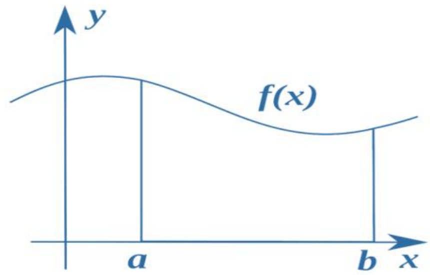
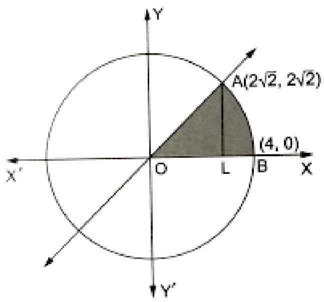
		
Q.19	A is false and R is true  Area=2 $\int_0^{\pi}$ Sin x dx = 4 sq.unit $\neq$ 3 sq.unit Assertion is false. Reason is true.	**(d)**
Q.20	Point of intersection of line and parabola are -1 and 4 Both A and R are true and R is the correct explanation for A.	**(a)**
Q.21	 Area=$-\int_{-1}^{0} -x^2 dx + \int_0^1 x^2\, dx = \frac{2}{3}$ sq.unit Assertion is true Reason is also true but R is not the correct explanation for A.	**(b)**
Q.22	Both A and R are true and R is the correct explanation for A.	**(a)**
Q.23	Both A and R are true and R is the correct explanation for A.	**(a)**

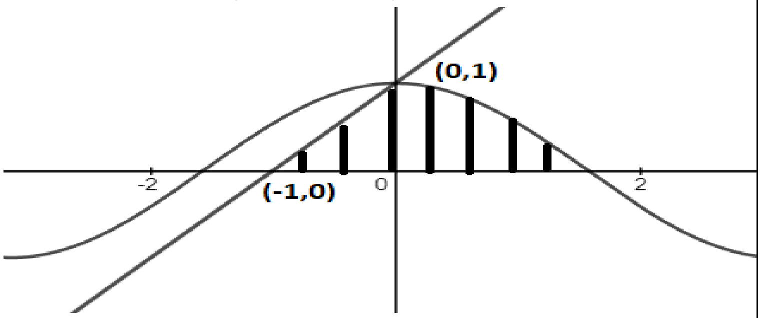
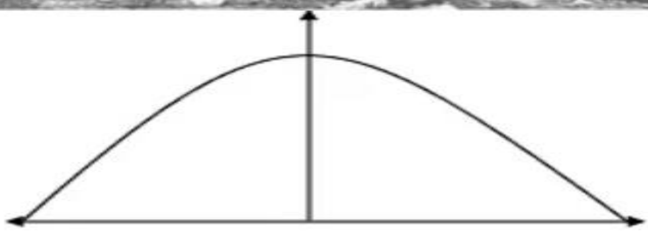
Q.24	A is false and R is true Assertion is false because Area $=\int_0^{ae} y$ dx. Reason is true	**(d)**
Q.25	Both A and R are true and R is the correct explanation for A.	**(a)**

CASE STUDY BASED QUESTIONS

1.	In a class room, teacher explains the properties of a particular curve by saying that this particular curve has beautiful up and downs. It starts at 1 and heads down until π radian, and then heads up again and closely related to sine function and both follow each other, exactly $\frac{\pi}{2}$ radians apart as shown in figure  Based on the above information, answer the following question a) Name the curve about which teacher explained in the classroom b) Area of curve explained in the passage from 0 to $\frac{\pi}{2}$ is c) Area of curve discussed in classroom from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ is or Area of curve discussed in classroom from 0 to 2 π
2	Graphs of two function f(x) =sin x and g(x) = cos x is given below.  Based on the above information, answer the following question a) In [0, π] the curves f(x) =sin x and g(x) = cos x intersect at x=? b) Value of $\int_0^{\frac{\pi}{4}} \sin x \; dx$ is c) Value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$ is **or** Value of $\int_0^{\pi} \sin x \; dx$ is

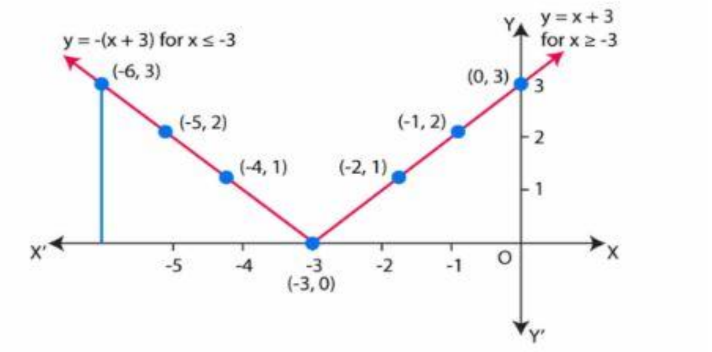
3	Location of three houses of a society is represented by the points A (-1, 0), B (1,3) and C (3,2), D(3,0) as shown in the figure. Based on the above information, answer the following question  a. Equation of line AB is b. Equation of line BC is c. Area of ΔABC is
4	Consider the following equation of curve y^2=4x, and straight line x + y= 3. Based on the above information, answer the following question i) The line x + y=3 cuts the x axis and y axis respectively at Or Points of intersection of two given curves is ii) Which of the following shaded portion represent the area bounded by given curves?  iii) Value of the integral $\int_{-6}^{2}(3-y)dy$
5	In the figure O (0, 0)is the centre of the circle . The line y=x meets the circle in the first quadrant at the point B. Based on the above information, answer the following question i. The equation of the circle is ii. The co-ordinate of B is iii. Find the area of shaded region 

6	A mirror in the shape of an ellipse is represented by $\frac{x^2}{9}$ $+\frac{y^2}{4}$=1 was hanging on the wall. Sanjeev and his daughter were playing with football inside the house, even his wife refused to do so. All of a sudden, football hit the mirror and got a scratch in the shape o line represented by $\frac{x}{3}+\frac{y}{2}$=1.  Based on the above information, answer the following question a. Find the point(s) of intersection of ellipse ant scratch (straight line). b. Area of smaller region bounded by the ellipse and line is represented by  c. Find the value of $\frac{2}{3}\int_0^3 \sqrt{9-x^2}$ dx
7	Area bounded by the curve y=f(x) , the x axis and between the ordinates at x=a and x= b is given by Area $=\int_a^b y\,dx$ $=\int_a^b f(x)\,dx$  Based on the above information, answer the following questions. 1. Draw the graph of f(x)= |x+3|, what does the value of the definite integral on the graph represents? 2. Find the value of $\int_{-6}^{0} \lvert x+3 \rvert$ dx
8	The sector of a circle bounded by the circle x^2+y^2=16 and the line y=x in the first quadrant  Based on the above information, answer the following questions. a. Point of intersection of both the given curve is b. The value of the integral $\int_0^{2\sqrt{2}} x$dx c. The value of the integral $\int_{2\sqrt{2}}^{4} \sqrt{(16-x^2)}$dx or Area bounded by the two given curve

9	Consider the following equations of curves y=cos $x$and y= x+1 . Based on the above information, answer the following questions.  a) The curves y=cos $x$and y= x +1 meet at b) Value of the integral $\int_0^{\frac{\pi}{2}} cosx\ dx$is c) The area bounded by y=x+1 and y=cos x and the x axis is
10	The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure   Based on the above information, answer the following question i. The equation of parabola designed on the bridge is ii. The value of integral $\int_{-50}^{50} \frac{x^2}{250}$dx is iii. The area formed between x^2 = 250y, y axis, y=0 and y=10 is

SOLUTIONS

1	Solution:1 a) Cosine b) Required Area $=\int_0^{\frac{\pi}{2}} \cos x\, dx$ $= [\sin\frac{\pi}{2} - \sin 0]$ =1 sq unit c) Required Area $=\left\lvert\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\, dx\right\rvert = \left\lvert \sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right\rvert$ =2 sq. Unit **or** Required Area $=\int_0^{2\pi} \cos x\, dx$ $= \int_0^{\frac{\pi}{2}} \cos x\, dx + \left\lvert\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\, dx\right\rvert +$ $\int_{\frac{3\pi}{2}}^{2\pi} \cos x\, dx$ = 1+2+1 = 4 sq.unit
2	Solution:2 a) For point of intersection, we have sin x = cos x , $\frac{sinx}{cosx}$=1 ,→ $\tan x = 1 \Rightarrow$ x= $\frac{\pi}{4}$ b) $\int_0^{\frac{\pi}{4}} sinx\ dx$= (-)[$\cos\frac{\pi}{4} - \cos 0] = 1$-$\frac{1}{\sqrt{2}}$ c) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cosx\ dx$= [$\sin\frac{\pi}{2} - \sin\frac{\pi}{4}] = 1$- $\frac{1}{\sqrt{2}}$ **or** $\int_0^{\pi} \sin x\ \ dx$ =(-)[$\cos\pi - \cos 0] = 2$
3	Solution:3 a. Equation of line AB is y= $\frac{3}{2}$(x+1) b. Equation of line BC is y=$\frac{-1}{2}$x+$\frac{7}{2}$ c. Area of ΔABC = area of region ABCD-Area of ΔACD $=\int_{-1}^{1} \frac{3}{2}(x+1)\, dx + \int_1^3 (\frac{-X}{2} + \frac{7}{2}) dx - \int_{-1}^{3} \frac{(x+1)}{2} dx$ = 3+5-4 = 4 sq units
4	Solution:4 i. The line x + y=3 cuts the x axis and y axis at (3,0)and (0,3)respectively [since at x-axis, y=0 and y-axis ,x=0] Or Equation of curve y^2=4x, and x + y=3 y^2 =4 $(3 - y)$ Required point of intersection are (1, 2), (9,-6) ii. b. iii. $\int_{-6}^{2} (3 - y) dy$= 40

5	Solution :5 i. equation of the circle is $x^2 + y^2 = 32$ ii. co-ordinate of B is (4,4) iii. Area of shaded region= Area of $\Delta OBM +$ Ar(BAMB) $=\int_0^4 xdx + \int_4^{4\sqrt{2}} \sqrt{(32 - x^2)}dx$ $=8 + 4\pi - 8$ $=4\pi\ sq\ units$
6	Solution :6 a. The point of intersection is (3,0),(0,2) b. b c. The value is = $\frac{2}{3}\int_0^3 \sqrt{9 - x^2}$ $=\frac{2}{3}(0+\frac{9}{2}\sin^{-1} 1\text{-}0)$ $= \frac{3\pi}{2}$
7	Solution : 7 1. y=$f(x) =$  $\begin{cases} -(x+3), & x+3 < 0 \\ x+3, & x+3 \geq 0 \end{cases}$ let's draw the graph is The value of integral represents the area bounded by the curve, the x ax and between ordinates at x=a, x=b 2. $\int_{-6}^{0} \|x + 3\|$ dx = $\int_{-6}^{-3} -(x + 3)$ dx + $\int_{-3}^{0}(x + 3)$ dx $= \frac{9}{2} + 9 + 0 + 0 + \frac{9}{2} - 9 = 9$ sq. Units

8	Solution:8 a. We have x^2+y^2=16 Point of intersection of $(2\sqrt{2},2\sqrt{2})$ b. $\int_0^{2\sqrt{2}} x$dx = 4 sq units c. $\int_{2\sqrt{2}}^{4}\sqrt{(16-x^2)}$dx=2($\pi$-2) Or Required area = area (OLA)+area(BAL)=2π sq units
9	Solution :9 d. The curves y=cos $x$and y=x+1 meet at (0,1) e. Value of the integral $\int_0^{\frac{\pi}{2}} cosx\ dx$ = (sin $\frac{\pi}{2}$ $-$sin 0) = 1 f. Area of the shaded region= $\int_{-1}^{0} x+1$ dx+$\int_0^{\frac{\pi}{2}} cosx\ dx$ =$\frac{3}{2}$ sq units
10	Solution:10 i. Parabola equation $x^2 = -4$ay It is given that height 10 feet is at the middle of the bridge. So a point on parabola is (50,-10). By putting point in above parabola equation. 50^2 = -4 a (-10) a=$\frac{125}{2}$ $hence\ required\ equation\ ,$ x^2 = - 250y ii. $\int_{-50}^{50}\frac{x^2}{250}$dx = $\frac{1000}{3}$ sq units iii. $\int_0^{10} 2\sqrt{(250y)}\ dy = \frac{2000}{3}$ sq unit

CHAPTER: DIFFERENTIAL EQUATIONS

ASSERTION REASONING BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true

1	**Assertion (A):** Order of a differential equation represents the number of independent arbitrary constants in the general solution. **Reason (R):** Degree of a differential equation represents the number of family of curves. **Answer:** B **Theoretical explanation is required**
2	**Assertion (A):** The order of the differential equation whose general solution is $y = c_1 cosx + c_2 sin^2x + c_3 e^{2x+c_4}$ is 3. **Reason (R):** The total number of arbitrary parameters in the given general solution in the assertion is 4.
3	**Assertion (A): Assertion (A)**: The differential equation of the family of curve represented by $y = Ae^x$ is given by $\frac{dy}{dx} = y$. **Reason (R):** $\frac{dy}{dx} = y$ is valid for every member of the given family.
4	**Assertion (A):** The degree of the differential equation $\frac{d^3y}{dx^3} + 3(\frac{dy}{dx}) = x^2 \log(\frac{d^2y}{dx^2})$ is not defined. **Reason (R):** If the differential equation is a polynomial in terms of its derivatives, then its degree is defined.
5	**Assertion (A):** Order of differential equation $(\frac{dy}{dx})^3 + \frac{d^2y}{dx^2} = 5x$ is 1 **Reason (R):** Order of the differential equation is the order of the highest order derivative present in the equation
6	**Assertion (A):** The differential equation $(\frac{dy}{dx})^3 + 2y^{\frac{1}{2}} = \boldsymbol{x}$ is of order 1 and degree 3 **Reason (R):** The order and degree of differential equation $\frac{d^3y}{dx^3} = \sqrt{\frac{dy}{dx} + 5}$ are 3 and 1 respectively
7	**Assertion (A):** Degree of differential equation**: x- cos**$(\frac{dy}{dx})$**=0** is 1. **Reason (R):** Differential equation **x-cos**$(\frac{dy}{dx})$**=0** can be converted in the polynomial equation of derivative.
8	**Assertion (A):** The solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ with x=1 and y=1 is x = y **Reason (R):** Separation of variable method can be used to solve the differential equation.
9	**Assertion (A):** The differential equation $\frac{dy}{dx} = \frac{x + \sqrt{y^2 - x^2}}{x}$ is homogeneous equation.

	Reason (R): $f(\lambda x, \lambda y) = \lambda^n f(x, y)$ for homogeneous equation.
10	**Assertion (A):** $\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} + \tan x = 0$ is not a linear differential equation **Reason (R):** A differential equation is said to be linear if dependent variable and its differential coefficients occur in first degree and are not multiplied together.
11	**Assertion (A):** $\frac{dy}{dx} = \frac{x^2 - xy + y^2}{x^2 - xy}$ is a homogeneous differential equation. **Reason (R):** The function $F(x, y) = \frac{x^2 - xy + y^2}{x^2 - xy}$ is homogeneous
12	**Assertion (A):** $\frac{dy}{dx} + x^2y = 2x$ is a first order linear differential equation **Reason (R):** If P and Q are functions of x only or constant then differential equation of the form $\frac{dy}{dx} + \boldsymbol{Py} = \boldsymbol{Q}$ is a first order linear differential equation
13	**Assertion (A):** If p and q are the degree and order of differential equation $(\frac{d^3y}{dx^3})^2 + \frac{d^2y}{dx^2} = 4$ respectively, then p = 2, and q = 3. **Reason (R):** and 2p – 3q = -5
14	**Assertion (A):** General solution of the differential equation $\log(\frac{dy}{dx}) = 2x + y$ is $-e^{-y} = \frac{1}{2} e^{2x} + C$. **Reason (R):** Degree of differential equation $\log(\frac{dy}{dx}) = 2x + y$ is 1.
15	**Assertion (A):** The differential equation $\frac{dy}{dx} = 1 + \frac{y}{x}$ is homogenous differential equation. **Reason (R):** For a homogenous equation $\frac{dy}{dx} = f(\frac{y}{x})$.
16	**Assertion (A):** To solve the differential equation $\frac{dy}{dx} = \sin(x + y)$, we first substitute x + y= t **Reason (R):** if x + y = t, then $\frac{dy}{dx} = \frac{dt}{dx} - 1$.
17	**Assertion (A):** $g(x,y) = xy^{\frac{1}{2}} + yx^{\frac{1}{2}}$ is a homogeneous function of degree $\frac{3}{2}$. **Reason (R):** A function is called homogeneous function of degree n if $h(\lambda x, \lambda y) = \lambda^n h(x, y)$, where $\lambda \neq 0$.
18	**Assertion (A):** The solution of the differential equation $\frac{dy}{dx} + y = 1$, with y=0 at x = 0 is $y = 1 - e^{-x}$ **Reason (R):** The given differential equation is a linear differential equation.
19	**Assertion (A):** The solution of the equation $3y \frac{dy}{dx} + 4x = 0$ represents family of ellipses. **Reason (R):** Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
20	Consider the differential equation $(xy-1) \frac{dy}{dx} + y^2 = 0$ **Assertion (A):** The solution of the equation is xy = logy + C

	Reason (R): The given differential equation can be expressed as $\frac{dx}{dy} + Px = Q$, whose integrating factor is log y.
21	A curve C has the property that its initial ordinate of any tangent drawn is less than the abscissa of the point of tangency by unity. **Assertion (A):** Differential equation satisfying the curve is linear **Reason (R):** Degree of differential equation may be one
22	**Assertion (A):** The differential equation $y^3\,dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$. **Reason (R):** All differential equation of first order and first degree becomes homogeneous if we put $y = tx$.
23	**Assertion (A):** The solution of $ydx - xdy + y^2\,dx = 0$ is $\frac{x}{y} + x = c$ **Reason (R):** $d\left(\frac{x}{y}\right) = \frac{(ydx - x.\,dy)}{y^2}$
24	**Assertion (A):** The equation of the curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$. **Reason (R):** The solution of differential equation $\frac{d^2y}{dx^2} - y = 0$ is $y = c_1e^x + c_2e^{-x}$
25	**Assertion (A):** The Integrating factor of the given differential equation $\boldsymbol{\frac{dy}{dx} - \frac{2y}{x} = 2x^2\ is\ \frac{1}{x^2}}$ **Reason (R):** The integrating factor of linear differential equation $\boldsymbol{\frac{dy}{dx} + Py = Q\ is\ e^{\frac{dP}{dx}}}$ where P and Q are functions of x or constants

ANSWERS & EXPLANATION

1.	**Answer:** B **Theoretical explanation is required**
2.	**Answer :** C $$y = c_1cosx + c_2sin^2x + c_3e^{2x}\ \ e^{c_4}$$ Where c_3 and e^{c_4} are considered as one constant, then equation becomes $$y = c_1cosx + c_2sin^2x + ke^{2x}$$ Clearly, it has 3 arbitrary constants
3.	**Answer : (a)** $y = Ae^x$, where A is arbitrary constant So, $\frac{dy}{dx} = Ae^x = y$ For different values of A, the curve belongs to the same family
4.	**Answer:** (a)

	The given DE can't be expressed in a polynomial in terms of its derivatives and its should not be associated with the logarithmic function.
5.	**Answer:** (d) Here, the highest order derivative present in the given DE is 2 and so its order is 2
6.	**Answer: (c)** The given DE should free from radical sign and expressed in a polynomial in terms of its derivatives. The differential equation $\frac{d^3y}{dx^3} = \sqrt{\frac{dy}{dx} + 5}$ Squaring on both the sides, then $$\left(\frac{d^3y}{dx^3}\right)^2 = \left(\sqrt{\frac{dy}{dx} + 5}\right)^2$$ $$\left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{dy}{dx} + 5\right)^2$$ **Order=3 and Degree=2**
7.	**Answer: (a)** The given DE is x- cos($\frac{dy}{dx}$)=0 $$-cos\left(\frac{dy}{dx}\right) = -x$$ $$cos\left(\frac{dy}{dx}\right) = x$$ $$\left(\frac{dy}{dx}\right) = \cos^{-1} x$$ If the differential equation is a polynomial in terms of its derivatives, then its degree is defined and it is one.
8.	**Answer:** (a) $\frac{dy}{dx} = \frac{y}{x}$ By applying, Separation of variable method $$\frac{dy}{y} = \frac{dx}{x}$$ Applying integral on both the sides, then logy=log x + log c y=cx , then with x=1 and y=1, the solution is x = y.
9.	**Answer:** (a) $$\frac{dy}{dx} = \frac{x + \sqrt{y^2 - x^2}}{x}$$ $$f(x,y) = \frac{x + \sqrt{y^2 - x^2}}{x}$$ $$then\ f(kx,ky) = \frac{kx + \sqrt{(ky)^2 - (kx)^2}}{kx} = k^0\left(\frac{x + \sqrt{y^2 - x^2}}{x}\right) = f(x,y)$$

	If the function is homogeneous then the given DE is a homogeneous differential equation.
10.	**Answer: (a)** **Theoretical explanation is required**
11.	**Answer:** (a) $$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{x}^2-\mathrm{xy}\ +\ \mathrm{y}^2}{\mathrm{x}^2-\mathrm{xy}}$$ $$f(x,y)=\frac{\mathrm{x}^2-\mathrm{xy}\ +\ \mathrm{y}^2}{\mathrm{x}^2-\mathrm{xy}}$$ $$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{x}^2-\mathrm{xy}\ +\ \mathrm{y}^2}{\mathrm{x}^2-\mathrm{xy}}$$ $$f(x,y)=\frac{\mathrm{x}^2-\mathrm{xy}\ +\ \mathrm{y}^2}{\mathrm{x}^2-\mathrm{xy}}$$ $$then\ f(kx,ky)\ =\frac{(\mathrm{kx})^2-(\mathrm{kx})(\mathrm{ky})\ +\ (\mathrm{ky})^2}{(\mathrm{kx})^2-(\mathrm{kx})(\mathrm{ky})}=k^0\left(\frac{\mathrm{x}^2-\mathrm{xy}\ +\ \mathrm{y}^2}{\mathrm{x}^2-\mathrm{xy}}\right)=f(x,y)$$ If the function is homogeneous then the given DE is a homogeneous differential equation.
12.	**Answer:** (a), The given DE can be expressed in the form of $\frac{\mathrm{dy}}{\mathrm{dx}}$ + $\boldsymbol{Py}$ = $\boldsymbol{Q}$ where P=x^2 and Q=2x
13.	**Answer:** (b) Here, the highest order derivative is 3 and its power is 2 Therefore, Order=q=3 and degree=p=2.
14.	**Answer: (b)** The given DE is log($\frac{\mathrm{dy}}{\mathrm{dx}}$)=2x+y expressed in the form of $\frac{dy}{dx}=e^{2x+y}=e^{2x}\ e^{y}$ Clearly, it is polynomial in differential coefficients. So Degree is one By applying variable and separable method $e^{-y}dy=e^{2x}dx$ Apply integral on both the sides $\int e^{-y}dy=\int e^{2x}dx$ Required solution is $-\mathrm{e}^{-\mathrm{y}}=\frac{1}{2}\ \mathrm{e}^{2\mathrm{x}}+\mathrm{C}$.
15.	**Answer:** (a) **Theoretical Explanation is required.**
16.	**Answer:** (a) The given DE can be solved by substituting x + y= t (reducible to variable and separable method) only.
17.	**Answer:** (d) Let $g(x,y)=xy^{\frac{1}{2}}+yx^{\frac{1}{2}}$ For homogeneous function, we have to write $g(kx,ky)=(kx)(ky)^{\frac{1}{2}}+(ky)(kx)^{\frac{1}{2}}$

	$= kk^{\frac{1}{2}}\left(xy^{\frac{1}{2}}\right) + kk^{\frac{1}{2}}\left(yx^{\frac{1}{2}}\right)$ $= k^{\frac{3}{2}}(xy^{\frac{1}{2}} + yx^{\frac{1}{2}})$ $= k^{\frac{3}{2}}\, g(x, y)$
18.	**Answer:** (a) The solution of the differential equation $\frac{dy}{dx} + y = 1$ **It is in the form of** $\frac{dy}{dx} + Py = Q$ **Then, I.F=**$\boldsymbol{e^{\int 1\, dx} = e^{x}}$ **Solution is y**$\boldsymbol{e^{x} = \int 1.e^{x}\, dx + C}$ **y**$\boldsymbol{e^{x} = e^{x} + C}$ when y=0 at x = 0 then C=-1 and the solution is y = 1-e^{-x}
19.	**Answer:** (a) The given differential equation 3y $\frac{dy}{dx}$ + 4x = 0 can be expressed as 3y dy=-4x dx Apply integral on both the sides, we have $$\frac{3y^2}{2} = \frac{-4x^2}{2} + c$$ $4x^2 + 3y^2 = C$ represents family of Ellipses
20.	**Answer: (c)** $$\frac{dy}{dx} = \frac{-y^2}{xy - 1}$$ $$\frac{dx}{dy} = \frac{1 - xy}{y^2}$$ $$\frac{dx}{dy} = \frac{1}{y^2} - \frac{xy}{y^2}$$ $$\frac{dx}{dy} + \frac{1}{y}x = \frac{1}{y^2}$$ Hence, P=$\frac{1}{y}$ and Q=$\frac{1}{y^2}$ Then, I.F.=$e^{\int \frac{1}{y} dy} = e^{log y} = y$ The solution is xy=$\int \frac{1}{y^2} X\, y\, dy + c$ xy=logy+c
21.	**Answer: (b)** We know that, equation of tangent $Y - y_1 = m(X - x_1)$ $at\ the\ point$ (x_1, y_1) By using the condition initial ordinate i.e., X=0 then it becomes $Y - y_1 = m(-x_1)$ $Y = y_1 - mx_1$

	According to the question, $y_1 - mx_1 = x_1 - 1$ $$mx_1 - y_1 = 1 - x_1$$ Replace the variables and m=$\frac{dy}{dx}$ $x\frac{dy}{dx} - y = 1 - x$ $\frac{dy}{dx} + \left(\frac{-1}{x}\right)y = \frac{1-x}{x}$
22.	**Answer: (c)** $The\ given\ DE\ can\ be\ written\ as\ \frac{dy}{dx} = \frac{-(x+y^2)}{y^3}$ Substitute $y^2 = t$ and $2y\frac{dy}{dx} = \frac{dt}{dx}$ Hence, the given DE can be written as $\frac{dt}{dx} = \frac{-2(x+t)}{t}$
23.	**Answer: (a)** y dx – xdy + y^2 dx= 0 $= \frac{ydx - xdy}{y^2}$ = –1dx ⇒ $d(\frac{x}{y})$ = -1dx ⇒ $\frac{x}{y} + x = c$
24.	**Answer: (b)** Given DE is $\frac{dy}{dx} = x + \frac{1}{x^2}$ Integrating on both the sides $y = \frac{x^2}{2} - \frac{1}{x} + c\ by\ substituting\ x = 3\ and\ y = 9\ then\ c = \frac{29}{6}$ The required solution is $y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$ after simplification $6xy = 3x^3 + 29x - 6$. $y = c_1e^x + c_2e^{-x}$ $\frac{dy}{dx} = c_1e^x - c_2e^{-x}$ $\frac{d^2y}{dx^2} = c_1e^x + c_2e^{-x}$ $\frac{d^2y}{dx^2} = y$
25.	**Answer: (C)** The given differential equation $\boldsymbol{\frac{dy}{dx} - \frac{2y}{x} = 2x^2}$ and compared with $\frac{dy}{dx} + Py = Q$ then we have P=$\frac{-2}{x}$ and Q=$2x^2$ Then, I.F.= $e^{\int\frac{-2}{x}dx} = e^{-2logx} = e^{logx^{-2}} = x^{-2} = \frac{1}{x^2}$

CASE BASED QUESTIONS

1	**Case Study Question 1:** A police cruiser, approaching a right-angled intersection from north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 km. north of the intersection and the car is 0.8 km to the east, the police determine with radar that the distance then and the car is increasing at 20 km/h. Suppose that the cruiser is moving at 60 km/h at the instant of measurement. **Q.1** If s is the distance between car and cruiser at time t, x = position of car at time t and y position of cruiser at time t then find the velocity of the cruiser. **Q.2** Find the moment i.e., $\frac{d^2s}{dt^2}$ if $\frac{d^2x}{dt^2} = -40$ and $\frac{d^2y}{dt^2} = 50$ (in km/h^2)
2	**Case Study Question 2:** A spherical drop of liquid evaporates at a rate proportional to its surface area, if the radius initially is 5mm and 5 minute later the radius is reduced to 2mm. **On the basis of above information, answer the following questions-** **Q.1** Write the differential equation for the above situation. **Q.2** Find the rate with which the surface area changes after 5 minutes. **Q.3** Find the rate of evaporation after, 5 minutes
3	**Case Study Question 3:** Differential equation $\frac{dy}{dx}$=f(x) g(y) can be solved by separating variable $\frac{dy}{g(y)}$ = f(x) dx. **On the basis of above information, answer the following questions:** **Q.1** Find the equation of the curve to the point (1, 0) which satisfies the differential equation $(1 + y^2)\,dx - xy\,dy = 0$. **Q.2** Find the solution of the differential equation $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0$ **Q.3** If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then find the equation of the curve.
4	**Case Study Question 4:** A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their

	inlets are closed and the water if released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is release, the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B. Let V_A and V_B represents volume of reservoir A and B at any time t, then: **On the basis of above information, answer the following questions:** **Q.1** If after 1/2 an hour $V_A = kV_B$, then find the value of k. **Q.2** After how many hours do both the reservoirs have the same quantity of water?
5	**<u>Case Study Question 5:</u>** For certain curves $y = f(x)\ satisfying\ \frac{d^2y}{dx^2} = 6x - 4$ and f(x) has local minimum value 5 when x=1. **On the basis of above information, answer the following questions:** Q.1 Find the number of critical points for $y = f(x)\ for\ x \in [0,2]$ Q.2 Find the Global minimum value of $y = f(x)\ for\ x \in [0,2]$ Q.3 Find the Global maximum value of $y = f(x)\ for\ x \in [0,2]$

ANSWERS

1. Solutions:

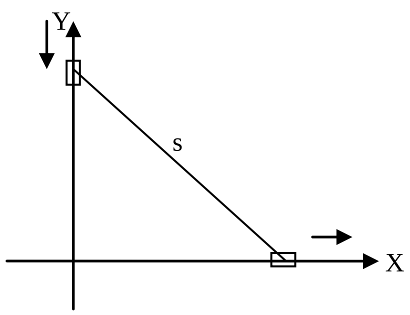
1. **[C]** At the instant in question x = 0.8, y = 0.6,

$$\frac{dy}{dt} = -60 \text{ km/h}$$

$$\frac{ds}{dt} = 20 \text{ km/h},\ s^2 = x^2 + y^2$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{ds}{dt} = \frac{1}{\sqrt{x^2+y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) \qquad \text{... (i)}$$



2. Sol.[B] Differentiating eqn. (i) $\frac{d^2s}{dt^2} = \frac{1}{\sqrt{x^2+y^2}}$

	$$\left[\left(1-\frac{x}{x^2+y^2}\right)\left(\frac{dx}{dt}\right)^2+\left(1-\frac{y}{x^2+y^2}\right)\left(\frac{dy}{dt}\right)^2\right.$$ $$\left.+x\frac{d^2x}{dt^2}+y\frac{d^2y}{dt^2}-\frac{2xy}{x^2+h^2}\frac{dx}{dt}\frac{dy}{dt}\right]$$ Putting the given value $\frac{d^2s}{dt^2}=6450.$
2.	**Solutions:** 1. $\frac{dv}{dt}==-k(4\pi r^2)\ldots..(1)$ where $0<k\in R$ 2. 4.8 π sq. mm/min 3. 9.6 π
3.	**Solutions:** **1.** $x^2 - y^2 = 1$ **2.** $\tan^{-1} y + \sin^{-1} x = c$ **3.** $e^{\frac{(1-x)^2}{2}}-1.$
4.	**Solutions:** **1.** $\sqrt{3}$ **2.** $\log_{4/3} 2$ hrs.
5.	**Solutions:** **1. 2** **2. 5** **3. 7**

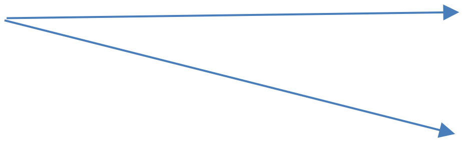
CHEPTER: VECTOR ALGEBRA

ASSERTION & REASONING QUESTIONS

Instructions

In the following questions , a statement of assertion (A) is followed by a statement of Reason (R) . Choose the correct answer out of the following choices

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

1	A : The vectors in the figure are coinitial vectors .  R: Two or more vectors having the same initial point are called coinitial vectors.
2	A: The vectors $3\hat{\imath}+2\hat{\jmath}-4\hat{k}$ are $3\hat{\imath}-2\hat{\jmath}-4\hat{k}$ are collinear vectors. R: Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.
3	A: The vectors $3\hat{\imath}+2\hat{\jmath}-4\hat{k}$ and $3\hat{\imath}+2\hat{\jmath}-4\hat{k}$ are equal vectors. R: Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points.
4	A:The vector $\hat{\imath}+\hat{\jmath}-\hat{k}$ is a unit vector. R: A vector whose magnitude is unity (i.e., 1 unit) is called a unit vector.
5	A:The vector $-2\hat{\imath}-2\hat{\jmath}-2\hat{k}$ is a scalar multiple of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ R: A vector whose magnitude is unity (i.e., 1 unit) is called a unit vector.
6	A:For the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$, a set of direction ratios are $1,-2,-2$ R: If $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$, then a_1 , a_2 , a_3 are also called direction ratios of $\vec{a}$
7	A: The vector $\frac{\hat{\imath}-2\hat{\jmath}-2\hat{k}}{3}$ is a unit vector in the direction of the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$ R: The unit vector in the direction of $\vec{a}$ is given by $\frac{\vec{a}}{\lvert\vec{a}\rvert}$
8	A: The direction cosines of the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$ are $1,-2,-2$ R: The magnitude r, direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l=\frac{a}{r}$, $m=\frac{b}{r}$, $n=\frac{c}{r}$
9	A: The direction cosines of the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$ are $\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$ R: : The magnitude r, direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l=\frac{a}{r}$, $m=\frac{b}{r}$, $n=\frac{c}{r}$

10	A: The vector joining the points P(2, 3, 0) and Q(– 1, – 2, – 4) is $\overrightarrow{PQ} = -3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$ R: If P (x_1, y_1, z_1) and Q(P (x_2, y_2, z_2)) are any two points, then magnitude of the vector $\overrightarrow{PQ}$ is given by $\left\|\overrightarrow{PQ}\ \right\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
11	A: The mid point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2) is R(3,2,1). R: The position vector of the point R which divides P and Q internally in the ratio of m : n is given by $\overrightarrow{OR} = \frac{m\vec{b}+n\vec{a}}{m+n}$
12	A: $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$ are perpendicular to each other R: Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a} \times \vec{b} = \vec{0}$
13	A: $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ are perpendicular to each other R: Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a}.\vec{b} = 0$
14	A:If $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$, then projection of $\vec{a}$ on $\vec{b}$ is 4. R: Projection of the vector $\vec{a}$ on $\vec{b}$ is given by $\vec{a}.\hat{b}$.
15	A: If $\vec{a} = \hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 3\hat{\imath} + \hat{k}$, then $\|\vec{a}\| = \left\|\vec{b}\right\|$. R: Two vectors are equal if their magnitudes are equal.
16	A: The angle between the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is $\frac{2\pi}{3}$. R: Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a}.\vec{b} = 0$
17	A: The angle between the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is $\frac{2\pi}{3}$. R: The angle between two vectors $\vec{a}$ and $\vec{b}$ is given by $\cos^{-1}\left(\frac{\vec{a}.\vec{b}}{\|\vec{a}\|\ \|\vec{b}\|}\right)$
18	A: The area of the parallelogram whose adjacent sides are $\hat{\imath} + \hat{k}$ and $2\hat{\imath} + \hat{\jmath} + \hat{k}$ is $\sqrt{3}$. R: Area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}\left\|\vec{a} \times \vec{b}\right\|$
19	A: The vector in the direction of the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 9 units is $3\left(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}\right)$. R: The vector in the direction of the vector $\vec{a}$ that has magnitude k units is $k\vec{a}$.
20	A: Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\|\vec{a}\| = 3$, $\left\|\vec{b}\right\| = \frac{\sqrt{2}}{3}$ and angle between them is $\frac{\pi}{4}$, then $\vec{a} \times \vec{b}$ is a unit vector. R: If $\vec{a}$ and $\vec{b}$ are two vectors then $\left\|\vec{a} \times \vec{b}\right\| = \|\vec{a}\|\left\|\vec{b}\right\| sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.
21	A: The vectors $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} - 6\hat{\jmath} + 6\hat{k}$ are parallel vectors. R: Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a}.\vec{b} = 0$
22	A: If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, then $\|\vec{a}\| = \left\|\vec{b}\right\|$ R: If $\|\vec{a}\| = \left\|\vec{b}\right\|$, then it necessarily implies that $\vec{a} = \pm\vec{b}$.
23	A: The value of $\hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{\imath} \times \hat{k}\right) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$ is 1. R: Dot product and cross product are commutative .
24	A: : If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ then $\vec{a}.\vec{b} = -9$ R: If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

25	A: $\left(\hat{\imath}-2\hat{\jmath}+2\hat{k}\right)\times\left(\lambda\,\hat{\imath}-2\lambda\hat{\jmath}+2\lambda\hat{k}\right)=\vec{0}$ R: Two vectors $\vec{a}$ and $\vec{b}$ are parallel to each other if $\vec{a}\times\vec{b}=\vec{0}$

ANSWERS

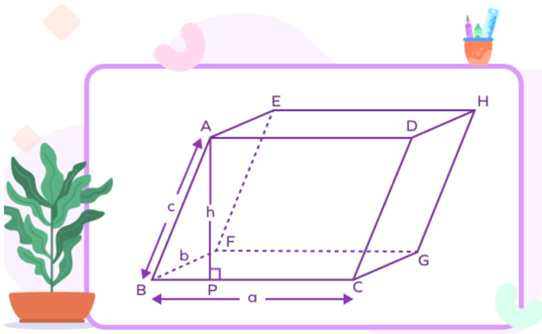
1	Ans: a) Solution : As the given vectors are starting from same point they are coinitial vectors
2	Ans: d) Solution: The given vectors $3\hat{\imath}+2\hat{\jmath}-4\hat{k}$ are $3\hat{\imath}-2\hat{\jmath}-4\hat{k}$ are not collinear.
3	Ans: a) Solution: The vectors $3\hat{\imath}+2\hat{\jmath}-4\hat{k}$ and $3\hat{\imath}+2\hat{\jmath}-4\hat{k}$ are equal vectors if they have the same magnitude and direction
4	Ans: d) Solution: Magnitude of $\hat{\imath}+\hat{\jmath}-\hat{k}$ is $\sqrt{3}$
5	Ans: b) Solution: $-2\hat{\imath}-2\hat{\jmath}-2\hat{k}=-2(\hat{\imath}+\hat{\jmath}+\hat{k}\,)$ R is also true, but R is not correct explanation of A.
6	Ans: a) Solution : Both A and R are true statements.
7	Ans: a) Solution: : Both A and R are true statements
8	Ans: d) Solution: The direction ratios of the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$ are $1,-2,-2$
9	Ans: a) Solution: Both A and R are true statements.
10	Ans:b) Solution: Both A and R are true statements.
11	Ans: a) Solution: Both A and R are true statements.
12	Ans: c) Solution: $\vec{a}.\vec{b}=0$, therefore $\vec{a}$ and $\vec{b}$ are perpendicular to each other
13	Ans: d) Solution : $\vec{a}.\vec{b}$ =10 , $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a}.\vec{b}=0$
14	Ans: a) Solution: Projection of $\vec{a}=\hat{\imath}+3\hat{\jmath}+4\hat{k}$ on $\vec{b}=2\hat{\imath}+2\hat{\jmath}+\hat{k}$ is 4
15	Ans: c) Solution: Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points
16	Ans : b) Solution: Both A and R are true statements, but R is not the correct explanation of A.
17	Ans: a) Solution: Both A and R are true statements
18	Ans: c)

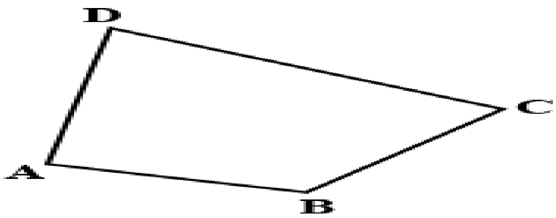
	Solution: Area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $\lvert\vec{a}\times\vec{b}\rvert$
19	Ans: c) Solution: The vector in the direction of the vector $\vec{a}$ that has magnitude k units is $k\hat{a}$
20	Ans: a) Solution: Since $\lvert\vec{a}\times\vec{b}\rvert = 1$, $\vec{a}\times\vec{b}$ is a unit vector
21	Ans: b) Solution: Both A and R are true statements , but R is not the correct explanation of A
22	Ans: c) Solution: $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert$ need not imply $\vec{a} = \pm\vec{b}$, e g : $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$
23	Ans: c) Solution: $\hat{\imath}.(\hat{\jmath}\times\hat{k}) + \hat{\jmath}.(\hat{\imath}\times\hat{k}) + \hat{k}.(\hat{\imath}\times\hat{\jmath}) = \hat{\imath}.\hat{\imath} + \hat{\jmath}.(-\hat{\jmath}) + \hat{k}.\hat{k} = 1$
24	Ans: a) Solution: $\vec{a}.\vec{b} = -9$
25	Ans: a) Solution: $(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) \times (\lambda\hat{\imath} - 2\lambda\hat{\jmath} + 2\lambda\hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 2 \\ \lambda & -2\lambda & 2\lambda \end{vmatrix} = \vec{0}$

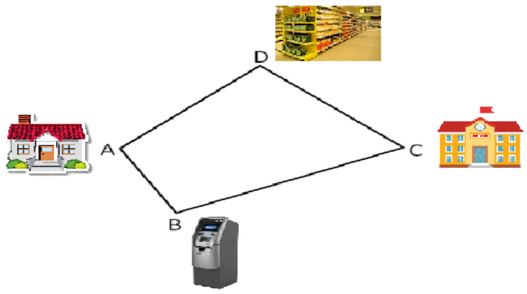
CASE STUDY QUESTIONS

Q.NO.	
1	A monkey starts moving from a point A(0,0,0) to 5 meters away a point B in east direction then 6 meters in north direction to a point C and then climbs a tree of 10 meters to a point D.  From above information answer the questions given below: (i) Find position vector of monkey. (ii) Find displace of monkey. (iii) Find unit vector of vector $\overrightarrow{AD}$,
2	Two toddlers are playing seesaw game in a garden. A student of XII class observing the positions of toddlers and finds the position of first

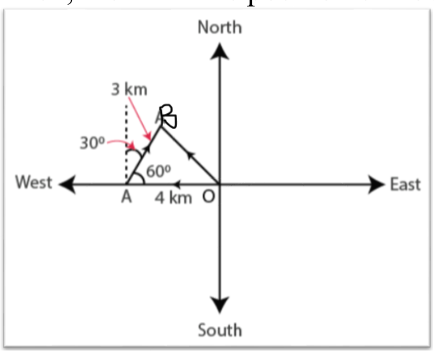
	toddler (3,4,2) and position of second toddler (5,6,0).  From above information answer the questions given below: (i) Find position vector of midpoint of seesaw. (ii) Find distance between both toddlers (iii) Find position vector of point on seesaw which divides seesaw beam in 2:1 .
3.	Three birds are sitting on tree at positions A(4,6,8) , B (6, 7 ,7) , C(5,6, 9) .A student of class XII wants apply vector algebra concept to find different component of triangle.  Solve the problem that he finds in following questions. (i) Find vector $\overrightarrow{AB}\ and\ \overrightarrow{AC}$. (ii) Find centroid of triangle. (iii) Find angle between vector $\overrightarrow{AB}\ and\ \overrightarrow{AC}$.
4.	. A student of class XII wants to find displace of a particle using the formula $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ and $\vec{a} = \frac{\vec{F}}{m}$ where $\vec{u} = 2\hat{\imath}\ m/s\ \ and\ mass\ of\ particle\ is\ 2\ kg.$ Force on the particle are as (newton unit)

	$\overrightarrow{F_1} = 2\vec{\imath} + 3\vec{\jmath} - \vec{k}$, $\overrightarrow{F_1} = 2\vec{\imath} + 2\vec{\jmath} - 3\vec{k}$  From the above informations solve the following questions (i) Find net force on object. (ii) Find acceleration of object . (iii) Find displacement in 2 second.
5.	There is a 9 meters tree in a field. Shape of field is in parallelogram whose two adjacent side are given $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = -2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.  From above informations solve following questions (i) Find area of field. (ii) Find height in vector form.
6.	Three adjacent side of a parallelopiped are represented by three vectors as shown in figure  $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$, $and\ \vec{c} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ From above information solve following questions (i) Find $\vec{a} \times \vec{b}$ (ii) Find projection of $\vec{c}$ on $\vec{a} \times \vec{b}$

	(iii) Find height h of parallelopiped.
7.	. A garden is in shape of quadrilateral. A student wants to finds its area by using vector algebra. points are given as A(0,1,1), B(2,3,-2), C(22,19,-5), and D(1,-2,1).  From above information solve following questions (i) Find $\overrightarrow{AC}$. (ii) Find $\overrightarrow{BD}$. (iii) Using this find area of quadrilateral.
8.	A student is observing the launch of Chandrayan- 3. He assumed a frame Of reference that is x-axis, y-axis and z-axis . he finds the launching pad A(5,10,0) in km units . The Chandranan 3 starts with net acceleration 5km/s^2 and he uses the formula $\vec{h} = 1/2\vec{a}t^2$.  Solve the following questions. (i) Find height of Chandrayan after 10 second. (ii) Find position vector of Chandrayan- 3 in 10 second from. (iii) What is angle of elevation by student if height of student neglected.
9.	. Tiranga point A(0,0,0) and Shivashakti point B(25,2,0)on moon are very important place for India’s unity and integrity. If Pragyan Rover is at a instant to a point C(0,1/2.0).  From these information solve the following questions (i) Find $\overrightarrow{AB}$.

	(ii) Find $\overrightarrow{AC}$. (iii) Find area of triangle ABC .
10.	Ritika starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to a ATM, from there to her daughter's school and then reaches the mall. In the diagram, A(1,1,1),B(-2,4,1),C(-1,5,5) and D(2,2,2)  are shown. From this information solve the following questions (i) Find distance between House A and ATM (B). (ii) Find distance between ATM (B) and school (C).
11	Anitha walks 4km towards west from her home and reaches her friend Geetu's house. Then together they walk 3km in a direction 30^0 east of north and reaches school .  Based on the above information, answer the following questions.
	Find Anitha's displacement from her house to Geetu's house?
	Find Geetu's displacement from her house to school?
	Find Anitha's displacement from her house to school?

	ANSWERS
1.	**SOLUTION:** Let X axis in east direction Y axis in north direction and Z axis in above direction (i) $\overrightarrow{AD} = 5\hat{\imath} + 6\hat{\jmath} + 10\hat{k}$ (ii) $\lvert\overrightarrow{AD}\rvert = \sqrt{25+36+100} = \sqrt{161}\, meter$. (iii) $\widehat{AD} = \frac{5\hat{\imath}+6\hat{\jmath}+10\hat{k}}{\sqrt{161}}$.
2.	**Solutions:** (i) $4\hat{\imath} + 5\hat{\jmath} + \hat{k}$. (ii) $\sqrt{(2)^2+(2)^2+(-2)^2} = \sqrt{12} = 2\sqrt{3}$ units (iii) $\vec{r} = \frac{13\hat{\imath}+16\hat{\jmath}+2\hat{k}}{3}$.
3.	**SOLUTION :** (i) $\overrightarrow{AB} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, $\overrightarrow{AC} = \hat{\imath} + \hat{k}$. (ii) (5,19/3,8) . (iii) $\cos^{-1}\frac{1}{\sqrt{12}}$.
4.	SOLUTION: (i) $\vec{F} = \vec{F_1} + \vec{F_2} = 4\hat{\imath} + 5\hat{\jmath} - 4\hat{k}$ (ii) $\vec{a} = \frac{\vec{F}}{m} = 2\hat{\imath} + 5/2\hat{\jmath} - 2\hat{k}$ (iii) $\vec{s} = 8\hat{\imath} + 5\hat{\jmath} - 4\hat{k}$.
5.	SOLUTION: (i) Area= $\lvert\vec{a}\times\vec{b}\rvert = \lvert-\hat{\imath}+2\hat{\jmath}+2\hat{k}\rvert$ =3 (ii) $9\frac{(-\hat{\imath}+2\hat{\jmath}+2\hat{k})}{3}$
6.	SOLUTION: (i) $\vec{a}\times\vec{b} = -5\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ (ii) Projection $=(\hat{\imath}+\hat{\jmath}+3\hat{k}).\frac{-5\hat{\imath}+3\hat{\jmath}+2\hat{k}}{\sqrt{38}} = \frac{4}{\sqrt{38}}$. (iii) Height= $\frac{4}{\sqrt{38}}$
7.	**SOLUTION:** (i) $\overrightarrow{AC} = 22\hat{\imath} + 18\hat{\jmath} - 6\hat{k}$ (ii) $\overrightarrow{BD} = -\hat{I} - 5\hat{J} + 3\hat{K}$ (iii) AREA= $\frac{1}{2}\lvert\overrightarrow{AC}\times\overrightarrow{BD}\rvert = \frac{1}{2}\sqrt{12940}$
8.	**SOLUTION:** (i) $\vec{h} = \frac{1}{2}5\times10^2\,\hat{k}$ or h= 250 km. (ii) $\vec{r} = (5\hat{\imath} + 10\hat{\jmath}\,) + 250\hat{k}$

	(iii) $\theta = \tan^{-1} 10\sqrt{5}$
9.	**Solution:** (i) $\overrightarrow{AB} = 25\hat{\imath} + 2\hat{\jmath}$ (ii) $\overrightarrow{AC}$=1/2$\hat{J}$. (iii) $Area = 25/2\hat{k}$.
10.	**Solution:** (i) $32\sqrt{2}$. (ii) $32\sqrt{2}$.
11	It is given that Let O be the position of Anitha’s house and B be the position of i Geetu’s house nd final positions of the girl, respectively. Then, the girl’s position can be shown as **Solution:** It is given that Let O and B be the initial and final positions of the girl, respectively. Then, the Anitha’s position can be shown as 
(i)	$\overrightarrow{OA}$ = - $4\hat{\imath}$
(ii)	$\overrightarrow{AB}$ = $\lvert AB\rvert\cos 60^0\,\hat{\imath} + \lvert AB\rvert\sin 60^0\,\hat{\jmath}$ $= \frac{3}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$
(iii)	$\overrightarrow{OB}$ = $\overrightarrow{OA} + \overrightarrow{AB}$ $= \frac{-5}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

CHAPTER: THREE DIMENSIONAL GEOMENTRY

ASSERTION & REASON QUESTIONS

Instructions

In the following questions , a statement of assertion (A) is followed by a statement of Reason (R) . Choose the correct answer out of the following choices

(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

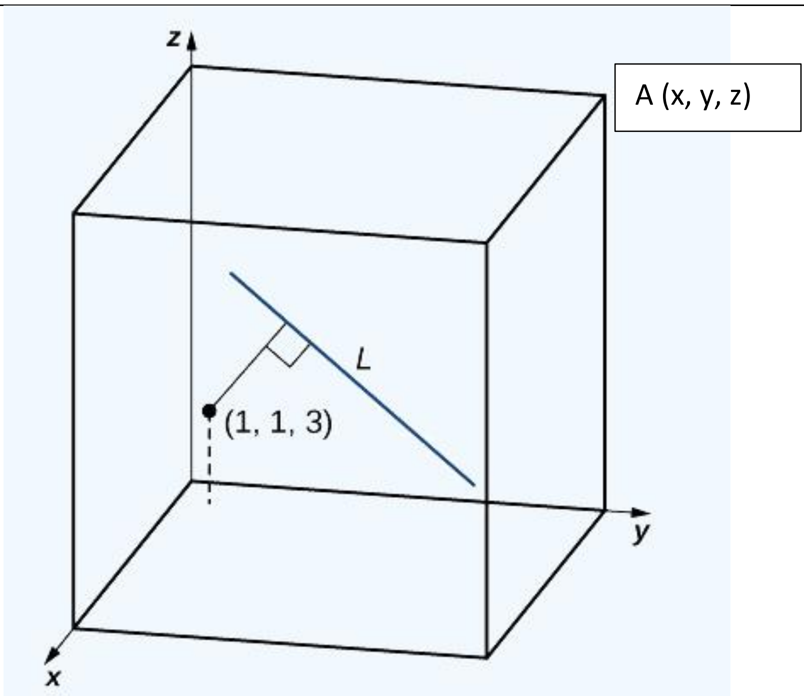
1	A line makes angle $\alpha, \beta and\ \gamma$ and with the X, Y & Z axes respectively . ASSERTION (A): $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$ REASON(R) : $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$
2	ASSERTION (A): The distance of the point P(a,b,c)` from the x-axis is $\sqrt{b^2+c^2}$ REASON(R) :Any point in the Y- axis is of the form (0,y,0)
3	ASSERTION (A): If the x-coordinate of a point P on the join of Q(2,2,1) and R(5,1,-2) is 4, then its z-coordinate is -1 REASON(R) :Equation of a line joining 2 points A(x_1,y_1,z_1) & B(x_2,y_2,z_2) is $\frac{x+x_1}{x_1+x_2} = \frac{y+y_1}{y_1+y_2} = \frac{z-z_1}{z_1-z_2}$
4	A line makes angle $\alpha, \beta and\ \gamma$ and with the X, Y & Z axes respectively ASSERTION: The direction ratios of x- axis are 0,0,1 REASON: The X- axis makes angles with coordinate axes are 0^o, 90^o &90^o respectively
5	A line makes the same angle θ with each of the x and z-axes and β with y-axis. ASSERTION : If $\sin^2\beta = 3\sin^2\theta\ then\ \cos^2\theta = \frac{3}{5}$ REASON: $cos^2\theta + cos2\beta + cos2\theta = 1$
6	**ASSERTION**: a,b, c are the direction ratios of a line p then the direction ratios of a line parallel to the line p is a,2b,3c **REASON:** Any three numbers which are proportional to the direction cosines of a line are called the *direction ratios* of the line.
7	ASSERTION: If a line has direction ratios 2, – 1, – 2, its direction cosines. are $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$ **REASON**: If *l*, *m*, *n* are direction cosines and *a*, *b*, *c* are direction ratios of a line, then $a = \lambda l$, $b= \lambda m$ and $c = \lambda n$, for any nonzero $\lambda \in R$
8	ASSERTION: The direction cosines of the line passing through the two points P(– 2, 4, – 5) and Q(1, 2, 3) are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$ **REASON:** The direction ratios of the line segment joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) may be taken as $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

9	**<u>ASSERTION:</u>** If a line makes angles 90°, 135°, 45° with the *x*, *y* and *z*-axes respectively, then its direction cosines are 0, $\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **<u>REASON:</u>** If a line makes angle $\alpha, \beta and\ \gamma$ and with the X, Y & Z axes respectively then its direction cosines are sin α,sinβ and $sin\gamma$
10	<u>ASSERTION:</u> The direction cosines of a line which makes equal angles with the coordinate Axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ **<u>REASON:</u>** $sin^2\alpha + sin^2\beta + sin^2\gamma = 1$
11.	ASSERTION: The points A (2, 3, – 4), B (1, – 2, 3) and C (3, 8, – 11) are collinear REASON: The direction ratios of AC are <1,5,-7>
12	<u>ASSERTION:</u> **If O is the origin, OP = 3 with direction ratios proportional to -1, 2, -2 then the coordinates of P are (-1/9, 2/9, -2/9)** **<u>REASON:</u>** The direction ratios of the line segment joining P (x_1, y_1, z_1) and Q(x_2, y_2, z_2) may be taken as $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$
13	**<u>Assertion (A)</u>**: $\frac{x}{1} = \frac{y-3}{-2} = \frac{z}{2}$ and $\frac{x-4}{2} = \frac{y+7}{2} = \frac{z-1}{2}$ are perpendicular. **<u>Reason (R)</u>**: The direction rations of parallel lines are proportional.
14	**<u>Assertion (A)</u>:** The shortest distance between the lines $r = 8i - 9j + 10k + \lambda(3i - 16j + 7k)$ and $r = 15i + 29j + 5k + \mu(3i + 8j - 5k)$ is given by 14 units. **<u>Reason (R)</u>**: The distance between the parallel lines $r = a_1 + \lambda b$ and $r = a_2 + \lambda b$ is given by $S.D = \frac{\vert b\times(a_2-a_1)\vert}{\vert b\vert}$
15	**<u>Assertion (A)</u>**: The part of lines given by $r = i - j + \lambda(2i + k)$ and $r = 2i - k + \mu(i + j - k)$ intersect. **<u>Reason (R)</u>**: Two lines intersect each other if they are not parallel and shortest distance is zero.
16	**<u>Assertion (A)</u>**: Vector form of the equation of a line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{3-z}{-1}$ is $r = (2i + j + 3k).\lambda(3i + 2j + k)$. **<u>Reason (R)</u>**: Cartesian equation of a line passing through the points (2, 1, 3) are parallel to the line $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z-4}{-2}$ $is\ 2x - 4 = y - 1 = 3 - z$.
17	**<u>Assertion (A)</u>**: Skew lines are the lines in space which are neither parallel nor intersecting and lie in different planes. **<u>Reason (R)</u>**: Angles between skew lines is angle between two intersecting lines drawn from any point parallel to each of the skew lines.
18	**<u>Assertion (A)</u>**: The numbers $\frac{11}{113}, \frac{12}{113}, \frac{132}{133}$ may represent the direction cosines of a line in space. **<u>Reason (R)</u>**: Numbers a, b, c represents direction cosines of a line $a^2 + b^2 + c^2 = 1$
19	**<u>Assertion (A)</u>**: The point A (1, 0, 7) is the mirror image of the point B (1, 6, 3) in the line $\frac{x}{1} = \frac{y-11}{2} = \frac{z-2}{3}$ **<u>Reason (R)</u>**: The line $\frac{x}{1} = \frac{y-11}{2} = \frac{z-2}{3}$ bisects the line segment joining point A (1, 0, 7) and B (1, 6, 3)
20	**<u>Assertion (A)</u>**: The point A (2, 9, 12), B (1, 8, 8), C (-2, 11, 8), and D (-1, 12, 12) are the vertices of a rhombus.

	Reason (R): AB=BC=CD=DA and AC≠BD
21	**Assertion (A)**: The shortest distance between the skew line $\frac{x+3}{-4}=\frac{y-6}{3}=\frac{z}{2}$ and $\frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$ is 9 **Reason (R)**: Two lines are skew if there exist no plane passing through them.
22	**Assertion (A)**: Two lines $r=(i+j+2k)+\lambda(2i-j+3k)$ and $r=i+j+\mu(6i-3j+9k)$ are parallel. **Reason (R)**: Two lines are parallel if the vector product of their direction vectors is zero.
23	**Assertion (A)**: Three lines $\frac{2x-3}{4}=\frac{y-2}{3}=\frac{4-z}{-2}$, $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-1}{-2}$, $3-x=\frac{2-y}{1}=\frac{z-2}{3}$ are mutually perpendicular. **Reason (R)**: Two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$
24	**Assertion (A)**: The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is 90° **Reason (R)**: Skew lines are lines in different planes which are parallel and intersecting.
25	**Assertion (A)**: If a line makes $\alpha, \beta\ and\ \gamma$ with the axes respectively then $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-1$ **Reason (R)**: $l^2+m^2+n^2=1$

CASE STUDY QUESTIONS

Q1	A football match is organised between students of class XII of two schools, say school A and school B. For which a team from each school is chosen. Remaining students of class XII of school A and B are respectively standing in the lines L_1 and L_2 respectively represented by the equations $\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{r}=5\hat{i}-2\hat{j}+0\hat{k}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$  Based on the above information answer the following Questions: (i) Write the Cartesian equation for the lines L_1 and L_2 (ii) Write the direction ratios of the lines L_1 and L_2 (iii) Find the angle between the lines L_1 and L_2 (OR) (iv) Find the direction ratios of a line perpendicular to both the lines L_1 and L_2
Q2	Consider the following diagram. Answer the following Questions



The equation of given line L is $\frac{x-3}{4}=\frac{y+1}{2}=\frac{z-3}{1}$ and given point M is (1,1,3)

(i) Write the vector equation of the given line L
(ii) Find the direction cosines of the line L
(iii)find the direction ratios of the line MA

(OR)

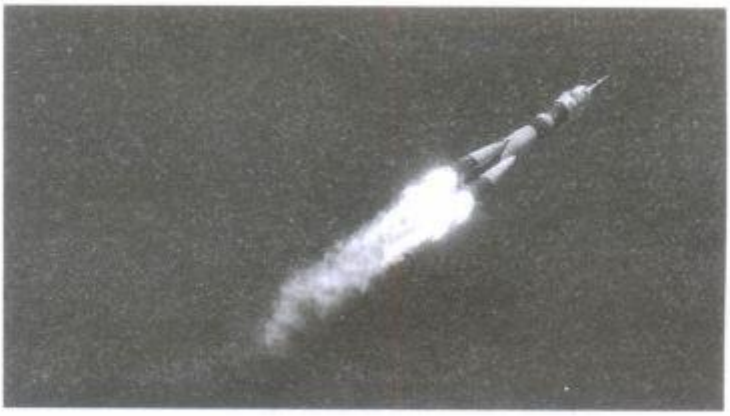
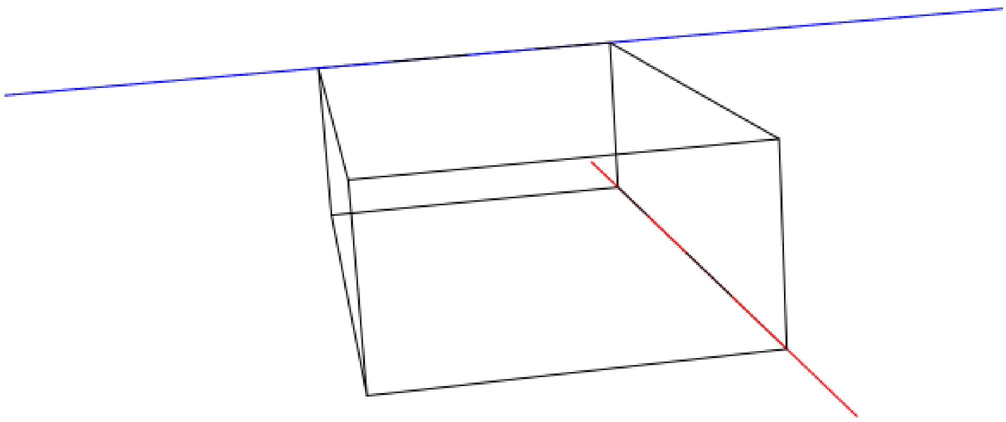
(iii) Find the length of the perpendicular MA

Q3 The Indian Coast Guard (lCG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a path L: $\frac{x-5}{3}=\frac{y-2}{2}=\frac{z+4}{-8}$. At an instant of time, the coordinates of the position of coast guard helicopter and boat are H(2, 3, 5) and B(1, 4, 2) respectively.

Based on the above information answer the following questions.
(i)Find the equation of a line joining the points H(2, 3, 5) and B(1, 4, 2)
(ii)Find the direction cosines of the line joining the points H(2, 3, 5) and B(1, 4, 2)
(iii)Find the vector and Cartesian equation of the line passing through the point H(2,3,5) and is in the direction of the vector $3\hat{i}+2\hat{j}-8\hat{k}$

(OR)

(iii) Find the equation of the line passing through the point B(1,4,2) and parallel to the

	line L: $\frac{3x-5}{3}=\frac{-y+2}{-2}=\frac{2z+4}{-8}$
Q4	The equation of motion of a rocket are: x = 2t, y = -4t, z = 4t, where the time 't' is given in seconds, and the distance measured is in kilometres.  Based on the above information, answer the following questions. (i) What is the path of the rocket? (ii) Does the point (1, -2, 2) satisfy the equation of path of rocket? Justify (iii) At what distance will the rocket be from the starting point (0, 0, 0) in 10 seconds? (OR) (iii)If the position of rocket at certain instant of time is (3, -6, 6), then what will be the height of the rocket from the ground, which is along the xy-plane?
Q5	Given two lines in the two-dimensional plane, the lines are equal, they are parallel but not equal, or they intersect in a single point. In three dimensions, a fourth case is possible. If two lines in space are not parallel, but do not intersect, then the lines are said to be **skew lines** (See the Figure). Based on the above information answer the following Questions.  The equations of given line are L_1:x=2s−1, y=s−1, z=s−4 and L_2:x=t−3, y=3t+8, z=5−2t (i)Write the above equations in standard form (OR) (i)check whether the lines are intersecting or not? (iii)What are the direction rations of L_1 and L_2 (iii)Are the lines perpendicular to each other? Justify your answer.

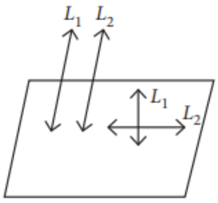
Q6 Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.

Based on the above information, answer the following questions.

(i) Find the shortest distance between the given lines.

(ii) Find the point at which the motorcycles may collide.

Q7 If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

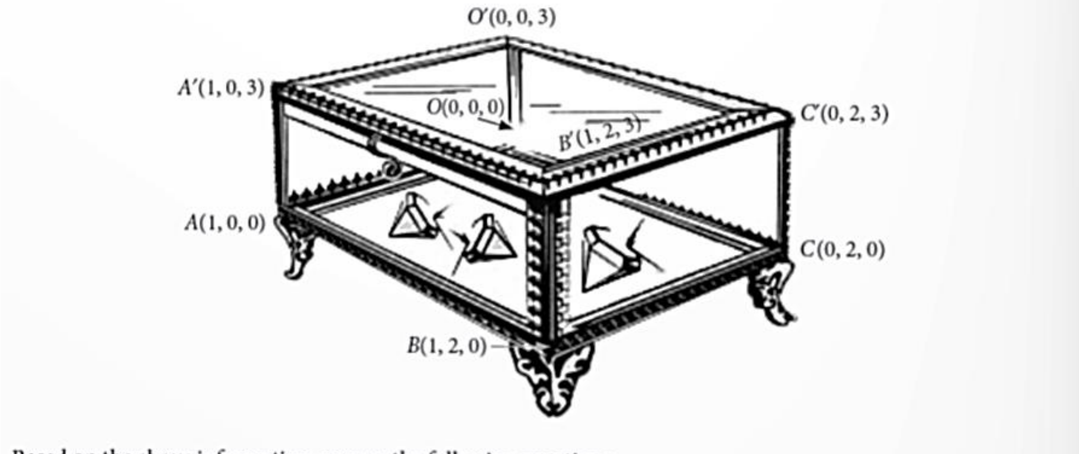


(i) If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, write the conditions for two lines to be perpendicular?

(ii) If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, write the conditions for two lines to be parallel?

(iii) Find the co-ordinates of foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining B(1, 4, 6) and C(5, 4, 4)

Q8 In a diamond exhibition, a diamond is covered in cubical glass box having coordinates $O(0, 0, 0)$, $A(1, 0, 0)$, $B(1, 2, 0)$, $C(0, 2, 0)$, $O'(0, 0, 3)$, $A'(1, 0, 3)$, $B'(1, 2, 3)$ and $C'(0, 2, 3)$.



Based on the above information, answer the following questions.

(i) Find the Direction ratios of OA?

(ii) Write the equation of diagonal OB' is?

(iii) Write the equation of plane OABC is?

(iv) Write the equation of plane O'A'B'C' is?

ANSWERS

ASSERTION & REASON QUESTIONS

1.	Using $sin^2\alpha = 1 - cos^2\alpha$ we get $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$ $\therefore$option (a)
2.	Both A & R are true and Reason (R) is a correct explanation of Assertion (A). $\therefore$option (b)
3.	Answer: Assertion (A) is true and Reason (R) is false. $\therefore$option (c)
4.	Answer: (d) Assertion (A) is false and Reason (R) is true.
5.	Answer: a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A). $cos^2\theta + cos2\beta + cos2\theta = 1 \Rightarrow 2cos2\theta + 1 - sin2\beta = 1$ $\Rightarrow 2cos^2\theta + 3cos2\theta - 3 = 0 \Rightarrow \cos^2\theta = \frac{3}{5}$
6.	(c) Assertion (A) is true and Reason (R) is false.
7.	(a)Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).
8.	(b)Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).
9.	(c) Assertion (A) is true and Reason (R) is false.
10.	(c) Assertion (A) is true and Reason (R) is false.
11.	(b)Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).
12.	(d) Assertion (A) is false and Reason (R) is true.
13.	Option (b) Solution: (A) is true as follows $1 \times 2 + (-2) \times 2 + 2 \times 1 = 0$. Therefore, the lines are perpendicular. (R) is true by definition but not the correct explanation.
14.	Option (b) Solution: (A) is true as follows. If $r = a + \lambda b; r = c + \lambda d$ then S.D = $\left\|\frac{(c-a).(b\times d)}{b\times d}\right\| = \frac{(7i+38j-5k).(24i+36j+72k)}{\sqrt{24^2+36^2+72^2}} = \left\|\frac{1176}{\sqrt{7056}}\right\| = 14\, units.$ (R) is true by definition but not the correct explanation.
15.	Option (a) `Solution: (A) is true as follows. If $r = a + \lambda b; r = c + \mu d$ then S.D = $\left\| \frac{(c-a).(b\times d)}{b\times d} \right\|$ $= \frac{(i+j-k).(i+j+2k)}{\sqrt{6}} = 0$ (R) is true and it is correct explanation.

16.	Option (b) Solution: (A) is true as equation of line passing through a point A(a) parallel to the given vector b is $r = a + \lambda b$ and its cartesian form is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$. (R) is true as follows. Equation line passing through (2, 1, 3) and parallel to the line $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z-4}{-2}$ is $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z-3}{-2}$ $= 2(x-2) = y-1 = -(z-3) => 2x-4 = y-1 = 3-z$ But not the correct explanation.
17.	Option (c) Sol: (A) is true by definition of skew lines. (R) is true as skew lines do not intersect, there is no angles between them.
18.	option (a) Solution: (A) is true as follows, $n^2 + (n+1)^2 = [n(n+1)+1]^2$ $11^2 + 12^2 + 132^2 = (11.12 + 1)^2$ (R) is true and correct explanation.
19.	option (b) Solution: (A) is true as follows, Mid point of AB is M (1, 3, 5). This lies on $\frac{x}{1} = \frac{y-11}{2} = \frac{z-2}{3}$ ---- (1) D.R of AB are (0, 6, -4) D.R of line (1) is (1, 2, 3) $1 \times 0 + 6 \times 2 + (-4) \times 3 = 0$; Therefore, AB is perpendicular to (1) (R) is true but not the correct explanation of (A)
20.	option (d) Solution: (A) is true. AB=$\sqrt{(2-1)^2 + (9-8)^2 + (12-8)^2} = 3\sqrt{2}$ Similarly, BC=CD=DA= $3\sqrt{2}$ Also AC=BD=6 Therefore, (R) is false.
21.	Option(b) Solution: (A) is true as follows, SD= $\left\lvert \frac{(-i+6j-7k).(i-4j+8k)}{\sqrt{81}} \right\rvert = 9\ units.$ (R) is true but not the correct explanation.

22.	option(a) Solution: (A) is true as $\frac{2}{6} = -\frac{1}{3} = \frac{3}{9} = \frac{1}{3}$ (R) is true as $a \times b = 0$ if a parallel b and is correct explanation.
23.	option(d) Solution: A is false, $\frac{x-\frac{3}{2}}{2} = \frac{y-2}{3} = \frac{z-4}{2}$; $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-1}{-2}$ $2 \times 1 + 3 \times 2 + 2 \times -2 \neq 0$ (R) is true by definition
24.	option(c) Solution: (A) is true, $\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = 0 => \theta = \pi/2$ (R) is false as skew lines do not intersect so there is no angle between them
25.	option(a) Solution: (A) is true, (R) is true and correct explanation of (A).

ANSWERS
CASE BASED QUESTIONS

1.	(i) $L1: \frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$ $L2: \frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6}$ (ii) (ii)The direction ratios of L1 are <1,2,2> , The direction ratios of L2 are <3,2,6> (iii) (iii) $\cos\theta = \frac{3+4+12}{\sqrt{9} \times \sqrt{49}} = \frac{19}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$ (OR) (iv) The direction ratios of a line perpendicular to both the lines L_1 and L_2 are <8,0,-4> or<2,0,-1>
2.	(i) $\vec{r} = 3\hat{i} - \hat{j} + 3\hat{k} + \lambda(4\hat{i} + 2\hat{j} + \hat{k})$ (ii) $\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{1}{\sqrt{21}}$ (iii) Direction ratios of MA are <4 λ +2,2 λ -2, λ > Using perpendicular condition we get λ =0 Direction ratios of MA are <2,-2,0> or <1,-1,0> (OR) (iii) Coordinates of foot of perpendicular is (3,-1,3) . MA= $\sqrt{4+4+0} = 2\sqrt{2}$

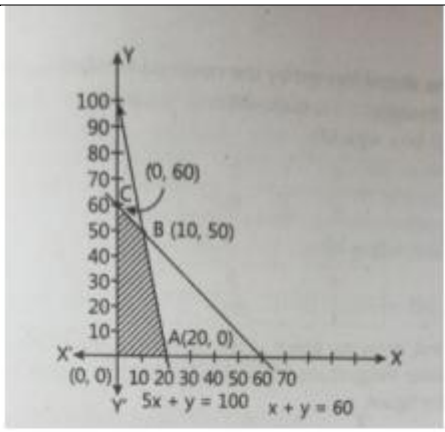
3.	(i) $\frac{x-2}{-1}=\frac{y-3}{1}=\frac{z-5}{-3}$ (ii) $\frac{-1}{\sqrt{11}},\frac{1}{\sqrt{11}},\frac{-3}{\sqrt{11}}$ (iii) $\vec{r}=2\hat{i}+3\hat{j}+5\hat{k}+\lambda(3\hat{i}+2\hat{j}-8\hat{k})$ & $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-5}{-8}$ (OR) (iii) $\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-2}{-4}$
4.	(i) Straight line $\frac{x}{2}=\frac{y}{-4}=\frac{z}{4}=t$ (ii) (ii) $Yes \because \frac{1}{2}=\frac{-2}{-4}=\frac{2}{4}$ (iii) Distance between (0,0,0) & (20, -40,40) is 60 km (OR) (iv) 6 km
5.	$L_1:\frac{x+1}{2}=\frac{y+1}{1}=\frac{z+4}{1}=s$ (i) L1: $L_2:\frac{x+3}{1}=\frac{y-8}{3}=\frac{-z+5}{2}=t$ (OR) (i)2s-t=-2, s-3t=9 and s+2t=9 . Solving last two questions we get t =0, s= 9. Substituting these values first equation doesn't satisfy the equation 2s-t=-2. Hence lines do not intersect (ii)<2,1,1> and <1,3.-2> $a_1a_2+b_1b_2+c_1c_2=0$ (iii) $2+3-2\neq 0. Hence\ are\ L_1\ and\ L_2\ not\ \perp$
6.	(i) Shortest distance = 0 (ii) The accident occurs at the point (1, -2, -1)
7.	(i) $l_1l_2+m_1m_2+n_1n_2=0$ (ii) $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$ (iii) (3, 4, 5)
8.	(i) <1, 0, 0> (ii) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (iii) $z=0$ (iv) $z=3$

CHAPTER: LINEAR PROGRAMMING

ASSERTION AND REASON BASED MCQS

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

Q.No	QUESTIONS
1	**Assertion (A):** Feasible region is the set of points which satisfy all of the given constraints and objective function too. **Reason (R):** The optimal value of the objective function is attained at the points on X-axis only.
2	**Assertion (A) :** For the constraints of linear optimizing function $Z = x_1 + x_2$ given by $x_1 + x_2 \leq 1$, $3x_1 + x_2 \geq 1$, there is no feasible region. **Reason (R):** $Z = 7x + y$, subject to $5x + y \leq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. Out of the corner points of feasible region (3, 0), $\left(\frac{1}{2}, \frac{5}{2}\right)$, (7, 0) and (0,5), the maximum value of Z occurs at (7, 0).
3	**Assertion (A):** Feasible region is the set of points which satisfy all of the given constraints. **Reason (R):** The optimal value of the objective function is attained at the points on X-axis only.
4	**Assertion (A):** It is necessary to find objective function value at every point in the feasible region to find optimum value of the objective function. **Reason(R):** For the constrains $2x+3y \leq 6$, $5x+3y \leq 15$, $x \geq 0$ and $y \geq 0$ cornner points of the feasible region are (0,2), (0,0) and (3,0).
5	**Assertion (A) :** For the constraints of linear optimizing function $Z = x1 + x2$ given by $x1 + x2 \leq 1$, $3x1 + x2 \geq 1$, $x \geq 0$ and $y \geq 0$ there is no feasible region. **Reason (R):** $Z = 7x + y$, subject to $5x + y \leq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The corner points of the feasible region are (0,3), (0,5) and ($\frac{1}{2}$., $\frac{5}{2}$)
6	**Assertion (A):** The maximum value of $Z = 11x+7y$ Subject to the constraints are $2x+y \leq 6$, $x \leq 2$, $x,y \geq 0$, occurs at the point (0,6). **Reason (R):** If the feasible region of the given LPP is bounded, then the maximum and minimum values of the objective function occurs at corner points.
7	**Assertion (A):** If an LPP attains its maximum value at two corner points of the feasible region then it attains maximum value at infinitely many points. **Reason (R):** If the value of the objective function of a LPP is same at two corners then it is same at every point on the line segment joining the two corner points.
8	Consider, the graph of constraints stated as linear inequalities as below: $5x+y \leq 100$, $x+y \leq 60$, $x,y \geq 0$.

	 **Assertion (A):** The points (10,50), (0,60) , (10,10) and (20,0) are feasible solutions. **Reason (R):** Points within and on the boundary of the feasible region represent feasible solutions of the constraints.
9	**Assertion (A):** The region represented by the set $\{(x,y): 4 \leq x^2 + y^2 \leq 9\}$ is a convex set. **Reason (R):** The set $\{(x,y): 4 \leq x^2 + y^2 \leq 9\}$ represents the region between two concentric circles of radii 2 and 3.
10	**Assertion (A):** For an objective function Z= 15x + 20y, corner points are (0,0), (10,0), (0,15) and (5,5). Then optimal values are 300 and 0 respectively. **Reason (R):** The maximum or minimum value of an objective function is known as optimal value of LPP. These values are obtained at corner points.
11	**Assertion (A):** For the LPP Z= 3x+2y, subject to the constraints $x+2y\leq 2$; $x, y \geq 0$ both maximum value of Z and Minimum value of Z can be obtained. **Reason (R):** If the feasible region is bounded then both maximum and minimum values of Z exists.
12	**Assertion (A):** The linear programming problem, maximize Z = x+2y subject to constraints $x-y \leq 10$, $2x + 3y \leq 20$ and $x \geq 0$; $y\geq 0$. It gives the maximum value of Z as 40/3. **Reason (R):** To obtain maximum value of Z, we need to compare value of Z at all the corner points of the shaded region.
13	**Assertion (A):** Consider the linear programming problem. Maximise Z = 4x+y Subject to the constraints $x+ y\leq 50$; $x+ y \geq 100$ and $x, y\geq 0$. Then, maximum value of Z is 50. **Reason (R):**If the shaded region is bounded then maximum value of objective function can be determined.
14	**Assertion (A):** The point (4,2) does not lie in the half plane of $4x + 6y - 24< 0$ **Reason (R):** The point (1,2) lies in the half plane of $4x + 6y - 24< 0$
15	The corner points of a feasible region determined by a set of constraints are P(1,6), Q(4,5), R(6,1) and S(5,2) and the objective function is Z = ax + 3by where a, b > 0 **Assertion (A):** The relation between a and b such that the maximum Z occur at P and Q is a = b **Reason (R):** The relation between a and b such that the maximum Z occur at P and Q is a = 3b
16	**Assertion (A) :** If the corner points of the feasible region for a linear programming problem are P(0,4), Q(1,4), R(4,1), and S(12,-1), then minimum value of objective function Z = 2x + 4y is at the point R(4,1) **Reason (R):** If the corner points of the feasible region for a linear programming problem are P(0,4), Q(1,4), R(4,1), and S(12,-1), then maximum value of objective function Z = 2x + 4y is 20
17	**Assertion (A) :** Constraints are inequations **Reason (R):** Linear inequalities related to variables involved, are known as constriants

<table>
<tr><td>18</td><td>Assertion (A) : Common solution of all the given constraints in LPP, is known as feasible region.
Reason (R): A linear function Z = ax + by, which has to be maximized or minimized is known as an objective functioned</td></tr>
<tr><td>19</td><td>The corner points of a feasible region are (0,0), (3,0) and (0,3) and the objective function Z = 4x +7y
Assertion (A) : Minimum value of Z is 12
Reason (R): Maximum value of Z is 21</td></tr>
<tr><td>20</td><td>Assertion (A) :$Z = 20x_1 + 20x_2$, subject to $x_1 \geq 0$, $x_2 \geq 2$, $x_1 + 2x_2 \geq 8$, $3x_1 + 2x_2 \geq 15$, $5x_1 + 2x_2 \geq 20$. Out of the corner points of feasible region (8, 0), $\left(\frac{5}{2}, \frac{15}{2}\right)$, $\left(\frac{7}{2}, \frac{9}{4}\right)$ and (0, 10), the minimum value of Z occurs at $\left(\frac{7}{2}, \frac{9}{4}\right)$.
Reason (R):
<table>
<tr><th>Corner Points</th><th>$Z = 20x_1 + 20x_2$</th></tr>
<tr><td>(8, 0)</td><td>160</td></tr>
<tr><td>$\left(\frac{5}{2}, \frac{15}{2}\right)$</td><td>125</td></tr>
<tr><td>$\left(\frac{7}{2}, \frac{9}{4}\right)$</td><td>115 minimum</td></tr>
<tr><td>(0, 10)</td><td>200</td></tr>
</table></td></tr>
</table>

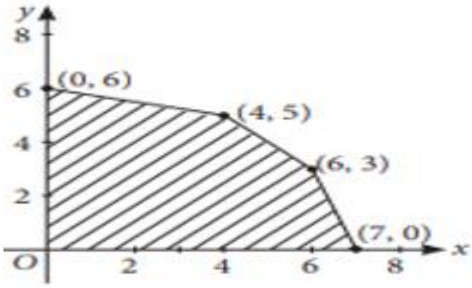
ANSWERS

Assertion and Reason based MCQs

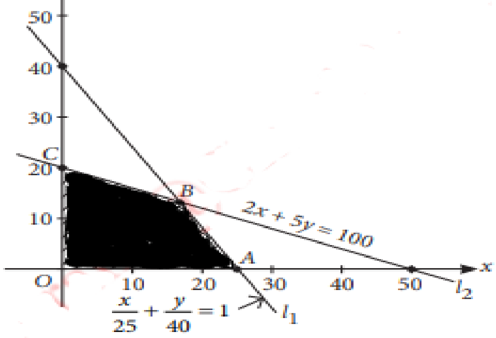
1	C	**2**	B	**3**	C	**4**	D	**5**	A
6	A	**7**	A	**8**	A	**9**	D	**10**	A
11	A	**12**	A	**13**	D	**14**	B	**15**	C
16	B	**17**	C	**18**	B	**19**	D	**20**	A

CASE BASED QUESTIONS

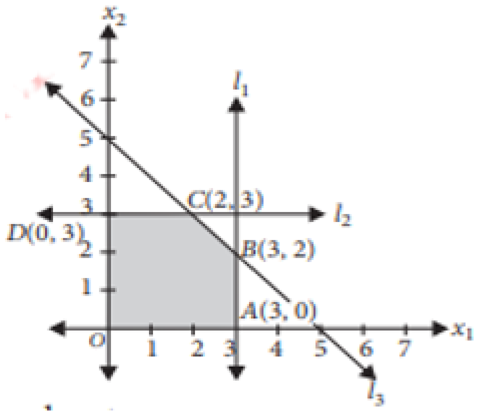
<table>
<tr><th>Q.NO</th><th>QUESTIONS</th><th>MARKS</th></tr>
<tr><td>1</td><td>
Linear programming is the method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as linear equations or in equations.
Based on the above information, answer the following questions.
(i) The optimal value of the objective function is attained at --------- points.
(ii) The solution of the inequality $3x + 4y < 12$ is -------------
(iii) Find the Maximum of $Z= 2x + 5y$ that occurs at the corner points of the feasible region.

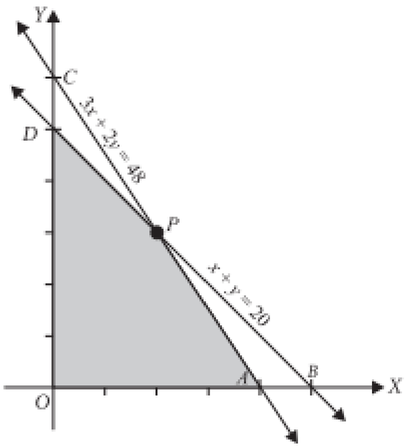
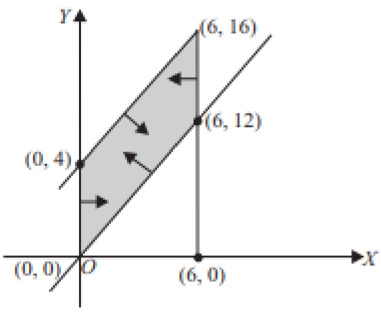


(OR)
The corner of the points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Find the condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20)
</td><td>1
1
2</td></tr>
<tr><td>2</td><td>
Deepa rides her car at 25 km/hr . She has to spend Rs. 2 per km on diesel and if she rides it at a faster speed of 40km/hr, the diesel cost increases to Rs. 5 per km. She has Rs.100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. the feasible region for the LPP is shown below.

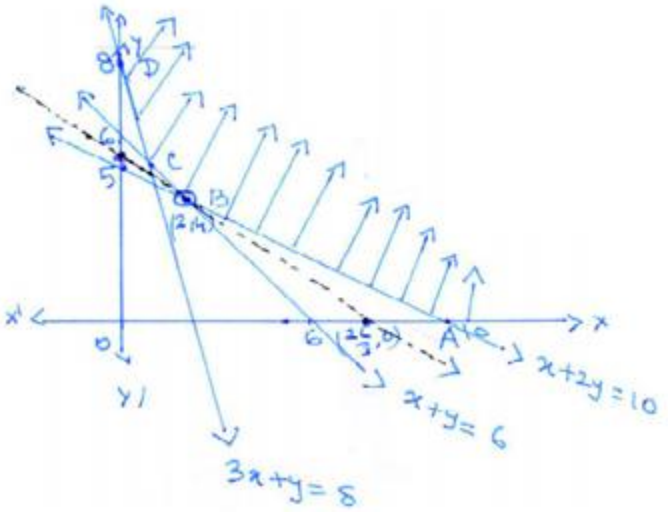
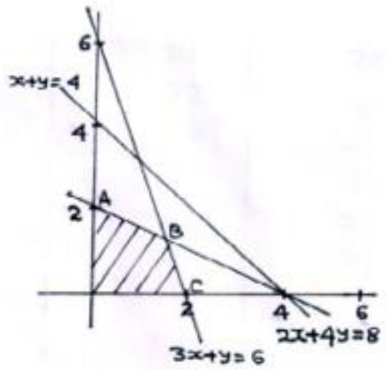


Based on the above information, answer the following questions.
(i) What is the point of intersection of line l_1 and l_2
(ii) What are the corner points of the feasible region shown in above graph?
(iii) If $Z = 6x - 9y$ be the objective function, then the maximum value of Z – the minimum value of Z
</td><td></td></tr>
</table>

3	Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z= 4x – 6y be the objective function. Based on the above information, answer the following questions. (i) Find the point at which the minimum of Z occurs and find the point at which the maximum of Z occurs. (ii) The feasible region determined by a system of linear inequalities is given below. Find the corner points of the feasible region and also write the corner points of the region bounded by the three lines l_1, l_2 & l_3 	**2** **2**
4	Suppose a dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him ₹ 360 and a manually operated sewing machine ₹ 240. He can sell an electronic sewing machine at a profit of ₹ 22 and a manually operated sewing machine at a profit of Rs.18. Based on the above information, answer the following questions. (i) Let x and y denotes the number of electronic sewing machines and manually operated sewing machines purchased by the dealer respectively. Write the inequality, if it is assumed that the dealer purchased at least one of the given machines (ii) Let the constraints in the given problem is represented by the following inequalities. $x + y \leq 20$, $360x + 240y \leq 5760$ $x, y \geq 0$ Then which one of the following points lie in its feasible region. (0, 24), (8, 12), (20, 2) (iii) Suppose the following shaded region APDO, represent the feasible region corresponding to mathematical formulation of given problem. Then write the corner points of the shaded region which lie on the Co- Ordinate axises. Also find the point P.	**1** **1** **2**

<table>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>5</td><td>Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
Based on the above information, answer the following questions.
(i) In solving the LPP : “minimize Z= 6x + 10y subject to the constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$” redundant constraints are
$x \geq 6$, $y \geq 2$ (or) $2x + y \geq 10$, $x \geq 0$, $y \geq 0$ (or) $x \geq 6$
(ii) The feasible region for a LPP is shown shaded in the figure. Let F = 3x – 4y be the objective function. Find the Maximum value of F.

</td><td>2

2</td></tr>
<tr><td>6</td><td>An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economic class. It is given that the number of executive class tickets is x and that of economy class ticket is y.
Based on above information answer the following questions:
(i) What is the maximum value of x+ y
(ii) What is the relation between x and y
(iii) What should be the objective function to get maximum Profit. What is the profit when x = 20 and y = 80</td><td>1
1
2</td></tr>
</table>

7	 The above feasible region is subject to the following constraints: : $x + 2y \geq 10$, $x + y \geq 6$, $3x + y \geq 8$, $x, y \geq 0$ (i) The feasible region is bounded or unbounded? (ii) If the objective function is Z= 3x + 5y, then how do you conclude the minimum or maximum value of the function in case of unbounded feasible region (iii) If an LPP admits optimal solution at two consecutive vertices of a feasible region, then the optimal solution occurs at----------------	**1** **2** **1**
8	The graph given below is subject to the following constraints : $2x + 4y \leq 8$, $3x + y \leq 6$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$  (i) Find the corner points of the feasible region (ii) Write equation of lines other than the Co-ordinate Axises which represent the sides of the feasible region.	**2** **2**
9	A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs10. Assuming that he can sell all the screws he manufactures. Assume that factory manufactures x screws of type A and y screws of type B. Based on the above information answer the following questions:	

	(i) Write the objective function to maximize the profit . (ii) What are number of screws of each type so that profit is maximum ? (iii) What is the maximum profit ?	**1** **2** **1**
10	A farmer uses two types of fertilizers F1 and F2 in his field. F1 consists of 10% nitrogen and 6%phosphoric acid and F2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F1 cost Rs 6/kg and F2 costs Rs 5/kg. (i) Determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. (ii) What is the minimum cost?	**2** **2**
11	A doctor suggests a patient two types tablets X and Y in order to supplement daily diet and patient wishes to take some X and some Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as below: <table><tr><th>Tablets</th><th>Iron</th><th>Calcium</th><th>Vitamin</th></tr><tr><td>X</td><td>6</td><td>3</td><td>2</td></tr><tr><td>Y</td><td>2</td><td>3</td><td>4</td></tr></table>The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is Rs 2 and Re 1 respectively. **(i)** How many tablets of each should the patient take in order to satisfy the above requirement at the minimum cost? **(ii)** What will be the minimum cost ?	**2** **2**
12	A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set. Assume that company makes x number of black and white T.V. and y number of colour T.V. Based on the above information answer the following questions : (i) Write the objective function representing this LPP and Write the constraints for this LPP. (ii) How many sets of each type should be produced so that the company has maximum profit?	**2** **2**

13	According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives < 225 a day and a woman receives < 200 a day. Assume that x and y the number of men and women helpers. Based on the above information answer the following questions : (i) Write the objective function representing this LPP and Write the constraints for this LPP (ii) How many men and women helpers should be hired to keep the pay-roll at a minimum and What is the minimum value of Pay roll ?	**2** **2**

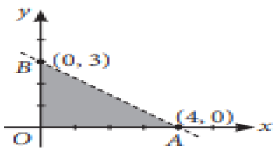
ANSWERS

CASE BASED QUESTIONS

1

i. When we solve an L.P.P. graphically, the optimal (or optimum) value of theobjective function is attained at corner points of the feasible region.

ii. From the graph of 3x + 4y < 12 it is clear that it contains the origin but not the points on the line 3x + 4y = 12.



iii. Maximum of objective function occurs at corner points

Corner points	**Z= 2x+ 5y**
(0,0)	0
(7,0)	14
(6,3)	27
(4,5)	33 ← **maximum**
(0,6)	30

(OR)

Value of $Z = px + qy$ at (15, 15) = $15p + 15q$
and that at (0, 20) = $20q$.
According to given condition, we have $15p + 15q = 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$

2

i. Let $B(x, y)$ be the point of intersection of the given lines
2x + 5y = 100 -----------(1) $\frac{x}{25} + \frac{y}{40} = 1 \Rightarrow$ 8x + 5y = 20 -----------------(2)
Solving (1) and (2), we get $x = \frac{50}{3}$, $y = \frac{40}{3}$ $\therefore$ The point of intersection B(x,y) = $\left(\frac{50}{3}, \frac{40}{3}\right)$

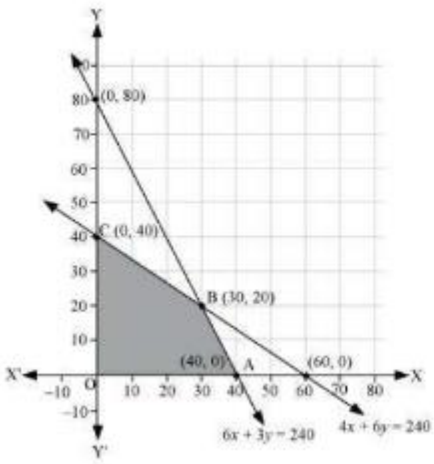
<table>
<tr><td></td><td>ii</td><td>the corner points of the feasible region shown in graph are (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$ (0, 20)</td></tr>
<tr><td></td><td>iii</td><td>
<table>
<tr><th>Corner Points</th><th>Value of $Z = 6x - 9y$</th></tr>
<tr><td>(0, 0)</td><td>0</td></tr>
<tr><td>(25, 0)</td><td>150 ← Maximum</td></tr>
<tr><td>$\left(\frac{50}{3}, \frac{40}{3}\right)$</td><td>–20</td></tr>
<tr><td>(0, 20)</td><td>–180</td></tr>
</table>
</td></tr>
<tr><td></td><td></td><td>
<table>
<tr><th>Corner Points</th><th>Value of $Z = 6x + 3y$</th></tr>
<tr><td>(0, 0)</td><td>0 ← Minimum</td></tr>
<tr><td>(25, 0)</td><td>150</td></tr>
<tr><td>$\left(\frac{50}{3}, \frac{40}{3}\right)$</td><td>140</td></tr>
<tr><td>(0, 20)</td><td>60</td></tr>
</table>
the maximum value of Z – the minimum value of Z = 150 – 0 = 150</td></tr>
<tr><td>3</td><td>i</td><td>
<table>
<tr><th>Corner Points</th><th>Value of $Z = 4x - 6y$</th></tr>
<tr><td>(0, 3)</td><td>$4 \times 0 - 6 \times 3 = -18$</td></tr>
<tr><td>(5, 0)</td><td>$4 \times 5 - 6 \times 0 = 20$</td></tr>
<tr><td>(6,8)</td><td>$4 \times 6 - 6 \times 8 = -24$</td></tr>
<tr><td>(0, 8)</td><td>$4 \times 0 - 6 \times 8 = -48$</td></tr>
</table>
Minimum value of Z is –48 which occurs at (0, 8).
Maximum value of Z is 20, which occurs at (5, 0).</td></tr>
<tr><td></td><td>ii</td><td>The corner points of the feasible region are $O(0, 0)$, $A(3, 0)$, $B(3, 2)$, $C(2, 3)$, D(0, 3</td></tr>
<tr><td>4</td><td>i</td><td>x + y > 0</td></tr>
<tr><td></td><td>ii</td><td>Since (8, 12) satisfy all the inequalities therefore (8, 12) is the point in its feasible region.</td></tr>
<tr><td></td><td>iii</td><td>The corner point which lies on X- axis is (16, 0) and Y- axis is (0, 20). The point P is (8,12)</td></tr>
<tr><td>5</td><td>i</td><td>When $x \geq 6$ and $y \geq 2$, then $2x + y \geq 2 \times 6 + 2$, i.e., $2x + y \geq 14$
Hence, $x \geq 0$, $y \geq 0$ and $2x + y \geq 10$ are automatically satisfied by every point of the region $\{(x, y) : x \geq 6\} \cap \{(x, y) : y \geq 2\}$</td></tr>
<tr><td></td><td>ii</td><td>
<table>
<tr><th>Corner Point</th><th>Value of $F = 3x - 4y$</th></tr>
<tr><td>(0, 0)</td><td>$3 \times 0 - 4 \times 0 = 0$ ← Maximum</td></tr>
<tr><td>(6, 12)</td><td>$3 \times 6 - 4 \times 12 = -30$</td></tr>
<tr><td>(6, 16)</td><td>$3 \times 6 - 4 \times 16 = -46$</td></tr>
<tr><td>(0, 4)</td><td>$3 \times 0 - 4 \times 4 = -16$</td></tr>
</table>
The maximum of F is 0</td></tr>
<tr><td>6</td><td>i</td><td>200</td></tr>
<tr><td></td><td>ii</td><td>$y \geq 4x$</td></tr>
<tr><td></td><td>iii</td><td>The objective function is Z = 1000x + 6000y.The profit when x = 20 and y = 80 is Rs. 68,000</td></tr>
<tr><td>7</td><td>i</td><td>The feasible region is unbounded</td></tr>
</table>

	ii	In case the feasible region is unbounded, (a) m is the minimum value of Z, if the open half plane determined by 3x + 5y < m has no point in common with the feasible region. Otherwise Z has no minimum value. (b) M is the maximum value of Z, if the open half plane determined by 3x + 5y > M has no point in common with the feasible region. Otherwise Z has no maximum value.
	iii	the optimal solution occurs at every point on the line joining these two points.
8	i	The corner points are O(0,0), A (0, 2), B (1.6, 1.2), C (2, .0)
	ii	2x + 4y= 8 and 3x + y = 6
9	i	Let the factory manufacture x screws of type A and y screws of type B on each day. Therefore, $x \geq 0$ and $y \geq 0$ The Objective function is to Maximize Z = 7x + 10y

9 ii The given information can be compiled in a table as follows.

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	4×60 =240
Hand Operated Machine (min)	6	3	4×60 =240

The constraints are $4x + 6y \leq 240$; $6x + 3y \leq 240$; $x \geq 0$ and $y \geq 0$



The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are asfollows.

Corner point	Z = 7x + 10y	
A (40, 0)	280	
B (30, 20)	410	Maximum
C (0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, **the factory should produce 30 packages of screws A and 20 packages of screws B** to get the maximum profit of Rs 410.

9 iii The maximum profit is Rs 410

10 Let the farmer buy x kg of fertilizer F1 and y kg of fertilizer F2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table asfollows.

	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs/kg)
F_1(x)	10	6	6
F_2(y)	5	10	5
Requirement (kg)	14	14	

The mathematical formulation of the given problem is

Minimize Z = 6x + 5y ... (1)

Subject to the constraints,

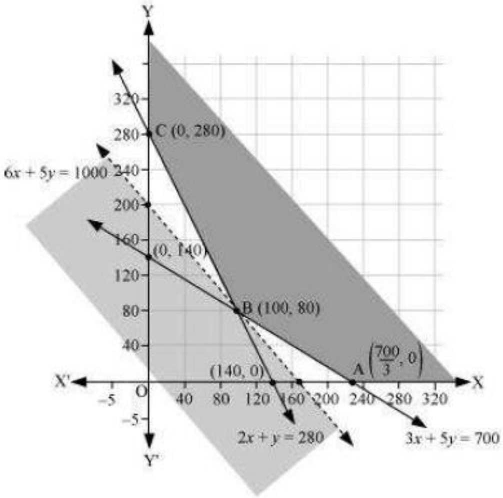
$$\frac{x}{10}+\frac{y}{20}\geq 14 \;=> 2x+y\geq 280$$

$$\frac{6x}{100}+\frac{10y}{100}\geq 14 => 3x+56y\geq 700$$

$$x, y\geq 0$$

The feasible region determined by the system of constraints is as follows.

i
ii



It can be seen that the feasible region is unbounded.

The corner points are, $A\left(\frac{700}{3},0\right)$,B (100,80), and (0,280)

The values of Z at these points are as follows.

Corner point	z = 6x + 5y	
$A\left(\frac{700}{3},0\right)$	1400	
B(100,80)	1000	Minimum
C(0, 280)	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, 6x + 5y < 1000, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with 6x + 5y < 1000

Therefore, 100 kg of fertiliser F1 and 80 kg of fertilizer F2 should be used to minimize the cost. The minimum cost is Rs 1000.

11

A person needs at least 18mg of iron.Hence 6x+2y ≥ 18

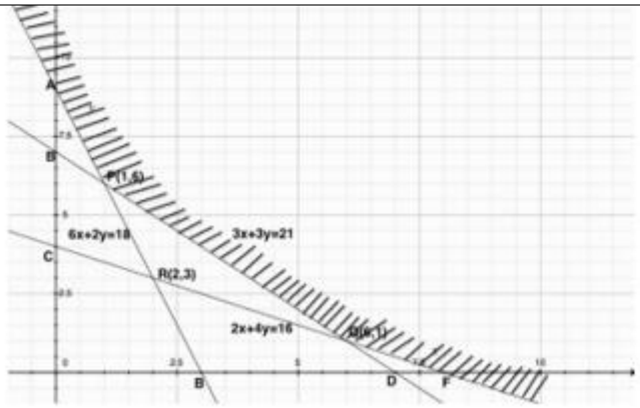
The person needs at least 21mg of calcium. Hence 3x+3y ≥21

The person needs at least 16mg of vitamins .Hence 2x+4y≥16

The price of each tablet of X is Rs.2 and tablet Y is Rs.1

Hence Z=2x+y

11 i



ii The corner points of the feasible region are A(0,9),P(1,6),Q(6,1),F(8,0)

12 i At A(0,9) ; Z=2×0+9=9

At P(1,6); Z=2×1+6=8

At Q(6,1); Z=2×6+1=13

At F(8,0); Z=2×8+0=16

Clearly the person should take one tablet of X and 6 tablet of Y to satisfy the requirement at the minimum cost.

Th minimum cost is Rs.8

Let x and y denote, respectively, the number of black and white sets and coloured sets made each week.

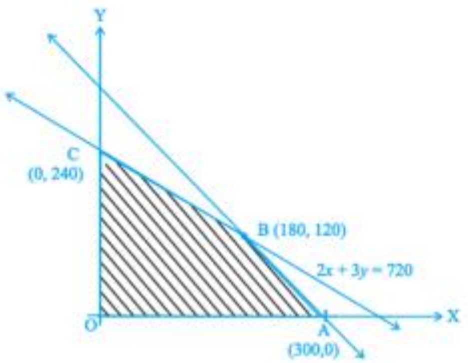
Thus, the mathematical formulation of the problem is

Maximise Z = $510x + 675y$

ii The constraints are : $x + y \leq 300$

$1800x + 2700y \leq 648000$, i.e., $2x + 3y \leq 72$, $x \geq 0$, $y \geq 0$

13



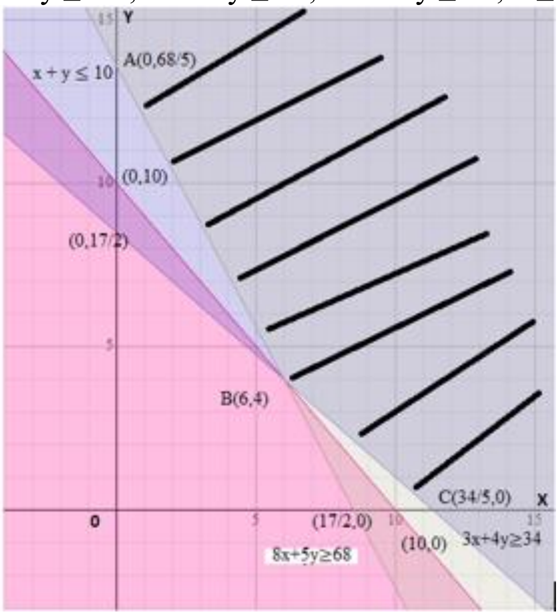
Since the feasible region is bounded, therefore maximum of Z must occur at the corner point of OBC.

Corner Points	Value of Z
(0,0)	0
(300,0)	153000

(180,120)	172800
(0,240)	162000

Thus, maximum Z is 172800 at the point (180, 120), i.e., the company should produce 180 black and white television sets and 120 coloured television sets to get maximum profit.

Let x be the man helpers and y be the woman helpers Pay roll: $Z = 225x + 200y$
Subject to constraints:
$x + y \leq 10$, $3x + 4y \geq 34$, $8x + 5y \geq 68$, $x \geq 0$, $y \geq 0$



Corner points areA (0,68/5),B(6,4) and C(34/5,0)
At A(0,68/5), Z = 2720
At B(6,4) , Z = 2150----------min
At C(34/5,0) Z = 2550

6 men and 4 women should be hired to keep the pay-roll at a minimum and the minimum value of Pay roll 2150

CHAPTER: PROBABILITY

ASSERTION-REASON BASED QUESTIONS

	In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.
1	Assertion (A) : Let A and B are independent events. If P(A) = 0.2 and P(B) = 0.1 then P(A∩B) = 0.02 Reason (R): For independent events A and B, P(A∩B) = P(A) x P(B)
2	Assertion (A) : Let A and B are independent events. If P(A) = 0.4 , P(B) = p and P(A∪B) = 0.6, then 3p = 1 Reason (R): For independent events A and B, P(A∩B) ≠ P(A) x P(B)
3.	Assertion (A) : Let A and B are independent events. If P(A) = p , P(B) = 2p and P(Exactly one of A,B) = $\frac{5}{9}$, then p = $\frac{3}{5}$. Reason (R): The value of p = $\frac{1}{3}$, $\frac{5}{12}$
4	Assertion (A) : Let A and $\bar{B}$ are independent events then, P($\bar{A} \cup B$) = 1 – P(A) P($\bar{B}$) Reason (R): P($\bar{A} \cup B$) = P(A $\cap \bar{B}$)
5	Assertion (A) : Let A and B are mutually exclusive events. If P(A) = $\frac{1}{2}$, P(B) = p and P(A∪B) = $\frac{3}{5}$, then p = $\frac{1}{5}$ Reason (R): For mutually exclusive events A and B, P(A∪B) = P(A) + P(B)
6	Assertion (A) : If A and B are any two events such that P(A) + P(B) – P(A∩B) = P(A), then P$\left(A/_{B}\right)$ = 1 Reason (R): For any two events A and B, P$\left(B/_{A}\right)$ = $\frac{P(A\cap B)}{P(A)}$.
7	Assertion (A): Given that the probability of 'Ajay' speaks truth is $\frac{4}{5}$. When a die is thrown once Ajay reports that 6 appears, then the probability that actually there was actually 6 appeared is $\frac{4}{11}$. Reason (R): Two events are independent then $P(A \cap B) = P(A) \times P(B)$

8	Assertion (A) : If A and B are any two events such that 2P(A) = P(B) $= \frac{5}{13}$, and $P(A/B) = \frac{2}{5}$ then $P(A\cup B) = \frac{11}{26}$ Reason (R): For any two events A and B, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
9	Assertion (A) : A die is thrown. If E is the event the number appearing is a multiple of 3 and F be the event the number appearing is even. The events E and Fare not independent. Reason (R): Two events E and F are independent then $P(E\cap F) = P(E) . P(F)$.
10	Assertion (A) : A speaks truth in 60% of the cases, while B in 90% of the cases. Then the percent of cases are they likely to contradict each other in stating the same fact is $\frac{21}{50}$ Reason (R): A and B are two independent events, then the probability of occurrence of at least one of A and B is $1 - P(\bar{A})P(\bar{B})$
11	Assertion (A) : A die is thrown twice and the sum of the numbers appearing is observed to be 6. Then the probability that the number 4 has appeared at least once is $\frac{2}{5}$. Reason (R): E and F are any two events, then $P(E/F) = \frac{P(E\cap F)}{P(F)}$.
12	Assertion (A) : For any two events A and B, if P(A) = 0.3, P(B) = 0.5 and $P(A/B) = 0.4$, then $P(B/A) = \frac{2}{5}$ Reason (R): For any two events A and B, $P(B/A) = \frac{P(A\cap B)}{P(A)}$
13	Assertion (A) : A and B are mutually exclusive events then $P(A/B) = 0$ Reason (R): A and B are mutually exclusive events then $A\cap B = \{\ \}$
14	Assertion (A) : If $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$ and $P(A\cup B) = \frac{5}{6}$, then the events A and B are independent. Reason (R): Events A and B are independent then $P(A\cup B) = P(A) \times P(B)$
15	Assertion (A) In answering an MCQ Test a student either knows the answer or guesses. Let the probability that he knows answer is $\frac{3}{4}$ and that of he guesses is $\frac{1}{4}$. Assume that a student guesses the answer will be correct with probability $\frac{1}{4}$. Then the probability that he knows the answer given that he answered it correctly is $\frac{12}{13}$. Reason (R): The probability of a sure event is 1
16	Assertion (A) : The mean of a number obtained in throwing a die having 3 on three faces ,2 on two faces and 5 on one face is 2. Reason (R) : Mean is also called Average, Expectation or Expected value as E

17

X	0	1	2	3	4	5	6
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Assertion (A) : Value of C is $\frac{1}{10}$

Reason (R): Using the formula $\sum P_i = 1$,

18

Assertion (A): The Mean number of tails in three throws of a coin is 1.5
Reason (R) : The mean of probability distribution is $\sum(x_i p_i)$

19

Assertion (A) : A Random variable X can take the values 0,1,2 and
P(X=0) = P (X =1) = P.

Reason (R): E(X) = E(X^2) =1/2
E (x) $= \sum x\, p(x)$, and E (x^2) $= \sum(x^2 p\ (x\ ))$

20

Assertion (A) : For any two events A and B , $P\left(\frac{\bar{A}}{\bar{B}}\right)$ = 5/8 , given P ($\bar{A}$) = ½
P ($\bar{B}$) =2/3 and P (A ∩ B) =1/4.

Reason (R) : A and B are independent events.

21

Assertion (A) Suppose a girl throws a die. If she gets a composite number, she tosses a coin two times, otherwise she tosses a coin only once. If she gets exactly one head then the probability that she does not get a composite number in the throw of a die is $\frac{2}{3}$.

Reason (R): The Sample Space of the above experiment is $S = \{1H, 1T, 2H, 2T, 3H, 3T, 5H, 5T, 4HH, 4HT, 4TH, 4TT, 6HH, 6HT, 6TH, 6TT\}$

22

Assertion (A): If the outcome of one event does not affect the outcome of another event ,they are Independent events.

Reason (R) : P (A ∩ B) = P (A) P (B).

23

Assertion (A): The probability distribution of a random variable is given below and value of k is 1 / 32.

X	2	3	4	5
P(X)	5/k	7/k	9/k	11/ k

Reason (R) : Using the formula $\sum P_i = 1$.

24

Assertion (A): The Probability of getting an even number on the die and a spade card in a single event of throwing a die and selecting a card is 1 /8.
Reason (R); Both are not independent events.

25

Assertion (A): Mean of getting sixes in throwing pair of dice simultaneously is 1 /3
Reason (R); $0 \leq P\ (X\ ) \leq 1$

26	Assertion (A): Probability of drawing four kings , provided they are drawn successively from a deck of 52 cards is $\frac{1}{270721}$ Reason (R): P (A $\cap B \cap C \cap D$) $= P\ (A\ )\ \times P\ (\frac{A}{B})\ \times P\ \left(\frac{C}{A\cap B}\right)\ \times P\ \left(\frac{D}{A\cap B\cap C}\right)$
	ANSWERS
1	Answer: (a) Both A and R are true and R is the correct explanation of A. P(A∩B) = P(A) x P(B) = 0.2 x 0.1 = 0.02
2	Answer: A is true but R is false. P(A∪B) = P(A) + P(B) - P(A∩B) = P(A) + P(B) - P(A) x P(B) 0.6 = 0.4 + p - 0.4 p p = $\frac{1}{3}$ 3p =1
3	Answer: (d) A is false but R is true. p(1-2p)+(1-p)(2p) = $\frac{5}{9}$ p = $\frac{1}{3}$, $\frac{5}{12}$
4	Answer: A is true but R is false. P($\bar{A} \cup B$) = 1 - P(A ∩ $\bar{B}$) = 1 – P(A) P($\bar{B}$) (since A and $\bar{B}$ are independent)
5	Answer: (d) A is false but R is true. P(A∪B) = P(A) + P(B) $\frac{3}{5} = \frac{1}{2} + p$ P = $\frac{1}{10}$
6	Answer: (b) Both A and R are true but R is not the correct explanation of A. P(A) + P(B) – P(A∩B) = P(A) P(B) – P(A∩B) = 0 P(B) = P(A∩B) $\frac{P(A\cap B)}{P(B)} = 1$ P($^{A}/_{B}$) = 1
7	$E_1 \rightarrow Ajay\ speaks\ truth;\ E_2 \rightarrow Ajay\ does\ not\ speak\ truth$ $A \rightarrow Ajay\ reports\ 6\ appeared$ $P(E_1/A) = \frac{P(A/E_1)\ P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)\ P(E_2)}$

	$$\frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}=\frac{4}{11}$$ Assertion is true Reason is also true but it is not the correct reason. So the answer is (b)
8	Answer: (a) Both A and R are true and R is the correct explanation of A. $P(A/B) = \frac{2}{5}$ $\frac{P(A\cap B)}{P(B)} = \frac{2}{5}$ $P(A\cap B) = \frac{2}{13}$. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$ $= \frac{11}{26}$
9	Answer: (d) A is false but R is true. $P(E) = \frac{1}{3}$, $P(F) = \frac{1}{2}$, $P(E\cap F) = \frac{1}{6}$. $P(E).P(F) = \frac{1}{6}$ E and F are independent
10	Answer: (b) Both A and R are true but R is not the correct explanation of A. Required probability = $P(A\cap \bar{B}) + P(\bar{A}\cap B)$ $= \frac{6}{10}\times\frac{1}{10}+\frac{4}{10}\times\frac{9}{10}$ $= \frac{6}{100}+\frac{36}{100}=\frac{42}{100}=\frac{21}{50}$
11	Answer: (a) Both A and R are true and R is the correct explanation of A. E : number 4 appears at least once F : sum of the numbers appearing is 6 $P(E/F) = \frac{P(E\cap F)}{P(F)}$. $= \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$.
12	Answer: A is false but R is true. $P(A/B) = \frac{P(A\cap B)}{P(B)}$ $P(A\cap B) = 0.2$ $P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{0.2}{0.3} = \frac{2}{3}$
13	Answer: (a) Both A and R are true and R is the correct explanation of A. $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0}{P(B)} = 0$
14	Answer: (c) A is true but R is false. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $P(A\cap B) = \frac{1}{3}$

$P(A) \times P(B) = \frac{1}{3}$

$P(A \cap B) = P(A) \times P(B)$.

15

$E_1 \rightarrow student\ knows\ the\ answer;\ E_2 \rightarrow student\ guesses\ the\ answer$

$A \rightarrow student\ answered\ it\ correctly$

$$P(E_1/A) = \frac{P(A/E_1)\,P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)\,P(E_2)}$$

$$= \frac{1\times\frac{3}{4}}{1\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}} = \frac{12}{13}$$

Assertion is true

Reason is also true and it is the correct reason. So the answer is (a)

16

(b)

X	1	2	5
P(x)	3/6	2/6	1/6

Mean $=\sum x\,p(x)$ $=\frac{(3+4+5)}{6}$ = 12/6 =2

17

(a)

formula $\sum P_i$ =1, so 9C + 10 C^2 =1 , 10C^2 + 9C -1 =0

10 C(C +1) – 1 (C + 1) =0, (10C -1) (C + 1) =0

(C +1) $\neq$ 0. (10C -1) =0 GIVES C =1/10

18

(a)

x	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

Mean = $\frac{0+3+6+3}{8}$ = 12 / 8 = 3/2 =1.5

19

(a)

X	0	1	2
P(X)	P	P	1-2P

E (x) $=\sum x\,p(x)$ = E (x^2) $=\sum(x^2 p\,(x\,))$

P + 2 -4P = P + 4 – 8P, 4P=2. P=1/2

20

(c)

$$P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{p\,(\bar{A}\cap\bar{B}\,)}{P(\bar{B})} = \frac{(1-P(A\cup B)}{\overline{P(B)}} = \frac{\left(1-(P(A\,)+P(B)-P\,(A\cap B))\right)}{P(\overline{B})} =$$

$$\frac{(P(\bar{A})-\{1-P(\,\bar{B})\}+P\,(\,A\cap B))}{P\,(\bar{B}\,)} = \frac{\left(\frac{1}{2}-\left(1-\frac{2}{3}\right)+\frac{1}{4}\right)}{\frac{2}{3}} = \left(\frac{5}{12}\times\frac{3}{2}\right) = 5/8$$

21

$E_1 \rightarrow student\ knows\ the\ answer;\ E_2 \rightarrow student\ guesses\ the\ answer$

$A \rightarrow student\ answered\ it\ correctly$

	$P(E_1/A) = \frac{P(A/E_1)\,P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)\,P(E_2)}$ $= \frac{1\times\frac{3}{4}}{1\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}} = \frac{12}{13}$ Assertion is true Reason is also true and it is the correct reason. So the answer is (a)
22	(a)
23	(d) Using the formula $\sum P_i$ =1, $\left(\frac{32}{k}\right)$ =1 , k =32.
24	(c) P(A) be the probability of getting even number is 3/6 =1/2 P(B)be the probability of getting spade is 13/52 = ¼ P (A$\cap B$) = P (A) P (B) = 1/8
25	(b) <table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(x)</td><td>25/36</td><td>10/36</td><td>1/36</td></tr></table>E(x) = $\sum x\,p(x)$ = 12/36 = 1/3
26	(d) $\frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}\times\frac{1}{49} = \frac{1}{270725}$

CASE BASED QUESTIONS

1	In a bilateral hockey series of two matches between the countries India and Germany the probabilities of India Winning, Losing and Drawing are $\frac{1}{2}, \frac{1}{5}$ *and* $\frac{3}{10}$ resply. Each win, draw and loss gives the points 2, 1, 0 respectively to the team. Let X and Y denote total points scored by India and Germany after two games series. 

	Based on the above information answer the following questions: (i) Write the probability distribution of X. (ii) Find the probability of India winning the series (iii) What is the probability of India drawing the series?
2.	A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots,player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.  Based on the given information, answer the following questions. (i) Find the probability that A, B and C all will hit the target? (ii) What is the probability that any two of A, B and C will hit?
3.	Two persons A and B decided to play a game by using a die. They decided to throw the die alternately. They play with the condition that the person who gets 4 first will be the winner of the game.  Based on the given information, answer the following questions. (i) A wins the game, if A starts the game first

(ii) B wins the game, if A starts first.

4. In a play zone, Alina is playing crane game. It has 12 blue soft toys, 8 red soft toys, 10 yellow soft toys and 5 green soft toys. Alina draws two soft toys one after the other without replacement.

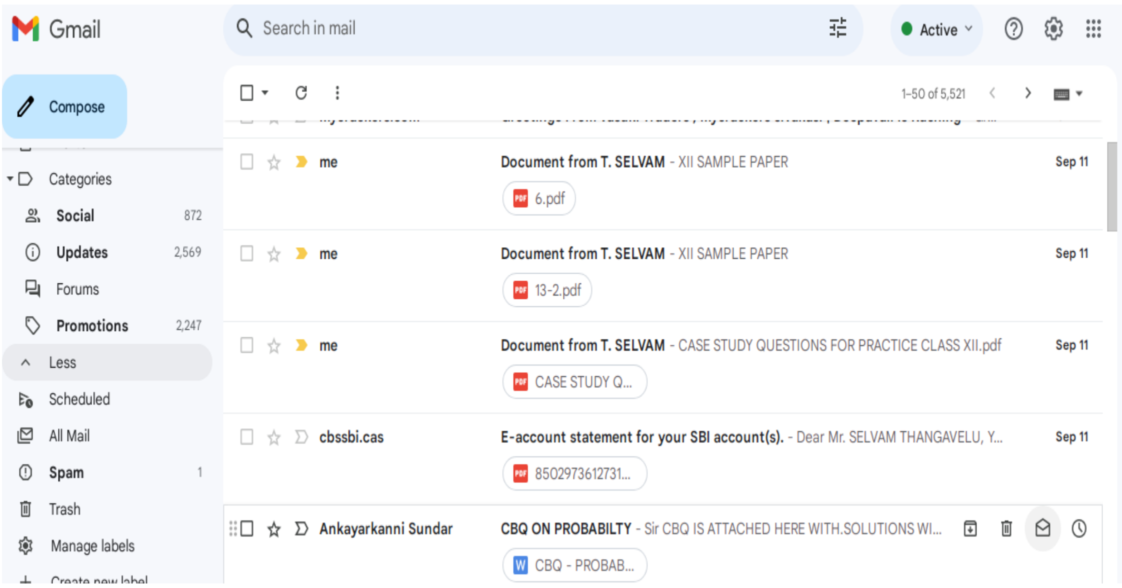
(i) What is the probability that the first soft toy is blue

and the second is green.

(ii). What is the probability that the first soft toy is green

and the second is not yellow.

5. Arun can detect spam e-mails in his inbox. It is found that the word “offer” occurs in 80% of the spam messages in his account. Also the word “offer” occurs in 10% of his desired e-mails. If 30% of the received e-mails are considered as spam.



Based on the above information answer the following questions:
(i) He received a new message which contains the word “offer”, what is the probability that it is a spam mail?
(ii) What is the probability of e-mails received with “offer” word?

6. Husband and wife appear in an interview for two vacancies in the same post. The probability of husband’s selection is $\frac{1}{7}$ and that of wife’s selection is $\frac{1}{5}$.

	 Based on the above information, answer the following questions. **(i)** Both of them will be selected **(ii)** Only one of them will be selected. **(iii)** None of them will be selected. (OR) At least one of them will be selected
7.	A fruit seller purchased Mango fruits from a farmer. The farmer told the vendor that he used chemical pesticide for 30% of the mango trees and organic pesticide for 70% of the mango trees during the flowering season to prevent the insects damaging the fruit. 80% of the flowers, which got chemical pesticide sprinkled, turned into fruits and 90% of the flowers, which got organic pesticide sprinkled, bore the fruits. Past experience shows that insects damage 20% of the fruits, sprinkled with chemical pesticide and only 10% of the fruits, with organic pesticide.  Based on the above information answer the following questions: (i) A consumer purchased a fruit from that shop and found to be not affected by the insects. What is the probability that the tree had been sprinkled with organic pesticide? (ii) What is the probability of selecting a fruit with insects inside the seed of it?
8.	A company has two plants to manufacture TVs. The first plant manufactures 70% of the TVs and the rest are manufactured by the second plant. 80% of the TVs manufactured by the first plant are rated of standard quality, while that of second plant

only 60% are of standard quality. One TV is selected at random.

Based on the above information answer the following;

(i) Find the probability that the selected TV is of standard quality.
(ii) Find the probability that the TV is of standard quality, given that it was made by 1st plant.

9. In a survey at Vande Bharat Train, IRCTC asked passengers to rate and review the food served in train. IRCTC asked 500 passengers selected at random to rate food according to price (low, medium ,or high) and food (1,2,3,or 4 stars). The results of this survey are presented in the two-way, or contingency, table below. The numbers in this table represent frequencies. For example, in the third row and fourth column, 30people rated the prices high and the food 4 stars.

Price/ rating	*	**	***	****	Total
Low	20	30	90	10	150
Medium	50	80	90	30	250
High	20	10	30	40	100
Total	90	120	210	80	500

(1) Find the probability that the passenger rates the prices medium?

	(2) Find the probability that the passenger rates the food 2 stars. (3) Suppose the passenger selected rates the price high. What is the probability that he rates the restaurant 1 star? (4) Suppose the passenger selected does not rate the food 4 stars. . What is the probability that she rates the prices high?
10.	Two friends A and B had gone for a shopping, and they came across a beautiful antique piece and both want to buy it. They asked shop keeper for another piece but not available in shop. Both of them decided to go to coffee shop to have coffee and toss a PAIR OF COINS, whosoever gets the pair of heads first will buy the antique piece, both shook hands and sat down for their luck , Answer the following if A starts (1) What is the probability of getting pair of heads? (2) What is the probability of getting only one head in a throw? (3) What is the probability that A gets pair of heads in third throw and wins the game? (4)What is the probability that B wins the game if A starts? (5) What is the probability that A buys the antique piece if A starts ? 
11.	During Examination, we need to reschedule on study hours and along with the study hours, we need quality revision of syllabus. In one such situation ,if X is a random variable which represents number of hours a student of class XII studied a particular subject per day, the probability distribution is given as,

X	0	1	2	3	>3
P(X)	0	K	3K	4K	0

	 (1) What is the value of K? (2) The probability that less than two hours time is given to a subject per day is: (3) What is the probability that two hours or three hours of time is given to a subject per day? (4) What is the probability that 3 hours or more than three hours of time is given to a subject per day?
12.	A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.  (1) When the doctor arrives late, what is the probability that he comes by metro? (2)When the doctor arrives late, what is the probability that he comes by cab? (3) When the doctor arrives late, what is the probability that he comes by bike?
13.	Suman was doing a project on a school survey, on the average number of hours spent on study by students selected at random. At the end of survey, Suman prepared the following report related to the data. Let X denotes the average number of hours spent on study by the students.

X	0	1	2	3	4	>4
P(X)	0.2	K x	K x	K (6-x)	k(6-x)	0

(1) Find the value of k?

(2)What is the probability that the average study time is not more than 1 hour?

(3)What is the probability that the average study time is atleast 3 hours?

(4)What is the probability that the average study time is exactly 2 hours?

14. Three persons A, B and C apply for manager post in a company ,the chances of selection is given by the ratio 1 : 2 : 4.The probability that if selected A , B and C can bring changes to improve profitability are 0.8, 0.5 and 0.3 respectively.

(1)If the changes does not take place Find The probability that C is selected as manager?

(2) What is the conditional probability that if change has taken place due to B?

ANSWERS

1. (i)

X	0	1	2	3	4
P(X)	$\frac{1}{5}\times\frac{1}{5}$	$2\left(\frac{3}{10}\times\frac{1}{5}\right)$	$2\left(\frac{1}{2}\times\frac{1}{5}\right)+\left(\frac{3}{10}\times\frac{3}{10}\right)$	$2\left(\frac{3}{10}\times\frac{1}{2}\right)$	$\left(\frac{1}{2}\times\frac{1}{2}\right)$

(ii) $P(India\ wins\ the\ series) = P(X=3)+P(X=4)$

$$= 2\left(\frac{3}{10}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)$$

$$= \frac{11}{20}$$

	(iii) $P(India\ draws\ the\ series) = P(X = 2)$ $= 2\left(\frac{1}{2} \times \frac{1}{5}\right) + \left(\frac{3}{10} \times \frac{3}{10}\right)$ $= \frac{29}{100}$
2.	Answer : (i) P(A) = $\frac{4}{5}$, P(B) = $\frac{3}{4}$, P(C) = $\frac{2}{3}$ P$(A \cap B \cap C) = P(A)P(B)P(C) = \frac{2}{5}$ (ii) P$(A \cap B \cap \bar{C})$ + P$(A \cap \bar{B} \cap C)$ $+ P(\bar{A} \cap B \cap C)$ P(A) P(B) P($\bar{C}$) + P(A)P($\bar{B}$)P(C) + P($\bar{A}$) P(B)P(C) = $\frac{13}{30}$
3.	Answer: P(Win) = $\frac{1}{6}$, P(lose) = $\frac{5}{6}$ (i) P(A wins) = $\frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \cdots\cdots\cdots$ $= \frac{1}{6} \times \frac{1}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11}$ (ii). P(B wins) = 1 - $\frac{6}{11} = \frac{5}{11}$.
4.	Let B, R,Y and G denote the events that soft toy drawn is blue, red, yellow and green respectively.(without replacement) B | R | Y | G | TOTAL 12 | 8 | 10 | 5 | 35 (i) $P(B \cap G) = P(B)P\left(\frac{G}{B}\right) = \frac{12}{35} . \frac{5}{34} = \frac{6}{119}$ (ii) $P(G \cap \bar{Y}) = P(G).P\left(\frac{\bar{Y}}{G}\right) = \frac{5}{35} . \frac{24}{34} = \frac{12}{119}$
5.	$(i) E_1 \rightarrow desired\ mail;\ E_2 \rightarrow spam\ mail$ $A \rightarrow selected\ mail\ contains\ the\ word\ "offer"$ $P(E_2/A) = \frac{P(A/E_2)\ P(E_2)}{P(A/E_1)P(E_1) + P(A/E_2)\ P(E_2)}$ $= \frac{\frac{80}{100} \times \frac{30}{100}}{\frac{80}{100} \times \frac{30}{100} + \frac{10}{100} \times \frac{70}{100}} = \frac{24}{31}$

	(ii) $(A) = P(A/E_1)P(E_1) + P(A/E_2)\,P(E_2)$ $= \frac{80}{100} \times \frac{30}{100} + \frac{10}{100} \times \frac{70}{100}$ $= \frac{31}{100}$
6.	Answer : (i) $\frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$ (ii) $\frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$ (iii) $\frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$ (iv) 1 - P(both will not be selected) = 1- $\frac{24}{35} = \frac{11}{35}$.
7.	(i) $E_1 \rightarrow Organic\ pesticide;\ E_2 \rightarrow Chemical\ pesticide$ $A \rightarrow not\ having\ insects\ in\ the\ seed\ of\ the\ fruit$ $P(E_1/A) = \frac{P(A/E_1)\,P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)\,P(E_2)}$ $= \frac{\frac{90}{100} \times \frac{70}{100}}{\frac{90}{100} \times \frac{70}{100} + \frac{80}{100} \times \frac{30}{100}} = \frac{63}{87}$ (ii) $Let\ B \rightarrow\ having\ insects\ in\ the\ seed\ of\ the\ fruit$ $P(B) = P(B/E_1)P(E_1) + P(B/E_2)\,P(E_2)$ $= \frac{10}{100} \times \frac{70}{100} + \frac{20}{100} \times \frac{30}{100}$ $= \frac{13}{100}$
8.	Answer : E_1 = TV produced by first plant E_2 = TV produced by second plant. A = Manufactured TV of standard quality. $P(E_1) = \frac{7}{10}\ , P(E_2) = \frac{3}{10}\ ,\ P\left(\frac{A}{E_1}\right) = \frac{8}{10}\ , P\left(\frac{A}{E_2}\right) = \frac{6}{10}$ (i) P(A) = $\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{6}{10} = \frac{74}{100} = 0.74$ (ii) $P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P(\frac{A}{E_1})}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P(\frac{A}{E_2})}$ = $\frac{56}{74}$ = 0.756.

9.	Number of passengers =500 (1) P (Rating Medium) = 250/500 =1/2 (2) P(Rating 2 stars) = 12/500 (3) P (Rating one star/selecting the price high) = 20/100 P (price high /does not rating 4 stars) =60/420
10.	(1) S = { HH,HT,TH,TT } P(getting two heads) = ¼ (2) P (getting only one head in a throw) =1/2 (3) P (A getting pair of heads in third throw) = $\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}$ (4) Ans : $\frac{3}{7}$ (5)Ans : $\frac{4}{7}$
11.	(1) We know $\therefore 0 + K + 3K + 4K + 0 = 1$ 8K=1, $= \frac{1}{8}$. (2) P (X < 2) = P (X =0) + P (X =1) =1K $= \frac{1}{8}$ (3) P (X =2) + P (X =3) = 3K + 4K = 7K = $\frac{7}{8}$ (4)P (X=3) + P(X =4) = 4K + 0 = $\frac{4}{8}$
12.	Let E be the event of coming late. A be the event of coming by cab. B be the event of coming by metro. C be the event of coming by bike. D be the event of coming by other transport. P (A) = 3/10 P (B) = 2/10 P (C) = 1/10 P (D) = 4/10 P (E/A) = 25/100 P (E/B) = 3/10 P (E/C) = 35/100 P (E/D) = 1/10 (1) P (E /B) = $\frac{P(B)P(\frac{E}{B})}{P(A)P\left(\frac{E}{A}\right)+P(B)P\left(\frac{E}{B}\right)+P(C)P\left(\frac{E}{C}\right)+P(D)P(\frac{E}{D})}$ $= \left(\frac{6/100}{\left(\frac{15}{200} + \frac{6}{100}+\frac{7}{200} + \frac{4}{100}\right)}\right) = \frac{6 \times 2}{42}$ = 2/7 (2) ANSWER : $\frac{5}{14}$ (3) ANSWER : $\frac{1}{6}$

13.	

P(X)	0.2	k	2k	3k	2k	0

(1) Using $\sum P(x) = 1$, 8k = 1 -0.2

8k = 0.8 , k = 1/10

(2) Probability of study time NOT more than one ; P(X=0) + P(X =1)

2/10 + 1/10 = 3/10

(3) P (X=3) + P(X =4) + P (X >4) = $\frac{5}{10}$

(4) P(X =2) = $\frac{1}{5}$

(5) P(X =1) + P(X=2) + P (X=3) + P(X =4) + P (X >4) ==1 – P(X=0)

= 1- 0.2 =0.8

14\. Let the events be described as below :

A: No change takes place

E1: Person A gets appointed

E2: Person B gets appointed

E3: Person C gets appointed.

The chances of selection of A, B and C are in the ratio 1: 2: 4 .

Hence, P(E1)=1/7 ,P(E2)=2/7 ,P(E3)=4/7

Probabilities of A,B and C introducing changes to improve profits of company are 0.8,0.5 and 0.3 respectively. Hence probability of no changes on appointment of A,B and C are 0.2,0.5 and 0.7 respectively.

Hence ,P(A|E1)=0.2= 2/10

P(A|E2)=0.5= 5/10

P(A|E3)=0.7= 7/10

(1)∴the required probability is

$$P\ (E_3/\ A\ ) = \left(\frac{P(E_3\ )P\left(\frac{A}{E_3}\right)}{\sum_1^3\left(P\ (E_i\ )\times P\left(\frac{A}{E_i}\right)\right)}\right) = \frac{\frac{28}{70}}{\left(\frac{2}{70}+\frac{10}{70}+\frac{28}{70}\right)} \qquad =28/40$$

= 7/10

∴ if no change takes place, the probability that it is due to appointment of C is 7/10

(2) E : Change takes place

$$P\left(\left(\frac{E_2}{E}\right) = \frac{P(E_2\ )P\left(\frac{E}{E_3}\right)}{\sum_1^3\left(P\ (E_i\ )\times P\left(\frac{E}{E_i}\right)\right)} = \left(\frac{\frac{2}{7}\times 5/10}{\left(\frac{8}{70}+\frac{10}{70}+\frac{12}{70}\right)}\right) = 10/30 \quad =1/3\right.$$